A Accounting for Inflation

A.1 The relationship between real and nominal interest rates[1]

If in period j, \$1 is invested in a bond which pays \$1+ρ_j in period $j+1$, then the nominal interest rate is ρ_j. The expected increase in real purchasing power from buying the bond is: $1+i_j = \frac{E(1/P_{j+1})(1+\rho_j)}{1/P_j}$ where P_j is the current level of the price index, $E(P_{j+1})$ is the expectation of the price level in period $j+1$ formed in period j, and i_j is the bond's expected rate of growth in purchasing power (its expected real return). \$1 has purchasing power $1/P_j$ in period j. Let z_j be the expected rate of growth of purchasing power of \$1: $z_j = \frac{E(1/P_{j+1}) - 1/P_j}{1/P_j}$ (note that if the price level was expected to increase, z would be negative).

Substituting this into the first equation gives: $(1+i_j) = (1+z_j)(1+\rho_j)$.

Now $1+z_j = \frac{E(1/P_{j+1})}{1/P_j} \approx \frac{P_j}{E(P_{j+1})} = \frac{1}{(1+\pi_j)}$ where π_j is the expected rate of inflation in period j (the expected percentage increase in the price index over the period). The relationship, therefore, between the real interest rate i and the nominal interest rate is: $(1+i)(1+\pi) = (1+\rho)$. This gives: $i = (\rho - \pi)/(1+\pi)$

Alternatively, taking logs gives $i^c = \rho^c - \pi^c$, where a ^{c} denotes the continuously compounded equivalents. For example, $i^c = ln(1+i)$. If an amount grows at rate i^c, continuously compounded, in one period it grows to $e^{ic} = 1+i$. $i^c < i$.

This formula slightly understates the real interest rate (as in general $E(1/P_{j+1}) > 1/E(P_{j+1})$ so that $(1+z_j)(1+\rho_j) > (1+\rho_j)/(1+\pi_j)$) but the understatement is small compared with the estimation error in inflation forecasts. Further, the formula is exact for realised values of the variables (that is, given actual π and ρ, the formula gives the realised real return, i).

1 Based on Sieper (1981, pp. 49-50).

Public policy analysis usually uses real dollars, with projected costs and benefits measured at today's prices. Real dollars adjust for inflation, they are nominal dollars deflated by a price index to account for changes in the general price level. Real dollars are really bundles of consumption (the basket of goods that goes into the price index). An inflation-adjusted dollar always has the same purchasing power (can buy the same bundle of goods).

The real rate of interest determines the relative value of goods received at different times in the future. The relative price of goods next period to goods now is $(1+i)$. More accurately, $(1+i)$ is the relative price of the basket of goods that goes into the price index used to measure inflation. Giving up one basket of goods today gives you $(1+i)$ baskets next period. Strictly speaking, when the relative prices of goods change, each good has its own real interest rate.

Using real dollars for future variables avoids having to estimate the future course of inflation. (which would be especially difficult if inflation varies).

Analysts should discount real flows (measured in one period's dollars) using a real rate of interest, and discount nominal flows using the nominal rate of interest. Both methods should result in the same net present value.[2]

2 For example, the present value of a perpetual stream of nominal payments (received at the beginning of the period) of amount \$I in the first period, which grows at (the inflation rate) π per year is:

$$I + \$I(1+\pi)/(1+\rho) + \$I(1+\pi)^2/(1+\rho)^2 + \$I(1+\pi)^3/(1+\rho)^3 + \ldots$$

$$= \$I[1 + (1+\pi)/(1+\rho) + (1+\pi)^2/(1+\rho)^2 + (1+\pi)^3/(1+\rho)^3 + \ldots]$$

$$= \$I/[1 - (1+\pi)/(1+\rho)] = \$I(1+\rho)/(\rho - \pi)$$

using the nominal interest rate to discount nominal flows. But we can also write

$$\$I + \$I(1+\pi)/(1+\rho) + \$I(1+\pi)^2/(1+\rho)^2 + \$I(1+\pi)^3/(1+\rho)^3 + \ldots$$

$$\$I + \$I/(1+i) + \$I/(1+i)^2 + \$I/(1+i)^3 + \ldots$$

But this is the an infinite stream of real payments of amount \$I per period, discounted at the real interest rate which has present value:

$$\$I[1 + 1/(1+i) + 1/(1+i)^2 + 1/(1+i)^3 + \ldots] \quad = \$I/[1 - 1/(1+i)] = \$I(1+i)/i$$

$$= \$I(1+i)(1+\pi)/(\rho - \pi) = \$I(1+\rho)/(\rho - \pi)$$

A.2 Converting nominal into real variables

Data on past monetary flows are usually in nominal (or current) dollars. Deflating (or inflating) with a price index (such as the Consumer Price Index or GDP deflator) converts nominal dollars into real, or inflation adjusted, dollars. For example, to convert last year's nominal dollars into this year's real dollars, multiply by the price index for this year and divide by the price index for last year.

A widely used price index is the Consumer Price Index (CPI), which measures the cost of purchasing households' average consumption bundle over time.

More generally, to express an amount $\$A_j$ of period j dollars in terms of period t dollars using the CPI we multiply by the CPI in period t and divide by the CPI in period j. That is, $\$A_j$ in period t dollars is $\$A_j(CPI_t/CPI_j)$.

For example, if period t occurs after period j $(t > j)$ and inflation between period j and t (π_{jt}) is positive, then $CPI_t/CPI_j=1+\pi_{jt} > 1$. We inflate the dollar amount because a period j dollar has a greater purchasing power than a period t dollar. It takes $CPI_t/CPI_j = 1 + \pi_{jt}$ period t dollars to buy the same amount as $1 in period j.

If period j is after period t, then $CPI_t/CPI_j=1/(1+\pi_{tj}) < 1$ if inflation is positive. That is, we deflate period j dollars when we express them in period t dollars, as period j dollars buy less than period t dollars.

If period j is in the future, and period t is now, then we need to multiply period j dollars by $E(CPI_t/CPI_j) = CPI_tE(1/CPI_j) = E(1/(1+\pi_{jt}) \approx 1/[1+E(\pi_{jt})]$ to convert them into period t dollars.

Economists estimate the CPI overstates the actual rate of increase in the cost of living by over 1 percentage point per year.[3] It gives too great a weight to the goods whose prices have risen the most because consumers substitute away from goods whose relative prices have increased towards goods whose relative prices have decreased. New goods are often only included once their price has fallen substantially, and they are widely consumed, so much of their price fall is not reflected in the CPI. Further, the CPI fails to fully allow for improvements in product quality.

The overstatement of the rate of decline in the purchasing power means that estimates of the real interest rate using the CPI to convert nominal into real measures tends to underestimate the real rate of interest.

3 See Boskin et al. (1998) and Gordon (2006)

A.3 The effect of expected inflation on real returns when nominal receipts are taxed[4]

If the variables are expressed as continuously compounded rates then the expected real before-tax interest rate is:

$$i^c = \rho^c - \pi^c$$

To simplify the equations, we will use the continuously compounded rates, but drop the superscripts. The rates are approximately the same as the per period rates.

When nominal returns ρ are taxed at rate τ, the after-tax expected real interest rate is: $r = (1 - \tau)\rho - \pi$

As set out in Sieper (1981), if the underlying after-tax rate of return is given[5], $r = \hat{r}$ and there is a single tax rate τ, and all interest receipts are taxable and all interest payments are tax deductible, then the nominal interest rate is $\rho = \frac{\hat{r} + \pi}{1 - \tau}$ which is the tax-adjusted Fisher effect, where nominal interest rates adjust to expected inflation so as to leave the expected after-tax interest rate constant. A 1 percentage point increase in expected inflation increases the nominal interest rate by $1/(1-\tau)$ percentage points. The interest rate rises by more than the increase in expected inflation.

Then the before-tax expected real rate of return is: $i = \rho - \pi = \frac{\hat{r} + \pi}{1 - \tau} - \pi = \frac{\hat{r} + \tau\pi}{1 - \tau}$

Even if the underlying after-tax real rate of return were constant, the expected before-tax real return is greater, would not be stable. It would increase with expected inflation and the tax rate. Every percentage point increase in expected inflation would increase it by $\tau/(1-\tau)$ of a percentage point. For example, if $\tau = 30$ per cent, $\tau/(1-\tau) = 0.43$. If $\tau = 50$ per cent, $\tau/(1-\tau) = 1$.

As shown in table A.1, the taxation of nominal receipts results in a large gap between the before- and after-tax real interest rate, especially at high tax rates.

For example, some authors suggest applying the empirical evidence to the Ramsey equation gives a real interest rate of 1.5 per cent. But the Ramsey equation produces a consumption rate, an after-tax rate. The before-tax investment rate would be higher.

4 Based on Sieper (1981)

5 For example, it may be tied down by the Ramsey equation, which is derived in appendix F and discussed in section 3.3.

Using the above equation for the before-tax return, for a tax rate of 50 per cent and with expected inflation of 3 per cent, the corresponding before-tax real return would be 6 per cent (see table A.1). The before-tax nominal interest rate would be 9 per cent.

Table A.1 How the before-tax real return varies with taxes, expected inflation and the after-tax return

Assuming all nominal returns are taxed at rate τ

	Tax rate					
	T=30 per cent			*T=50 per cent*		
	Expected rate of inflation					
After-tax return	2	3	4	2	3	4
1	2.3	2.7	3.1	4.0	5.0	6.0
1.5	3.0	3.4	3.9	5.0	6.0	7.0
2	3.7	4.1	4.6	6.0	7.0	8.0
4	6.6	7.0	7,4	10.0	11.0	12.0
6	9.4	9.9	10.3	14.0	15.0	16.0

Source: Author's calculations.

If v is the realised rate of inflation, then the realised real interest rate is: $\rho - v = \frac{\hat{r}+\pi}{1-\tau} - v$. If inflationary expectations are unbiased, then the average realised rate of inflation over a large number of observations would equal expected inflation and so the average observed before-tax real interest rate is:

$$\tilde{\rho} - \tilde{v} = \frac{\hat{r}+\tilde{v}}{1-\tau} - \tilde{v} = \frac{\hat{r}+\tau\tilde{v}}{1-\tau}$$ where ˜ represents an average over many observations.

Average realised real returns would depend on the historical pattern of inflationary expectations and the structure of taxes.

We can express the after-tax return as:

$$r = (1-\tau)(i+\pi) - \pi = (1-\tau)i - \tau\pi = (1-\tau)(\rho-\pi) - \tau\pi$$

That is, taxation (or deduction) of nominal returns (payments) is equivalent to a tax on the real return plus an additional term that arises because tax is paid on that part of the nominal return that compensates for expected inflation, equivalent to a capital levy per dollar lent (or subsidy per dollar of deductible borrowing).

In the Australian tax system, people face different marginal tax rates. For example, the marginal income tax rate is increased at higher levels of income, corporate rates may differ from personal rates and effective tax rates differ because people are subject to different benefit withdrawal rates. For a given before-tax interest rate, those with higher marginal tax rates receive a lower after-tax return. Further, some borrowers and savers face a zero marginal rate. Some savers are tax exempt. Although business enterprises and people borrowing for investment purposes can deduct interest payments, borrowing to finance personal consumption, the purchase of consumer durables or owner-occupied housing are not tax deductible and those borrowers pay the before-tax rate.

When tax rates differ across individuals, not only do their after-tax returns differ, but when expected inflation changes there is no adjustment in the before-tax nominal interest rate that would preserve the expected after-tax real interest rate of all lenders and borrowers. The implicit capital levy on savers (subsidy to borrowers) varies with their tax rate.

The effect of expected inflation on the nominal interest rate would depend on the proportions of savers and borrowers in each tax bracket and the responsiveness of their saving and borrowing to changes in the interest rate.

Because a large portion of interest receipts are taxed (and some payments deductible), before-tax nominal interest rates would tend to rise by more than the expected inflation rate, but by less than the tax-adjusted Fisher effect for the highest marginal rate. That is, if the before-tax real return in the absence of inflation $\hat{i}$ was fixed, we would expect: $\rho = \hat{i} + \alpha\pi$ where $1 \leq \alpha \leq 1/(1-\tau^M)$ where τ^M is the maximum tax rate. This gives $i = \rho - \pi = \hat{i} + (\alpha - 1)\pi$, so the expected before-tax real return would tend to increase with expected inflation.

The expected after-tax real return for a person with tax rate τ_j is $r_j = \rho(1-\tau_j) - \pi = (1-\tau_j)\hat{i} + \pi[\alpha(1-\tau_j) - 1]$. It would stay the same when expected inflation increased for a person with tax rate $\tau_j = (\alpha - 1)/\alpha$. For people with tax rates smaller than this, the after-tax real return would rise with expected inflation. For people with tax rates higher, it would fall. Even if interest rates in the absence of inflation, $r_j = (1-\tau_j)\hat{i}$, were stable, expected inflation would, in general, change the after-tax return and may increase it for some groups of taxpayers and decrease it for others, depending on their marginal tax rate.

Even if the assumptions that underlie the Ramsey equation hold, the tax system ensures the after-tax real rate of return will vary across individuals and will vary with expected inflation.

It would be good to have an estimate of α, the inflation adjustment factor for nominal interest rates. If we assume that inflationary expectations are unbiased, realised inflation v will be expected inflation plus an unbiased error term: $v = \pi + e$.

If the before-tax real return in the absence of inflation was a constant $\hat{i}$, then from our expression for the nominal interest rate $\pi = \frac{\rho - \hat{i}}{\alpha}$. Substituting this into our expression for realised inflation gives: $v = \frac{\rho}{\alpha} - \frac{\hat{i}}{\alpha} + e$. If we run a regression:

$v = \beta + \gamma\rho + e$ then $\alpha = 1/\gamma$ and $\hat{i} = -\beta/\gamma$

Although some empirical work has been done on the relationship between expected inflation and nominal interest rates in Australia, the researchers usually test whether the classic Fisher effect ($\alpha = 1$) holds.[6] For example, Inder and Silvapulle (1993) test whether realised real interest rate are constant over time, and find they are not. Hawtrey (1997) does the same and finds real interest rates were not constant before deregulation but that since deregulation, after-tax nominal rates (using the company tax rate) rise one for one with expected inflation. Olekahns (1996) has a more sophisticated expected inflation estimate than the one suggested here, and finds that a 1 percentage point increase in expected inflation increases the nominal interest rate by less than 1 per cent (from September 1969 to September 1993) (i.e. $\alpha < 1$). For the period after deregulation, he could not reject that $\alpha = 1$. If Australia were perfectly integrated with the international capital market, the classic Fisher effect would hold for debt instruments (bonds), and perhaps that is reflected in the findings after de-regulation.[7] Nevertheless, even if Australia could borrow as much as it liked at a given world interest rate, changes in world inflation levels could still affect real interest rates.

None of these studies run the regression here. In order to get a rough idea of the size of these parameters we ran this simple regression using quarterly data, from September 1970 to June 2008 and from March 1980 to the June 2008. Realised returns were negative throughout the 1970s, implying that inflation was consistently under-estimated.

Over the full period, we estimated that $\alpha = 1.65$ and $\hat{i} = -0.9$ per cent. For the second period $\alpha = 1.64$ and $\hat{i} = 1.7$ per cent. Both regressions imply the tax rate at which the after-tax return is invariant to changes in the rate of expected inflation is 39 per cent. If $\hat{i} = 1.7$ per cent, the after-tax real return for someone on that tax rate would be 1.0 per cent.

6 See Atkins (1989); Inder and Silvapulle (1993); Olekahns (1996) and Hawtrey (1997). These were all the papers about Australia referred to in the survey Coorey (undated).

7 See, appendix E, box E.1.

For those on lower tax rates the after-tax return would be higher and would increase with expected inflation. Those on higher tax rates receive a lower after-tax return, and it decreases with expected inflation.

All these results apply when inflation-indexed bonds are available, because the inflation adjustment payments are taxed annually as they accrue.[8] The after-tax realised real return on indexed bonds with a return i is $(1-\tau)(i+v)-v=(1-\tau)\hat{i}-\tau v$ which varies with the individual's tax rate and with realised inflation.

8 See Australian Office of Financial Management (2002, p. 6).