B Specification of model

Corrado, Hulten and Sichel (CHS 2006, pp. 4–9) outline the implications of capitalising intangibles expenditures compared with their current treatment as intermediate goods. This appendix outlines the production functions and accounting identities associated with the two approaches as specified by CHS (2006).

B.1 Intangibles treated as intermediate inputs

Suppose there are three goods produced, a consumption good C, a tangible investment good I, and an intangible good N. When the intangible is considered to be an intermediate good, it is an input to the other two goods (C and I), and labour L and tangible capital K are inputs to all three goods.

The production function and flow account for each of the three sectors is then

Intangible sector $N_t = F_N(L_{N_t}, K_{N_t}, t)$ $P_{N_t}N_t \equiv P_{L_t}L_{N_t} + P_{K_t}K_{N_t}$ (B.1)

Tangible sector $I_t = F_I(L_{I_t}, K_{I_t}, N_{I_t}, t)$ $P_{I_t}I_t \equiv P_{L_t}L_{I_t} + P_{K_t}K_{I_t} + P_{N_t}N_{I_t}$ (B.2)

Consumption sector $C_t = F_C(L_{C_t}, K_{C_t}, N_{C_t}, t)$ $P_{C_t}C_t \equiv P_{L_t}L_{C_t} + P_{K_t}K_{C_t} + P_{N_t}N_{C_t}$ (B.3)

where $L \equiv L_N + L_I + L_C$, $K \equiv K_N + K_I + K_C$, $N \equiv N_I + N_C$, and tangible capital accumulates according to $K_t \equiv I_t + (1\text{-}\delta_k)K_{t\text{-}1}$ with depreciation rate δ_k. The production functions are linked to the accounting identities by the assumption that each input is paid the value of its marginal product. In this case, N_t is both an output and an intermediate input to the production of the other products. N_t therefore nets out of the aggregate and does not appear separately in the GDP identity[1]

$$P_{Q'_t}Q'_t \equiv P_{C_t}C_t + P_{I_t}I_t \equiv P_{L_t}L_t + P_{K_t}K_t \quad \text{(B.4)}$$

[1] CHS (2006) adopts the convention that intermediates used by the industry that produced them are netted out of final output. They also omit chain weighting from the equations for simplicity of exposition.

The conventional sources of growth (SOG) framework allocates the output growth to the share-weighted input growth and a residual, multifactor productivity (MFP) growth.[2] The SOG equation is derived by logarithmic differentiation of (B.4):

$$\dot{Q}'_t = s'_{C_t}\dot{C}_t + s'_{I_t}\dot{I}_t \qquad (B.5)$$

$$= s'_{L_t}\dot{L}_t + s'_{K_t}\dot{K}_t + \dot{MFP}'_t$$

where $\dot{x}_t$ denotes the rate of growth of variable x_t.

Rearranging (B.5) MFP growth[3] is therefore

$$\dot{MFP}'_t = \dot{Q}'_t - s'_{L_t}\dot{L}_t - s'_{K_t}\dot{K}'_t$$

The expenditure shares $s'_{C_t} \equiv [P_{C_t}C_t]/[P_{C_t}C_t + P_{I_t}I_t]$ and $s'_{I_t} \equiv [P_{I_t}I_t]/[P_{C_t}C_t + P_{I_t}I_t]$

and income shares $s'_{L_t} \equiv [P_{L_t}L_t]/[P_{L_t}L_t + P_{K_t}K_t]$ and $s'_{K_t} \equiv [P_{K_t}K_t]/[P_{L_t}L_t + P_{K_t}K_t]$

are assumed to be equal to the corresponding output elasticities. Intangible input and output do not appear in this SOG equation.

B.2 Intangibles treated as capital

If the intangible is treated as capital, a different model applies. The output of the intangible, N_t, enters in the production functions of the consumption and tangible investment sectors as a cumulative stock rather than as an intermediate input. The intangible capital stock accumulates according to the perpetual inventory method $R_t \equiv N_t + (1-\delta_R)R_{t-1}$, in the same way as tangible capital. The equations for each of the sectors become

Intangible sector $N_t = F_N(L_{N_t}, K_{N_t}, R_{N_t}, t)$ $\quad P_{N_t}N_t \equiv P_{L_t}L_{N_t} + P_{K_t}K_{N_t} + P_{R_t}R_{N_t}$ (B.6)

Tangible sector $I_t = F_I(L_{I_t}, K_{I_t}, R_{I_t}, t)$ $\quad P_{I_t}I_t \equiv P_{L_t}L_{I_t} + P_{K_t}K_{I_t} + P_{R_t}R_{I_t}$ (B.7)

Consumption sector $C_t = F_C(L_{C_t}, K_{C_t}, R_{C_t}, t)$ $\quad P_{C_t}C_t \equiv P_{L_t}L_{C_t} + P_{K_t}K_{C_t} + P_{R_t}R_{C_t}$ (B.8)

2 This follows Solow (1957).

3 As is common practice, the continuous time variables are approximated with their discrete time counterparts. And a combined input index is computed as a Tornqvist index (a discrete approximation of a continuous Divisia index).

The balance equations are modified with $R \equiv R_I + R_C + R_N$ replacing $N \equiv N_I + N_C$.[4] The production functions are linked to the accounting identities by the assumption of marginal productivity pricing, as in the above case. The GDP identity is expanded to include the flow of new intangibles on the expenditure side and the flow of services from the intangible stock on the income side:

$$P_{Q_t} Q_t \equiv P_{C_t} C_t + P_{I_t} I_t + P_{N_t} N_t \equiv P_{L_t} L_t + P_{K_t} K_t + P_{R_t} R_t \quad \text{(B.9)}$$

The price P_{R_t} is the rental price associated with the services of the intangible stock and is a source of income that is not included in the conventional intermediate goods case (B.4).[5]

When intangible capital is treated in the same way as tangibles, the SOG equation becomes

$$\dot{Q}_t = s_{C_t} \dot{C}_t + s_{I_t} \dot{I}_t + s_{N_t} \dot{N}_t \quad \text{(B.10)}$$

$$= s_{L_t} \dot{L}_t + s_{K_t} \dot{K}_t + s_{R_t} \dot{R}_t + \dot{MFP}_t$$

4 In this second case, CHS (2006) expand the technology of the intangible producing sector to use its own stock of accumulated intangibles.

5 The rental price of tangible capital (P_{K_t}) will also change when intangibles are capitalised because of the change in the equalising rate of return (see appendix C). However, the same notation 'P_{K_t}' is maintained in both cases for simplicity.

In practice, total capital income is derived as the difference between total income and labour income, which is

$P_{K_t} K_t \equiv P_{Q'_t} Q'_t - P_{L_t} L_t$ when intangibles are expensed

$P_{K_t} K_t + P_{R_t} R_t \equiv P_{Q_t} Q_t - P_{L_t} L_t$ when intangibles are capitalised.

Since capitalising intangibles increases total income ($P_{Q_t} Q_t > P_{Q'_t} Q'_t$) and labour income is unchanged, total capital income increases. However, after the capitalisation of intangibles the rental price of *tangible* capital changes (as a result of the changed capital income, and the inclusion of intangible assets in the derivation of a new equalising rate of return). (P_{K_t} is derived from total capital income rather than measured directly, so when intangibles are expensed some capital income that is attributed to tangible capital is actually a return on intangibles that have not been counted as part of the capital stock.) Total capital income is split between tangibles and intangibles as follows

$[P_{K_t} K_t]/[P_{K_t} K_t + P_{R_t} R_t]$ for tangibles

$[P_{R_t} R_t]/[P_{K_t} K_t + P_{R_t} R_t]$ for intangibles.

where the expenditure shares are now

$$s_{C_t} \equiv [P_{C_t} C_t]/[P_{C_t} C_t + P_{I_t} I_t + P_{N_t} N_t], \qquad s_{I_t} \equiv [P_{I_t} I_t]/[P_{C_t} C_t + P_{I_t} I_t + P_{N_t} N_t] \qquad \text{and}$$
$$s_{N_t} \equiv [P_{N_t} N_t]/[P_{C_t} C_t + P_{I_t} I_t + P_{N_t} N_t]$$

and the income shares are now

$$s_{L_t} \equiv [P_{L_t} L_t]/[P_{L_t} L_t + P_{K_t} K_t + P_{R_t} R_t], \qquad s_{K_t} \equiv [P_{K_t} K_t]/[P_{L_t} L_t + P_{K_t} K_t + P_{R_t} R_t] \qquad \text{and}$$
$$s_{R_t} \equiv [P_{R_t} R_t]/[P_{L_t} L_t + P_{K_t} K_t + P_{R_t} R_t].$$

Rearranging (B.10) MFP growth[6] is therefore

$$\dot{MFP}_t = \dot{Q}_t - s_{L_t} \dot{L}_t - s_{K_t} \dot{K}_t - s_{R_t} \dot{R}_t$$

Comparing (B.5) and (B.10), not only the growth terms $\dot{N}_t$ and $\dot{R}_t$ change, but also all the income and expenditure shares.

6 This can also be expressed in labour productivity (LP) terms by rearranging (B.10)

$$\dot{Q} = s_L \dot{L} + (1 - s_L)(\dot{TK}) + \dot{MFP} \qquad \text{where } TK \equiv K + R \text{ and } s_L + s_K + s_R = 1$$
$$(\dot{Q} - \dot{L}) = s_L \dot{L} + (1 - s_L)(\dot{TK}) + \dot{MFP} - \dot{L}$$
$$= (1 - s_L)(\dot{TK}) - (1 - s_L)\dot{L} + \dot{MFP}$$
$$(\dot{Q} / L) = (1 - s_L)(\dot{TK} / L) + \dot{MFP}$$
$$\dot{LP} = \dot{KD} + \dot{MFP}$$

The labour composition effect can be separately identified (not shown above for simplicity). This results in the expression $\dot{LP} = (\dot{Q} / H) = \dot{KD} + \dot{MFP}^* + s_L \dot{L}_{SC}$, where labour (L) is separated into two components, L_{sc} the skill composition component and H the labour quantity (hours), and MFP^* is MFP adjusted for skill composition change.