I Discounting the distant future

I.1 Uncertainty about the appropriate rate — averaging discount factors

When there is uncertainty about what discount rate to use, Weitzman argues that the appropriate discount rate to calculate present value is lower the further in the future the payments are received.[1]

Assume that the relevant discount rate is constant for the whole period, but we simply are not sure what it is. It is uncertainty about the rate to use, not an uncertain interest rate that fluctuates over time. To put risk aside, assume risk neutrality. Weitzman shows that as the time horizon of the project increases, the appropriate discount rate falls, because it is discount factors, not discount rates, that should be averaged.

Although it may seem strange that the discount rate is persistent, yet unknown, there is genuine uncertainty about the correct discount rate, even when interest rates are known.

For example, say we are uncertain whether the discount rate should be 3 or 7 per cent per year and each is equally likely. The expected present value of a dollar received in one year's time is:

$$0.5*\$1/1.03 + 0.5*\$1/1.07 = 0.5*(\$0.97 + \$0.93) = \$0.953 = 1/1.0496,$$

which implies a discount rate of 4.96 per cent, close to the average of 3 and 7 per cent.

The expected present value of a dollar received in 100 years time is:

$$0.5*\$1/(1.03)^{100} + 0.5*\$1/(1.07)^{100} = 0.5*(\$1/19.2 + 1/867.7) = \$1/37.6 = 1/(1.037)^{100}$$

The implied discount rate is 3.7 per cent, much closer to 3 than to 7 per cent.

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1 So called gamma discounting. See Weitzman (2001).Many other authors present this argument, including Pindyck (2006, p. 5); Pearce et al. (2003, pp. 127-130) and Cowen (2008, pp. 12-13).

The expected present value of a dollar received in 200 years time is:

$$0.5*\$1/(1.03)^{200} + 0.5*\$1/(1.07)^{200} = 0.5*(\$1/369.36 + 1/752{,}931.6) = \$1/738.35 = =1/(1.034)^{100}$$

The implied discount rate is 3.4 per cent, even closer to 3.

That is, if we are uncertain about which discount rate to use, the appropriate 'certainty equivalent' discount rate declines over time. The further in the future the payment is received, the lower the discount rate we should use to calculate its present value. For payments in the distant future, we should be using discount rates from the lower end of the spectrum of possible values. The lower rates have a greater relative weight the further we look into the future.

For example, often a proposal's risk characteristics are not clear, nor is the rate of return for that risk class or the weights to put on different sources. Averaging of discount factors favours using the lower possible rates, such as a lower risk premium.

When the discount rate declines the further into the future a payment is received, it is called hyperbolic discounting. There is some empirical evidence that people discount in a hyperbolic fashion – but this evidence also shows very high short–term discount rates (17–30 per cent).[2]

But Gollier (2003) turns the Weitzman logic on its head. Consider a riskless public investment project of one unit, which generates a single payoff of $(1.04)^n$ in *n* years time. Column 2 in table I.1 lists the payoffs for payments received in 1100 and 200 years time.

The public project comes at the expense of private investment. We do not know what the return to private capital will be. Suppose there is a 50 per cent chance it will be 7 per cent and 50 per cent chance it will be 3 per cent. The expected payoffs that a unit of private investment would produce is set out in column 3 of table I.1.

According to the Weitzman logic, the 1 year public project should be rejected. Its return of 4 per cent is less than the certainty equivalent discount rate (4.96 per cent), so it has a negative net present value. But we should accept the 100 and 200 year projects — as the rate of return is 4 per cent, above the appropriate discount rates (which are near 3 per cent, at 3.7 and 3.4 per cent).

2 For example, Warner and Pleeter (2001) find the vast majority in their sample had discount rates of at least 18 percent. Frederick et al. (2002, table 1, p. 379) survey estimated rates and find they vary from negative to several thousand percent per year. Viscusi (2007, p. 228) finds people discount their own lives with real interest rates in the range 11 to 17 per cent.

Table I.1 **Payoffs from a one unit investment**

Time horizon	*Public project with a 4 per cent return*	*Expected payoff with equi-probable 3 or 7 per cent return*
1	1.04	0.5*1.03+ 0.5*1.07 = 1.05
100	1.04^{100} = 50.5	$0.5*1.03^{100}+0.5*1.07^{100}$= 9.6+433.9= 443.5 = 1.063^{100}
200	1.04^{100}= 2 550.75	$0.5*1.03^{200}+0.5*1.07^{200}$= 184.7+376 465.8 = 376 650.5 = 1.066^{200}

But with risk neutrality, if we maximise future value it is clearly better to invest in the market rather than the project. For the 100 year public project, the future value of the private investment is almost 9 times greater than the project and the expected return is 6.3 per cent (closer to 7 than 3 per cent, and well above the project's 4 per cent return). For the 200 year project the expected future value of the private investment is almost 150 times greater and the expected return 6.6 per cent. The certainty equivalent discount rate rises if we evaluate the project further in the future.

Gollier argues that different investment projects should be ranked according to their expected net future value and we should take a larger interest rate to discount long–term cash–flows with respect to short–term ones. As the time horizon of the project increases, we should be using interest rates from the upper end of the spectrum of possible values — the opposite of the Weitzman result.

Both results are correct about the effects of uncertainty about the discount rate. The further into the future a payment is received, the lower the discount rate used to calculate expected net present value (Weitzman). The further into the future we evaluate a project, the higher the discount rate used to calculate net future value (Gollier 2003). The expected net present value and expected net future value criteria can recommend different courses of action.

The paradox disappears if the investment problem is carefully specified. For example, Gollier and Weitzman (2009) show that in a more rigorous formulation of the problem, the two approaches give the same discount rate and the puzzle is resolved. They examine a model where a project decision must be made, then the interest rate is revealed and the consumer determines his optimal consumption path. Whether the cash flows are converted into present consumption or future consumption makes no difference to the discount rate so long as the values are adjusted by marginal utility at the relevant time period, which adjusts for the risk associated with financing the project.

In this case the risk-adjusted discount rate has properties that resemble the Weitzman recommendations. The rate is lower the further in the future the payment is received, and approaches the bottom of the interest rate distribution. In the case of log utility, the original Weitzman rule is correct.

I.2 The benefits of delay

Hyperbolic discounting is extremely favourable for projects, like global warming abatement, with immediate costs but benefits that are received in the distant future. The immediate costs would be discounted at a high discount rate. The future benefits are discounted at a low rate, raising the net present value of the project compared with a constant rate.

But we need to account for the benefits of delay. Even if a project has a positive net present value, delaying it for a year may increase the net present value.[3] If the project is undertaken immediately (in period *0*), the cost C_0 is borne in period *0* and a benefit B_0 is received in period *n*, far in the future. C_0 could be the present value in period *0* of a stream of costs and B_0 could be the present value in period *n* of a stream of benefits. If we delay the project one period (undertake it in period 1), then C_1 is the cost borne in period 1 and B_1 is the benefit received in period *n*. An alternative assumption is that the benefits will also be delayed by one period, but with hyperbolic discounting, that would make little difference to the period *0* present value of the benefits (the marginal discount rate from period *n* to period *n+1* is almost 0). If we use i_s to discount short-term costs and i_L to discount long term benefits, then it is efficient to delay the project for a year if the present value of starting the project now is less than the present value of waiting a year:

$$-C_1/(1+i_s)+B_1/(1+i_L)^n > -C_0+B_0/(1+i_L)^n \text{ or}$$

$$-C_1/(1+i_s) < C_0+(B_1-B_0)/(1+i_L)^n$$

But when the benefits are received far in the future (*n* is large), then the last term is negligible and the project should be delayed if $C_1/(1+i_s) < C_0$. That is, so long as costs grow at a slower rate than the short term discount rate. In the context of global warming abatement, say the policy is to reduce atmospheric concentration of CO_2 to some target level by 2100. The present value of the benefits from that are likely to be fairly constant. If the present value of the costs if we start today are more than the present value of the costs if we delay starting the project for one period, then delaying the delaying the project increases its value. The relevant discount rate is i_s, the short term discount rate.

3 See Layard (1994, pp. 43-44).

Although hyperbolic discounting increases the net present value of long term projects, it may also make delaying them attractive. Delay may have a further benefit if more information becomes available and some uncertainty about costs and benefits is resolved.

To find the best starting date for a project, its net present value for different starting dates should be compared. At minimum, the option of delaying for a year should always be considered.

I.3 Uncertainty about the path of rates

A different source of uncertainty is the lack of knowledge of the future paths of interest rates. There is a risk that interest rates may change.

Gollier models the interest rate process to determine the term structure of interest rates.[4] The result is time consistent. He shows the answer mainly depends upon the time horizon under scrutiny, the degree of relative prudence and the degree of resistance to intertemporal substitution. For commonly accepted levels of these indexes, the effect of uncertainty on the socially optimal discount rate may vary from 2 to 8 per cent. He finds that the discount rate to be used for long–lasting investments should be a decreasing function of their duration, because of the negative effect of accumulating the per period growth risk in the long run.

That is, uncertainty about future interest rates means we should use a lower than current real interest rate to discount costs and benefits received further in the future (the yield curve for real interest rates slopes down).[5]

Newell and Pizer (2003) model possible future time paths of interest rates, past on historical interest rate behaviour. Future rates can vary widely. They construct expected discount factors to calculate present value, which gives a lower discount rate for benefits received further in the future — again because the lower rates have a relative weight in the discount factor the further we look into the future.

4 See Gollier (2002; 2002a; 2004).

5 Note the usual yield curve slopes up – but that is for the yield on nominal bonds. The longer the term of a nominal bond, the greater is inflation risk and the higher the yield needed to compensate.

I.4 Real options

Long term projects should be treated as a problem of sequential decision making under uncertainty rather than a 'one-shot' benefit cost analysis.[6]

A project may have a positive net present value but still not be accepted right away. The firm may gain by waiting and accepting the project in a future period, for the same reasons that investors do not always exercise an option just because it is in the money. This is more likely to happen if the firm has the rights to the project for a long time, and the variance in project inflows is high.

When a project has a positive net present value, it is necessary, but not sufficient condition for an efficiency improvement. For example, say a project costs $20 and gives $1000 benefit in 101 years. Today the interest rate is 5 per cent, but next year it will change. There is a 50 per cent chance it will rise to 7 per cent and a 50 per cent chance it will fall to 3 per cent. It will then remain at this new level. The expected present value of the payment next year (as worked out in 'Uncertainty about the appropriate rate — averaging discount factors') is:

$$\$1000/(37.6) = \$1000/(1.037)^{100} = \$26.60.$$

The project has a positive net present value. If our choice was, invest today or never invest, we would invest today. Further, the uncertainty about the future rate (holding the expected value of the interest rate constant) increases the expected value of the project. If the interest rate stayed at 5 per cent, the project would be worth $\$1000/1.05^{100}= \$1000/131.5 = \$7.60$ next period and it would not be worth undertaking.

But what if we wait one year before deciding whether to invest. If the interest rate rises to 7 per cent, then the value of the benefit is: $\$1000/1.07^{100} = \$1000/867.7 = \$1.15$, less than the cost of the investment.

If it fell to 3 per cent, the value of the benefit next year is: $\$1000/1.03^{100} = \$1000/19.2 = \$52.03$.

We would only invest if the interest rate fell. Further, the value of the project is double compared with not waiting. The interest rate uncertainty creates a value to waiting for new information and an incentive to wait rather than invest now. If interest rates are uncertain, it may be worth waiting to see whether they rise or fall.[7]

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6 Weyant (2008, p. 78) .See also Lind (1995).

7 In the Weitzman example, the uncertainty about the interest rate never resolves itself. We go on being uncertain about the appropriate interest rate for the whole period.

With any irreversible investment under uncertainty, the timing of the investment is crucial. The decision is not only whether to invest, but when to invest, there can be benefits from delay. Even if the net present value of the project is positive, it still may be worth delaying it. When there is uncertainty over net benefits, flexibility is more valuable.

For most government projects, there is an option to delay and they are difficult to reverse. When the project involves making public policy or providing a public good, the government is usually a monopoly, and can choose when to start, unlike a competitive firm, which has to worry whether a competitor will jump in and make the investment.

The ability to delay an irreversible investment profoundly affects the decision to invest. This has been called real option theory: a firm with an opportunity to invest is holding an option – it has the right, but not the obligation, to buy an asset at some future time of its choosing. When it invests, it exercises its option. It gives up the possibility of waiting for new information to arrive that may change the desirability or timing of the expenditure. This lost option value is an opportunity cost that must be included as part of the cost of investment.[8]

As a result a positive net present value is a necessary, but not sufficient condition for a project to be efficient. The net present value must exceed the value of keeping the option alive. This option value can be large, explaining why firms often set hurdle rates three or four times the cost of capital. The 'trigger' value is not when the expected benefit becomes positive, but when it is sufficiently high. The trigger is higher when there is greater uncertainty, especially if future information would help resolve it. Ironically, a lower interest rate increases the value of waiting and the investment hurdle.

The option value of delay increases the appropriate discount rate to apply to government projects. Even a high discount rate is conservative when the project is an irreversible investment with uncertainty. Even if the net value of the project is positive, it still may be worth paying to delay it.[9] The option value of waiting is valuable when there is uncertainty and you may acquire some information. There may also be a benefit from delay simply by waiting to bear costs.[10]

8 Dixit and Pindyck (1995).

9 See Dixit and Pindyck (1994, chapter 1) and Dixit (1992).

10 See appendix I.2.

The value of delaying the project increases with:

- The sunk cost of the investment
- The degree of uncertainty over benefits.

The higher the degree of uncertainty, the greater the benefit from waiting. Uncertainty about net benefits increases the hurdle rate for the project. It is not enough that the net present value is positive, it must be larger than the option value of delay.

I.5 The risk premium for the distant future

There are reasons for using a lower risk premium for assets that pay off in the distant future. For example, Myers and Turnbull (1977) find that beta in the CAPM falls with asset life. Projects that pay off in the distant future have a lower beta, and so a lower risk premium.

The cash flow is decomposed into components, each with its own risk. They assume that the CAPM model holds each period and each component's beta is independent of time. Each component is like a distinct asset. The composite asset's beta is a weighted average of each component's own beta. The weights are the proportional contribution of the component to the composite asset's price. For example, a net benefit flow could be decomposed into the cost flow and benefit flow.[11]

Myers and Turnbull show that when the component's have different betas. The beta for the composite asset must be a declining function of asset life because asset life affects each component's weight. The expected annual cash flow generated by each component is held constant. But as the horizon is expanded, the present value of each stream increases, but not at the same rate. The cash flows of the stream having the higher beta are discounted at a higher rate and so the weight put on the high beta stream declines as asset life increases. The longer the horizon, the greater the proportion of the asset's price generated by the safer stream and the lower the asset's beta and so the lower the relevant discount rate for evaluating the project.

[11] See Jones (2008, pp. 151-53) for details.