3 Econometric model and estimation strategy

This chapter sets out the econometric model and estimation strategy used to estimate the factors that drive inter-temporal labour supply of married women in Australia.

3.1 The econometric model

This study explores labour supply in terms of the hours worked rather than participation or employment. Since working hours are censored at zero for those who do not work, the conventional model used is the Tobit model (Killingsworth 1983).

Although the Tobit models fit the censored nature of the dependent variable well, it has some limitations that need to be considered.

- The Tobit model relies on an implicit assumption that working hours vary continuously from zero (at a wage equal to reservation wages) to progressively larger positive hours (at wages greater than reservation wages) with no jumps or discontinuity (Killingsworth and Heckman 1986, p. 196; Killingsworth 1983, pp. 141–48).[1] While this assumption is consistent with labour supply theory, if a discontinuity is observed in working hours it introduced empirical problems (Killingworth 1983). This does not appear to be a concern in this study as observed working hours vary continuously between zero and larger positive hours (see figure 1 in section 4.3).
- The model treats labour force non-participation and unemployment as the same labour force state as both are represented by zero working hours. Thus, non-

[1] A violation of the continuous working hour assumption would suggest that zero working hour should be modelled as a separate decision process that differs from the decision process of generating positive working hours (Killingsworth and Heckman 1986, p. 196). Maddala (1992) makes a simular point using a different argument. According to Maddala (1992) zero working hour is not due to censoring since individuals cannot in principle working negative hours. The observed zero working hour is instead due to the decisions of individuals. As a result, the decision that produces the zero hour observations should be modelled separately.

participation and unemployment are assumed to be determined by the same decision process, although they may be affected by different driving forces.[2]

When interpreting the model estimation results, the limitations of the model and the associated assumptions should be kept in mind.

To take advantage of the panel nature of the HILDA data, the conventional Tobit model is augmented by including working hours lagged by one year as an explanatory variable. The resulting model is often called a dynamic Tobit model.

The model can be described as follows. For an individual *i* at time *t* the dynamic model can be expressed as:

$$y_{it}^{*} = \alpha y_{it-1} + x_{it}^{'}\beta + v_{it} \text{, for } t=1,\ldots,T;\ i=1,\ldots,N, \text{ and with} \quad (1)$$

$$y_{it} = \begin{cases} y_{it}^{*} & \text{if } y_{it}^{*} > 0 \\ 0 & \text{if } y_{it}^{*} <= 0 \end{cases}.$$

Where y_{it}^{*} and y_{it} are the latent and observed working hours of individual i respectively; x_{it} is a vector of observed variables that are expected to affect working hours of individual *i*; and v_{it} is an error term, capturing the unobserved factors that affect labour supply decision.

The lagged dependent variable y_{it-1} is included in the right hand side of equation (1) to capture the dynamic feature of working hours, in the sense that current working hours may, all other things being equal, also depend on past working hours. This dependence can be due to things such as the accumulation of skills derived from past work.

With the assumption that v_{it} follows the normal distribution with mean zero and variance σ_v^2 and is independent across individuals and over time for the same individual, equation (1) represents a conventional Tobit model which can be estimated consistently by pooling the panel data to form an enlarged dataset. For the remainder of this paper, this model is termed the 'pooled Tobit' model.

However, the assumption that v_{it} is independent over time for the same individual is violated if labour supply is affected by unobserved individual heterogeneity — that is, the characteristics of the individual not captured by the observed variables in the dataset do have an important influence over the individual's labour supply

[2] Such an assumption is commonly made in estimating a two-step wage equation with Heckman selection correction.

decisions. An example of this is an individual's preference to work, which is not directly captured by the observed variables. Other examples include an individual's level of motivation and/or their innate ability. As it would be reasonable to expect that these factor would influence labour supply decisions, it would also be reasonable to expect unobserved heterogeneity exists. In this situation, failure to control for unobserved individual heterogeneity would lead to the estimate for state dependence being biased upwards.

One method to overcome this would be to make use of measures or proxies of these variables, however these are not available in the data used. An advantage of panel data is that it provides a way to control for unobserved individual heterogeneity through decomposing the error term as:

$$v_{it} = \eta_i + \varepsilon_{it}, \tag{2}$$

where, η_i represents the unobserved time invariant individual effects and thus measures unobserved individual heterogeneity ε_{it} represents the unobserved transitory or time variant shocks/errors to labour supply, and is independent of the observed variables and η_i. In estimation ε_{it} is assumed to follow the normal distribution with mean zero and variance σ_ε^2, i.e. $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$.

The unobserved individual effects η_i can be assumed to be either random or fixed. A random effects assumption implies that η_i is uncorrelated with any of the observed variables included in the model. A fixed effects assumption allows η_i to be correlated with the observed variables. Since the dependent variable working hours is censored and the Tobit model is a non-linear model, it is not technically feasible to use the fixed effects estimator (Hsiao, 2003).[3] The model estimated using the random effects assumption is denoted as 'RE Tobit', where η_i is assumed to follow the normal distribution with mean zero and variance σ_η^2.

Given that unobserved individual factors such as motivation and innate ability are likely also to influence observed outcomes such as education levels and wages for a given education level some modification of the random effects assumption is desirable. One such modification is to use Mundlak's (1978) approach, and allow the unobserved individual effects to be correlated with observed variables through a linear form as denoted in equation (3).

[3] In a linear model with fixed effects, the fixed effects can be differenced out and thus do not cause any complication in estimation. But such a differencing approach does not apply to non-linear models. A non-linear model with fixed effects is generally unfeasible for estimation. Logit and Poisson models seem to be the only models that can incorporate fixed effects in estimation (Wooldridge 2002).

$$\eta_i = \bar{x}_i' \pi + \mu_i \quad (3)$$

With this specification, the unobserved individual effects are assumed to comprise two components, $\bar{x}_i = \frac{1}{T}\sum_{t=1}^{T} x_{i,t}$ and an error term $\mu_i \sim N(0, \sigma_\mu^2)$ which is uncorrelated with any observed variables and the transitory error ε_{it}.[4] Following Wooldridge (2002), this model is called a correlated random effects model (denoted as 'Cor. RE Tobit').

As discussed earlier, even in the absence of state dependence and unobserved individual heterogeneity, labour supply persistence may still be observed if unobserved transitory shocks to labour supply decisions are correlated over time. To control for this source of persistence, an autoregressive relationship between two adjacent transitory errors can be specified:

$$\varepsilon_{it} = \rho \varepsilon_{i,t-1} + \zeta_{it}, \quad (4)$$

where $\zeta_{it} \sim N(0, \sigma_\zeta^2)$ and is independent of μ_i and of the observed variables. The model that allows for correlated random effects and serially correlated transitory shocks is denoted as 'AR. Cor. Tobit'.

To summarise, four dynamic Tobit models are to be estimated, each relaxing assumptions of the previous one.

- *Model I Pooled Tobit*: a conventional Tobit model augmented by including the one-year lagged dependent variable in the right-hand side and being estimated with pooled data. This model assumes that there is no unobserved heterogeneity, working hours in the first wave are exogenous, and the error term is independent across individuals and over time for the same individual. As this model relies on the greatest number of restrictive assumptions, it may be regarded as a 'naïve' model.
- *Model II RE Tobit*: extends Model I to include unobserved individual effects that are assumed to be random and also to endogenize the initial condition using the Heckman (1981c) approach (see the following section). Unobserved transitory shocks to labour supply decisions are assumed to be independent over time for the same individual.

[4] Since time invariant variables cannot be separately identified from their means in the correlated random effects model, $\bar{x}_i'$ can include only the means of time variant variables. Specifically, this study includes the means of the variables for health, the number of children by age, and whether the individual lives in a capital city in the correlated random effects models.

- *Model III Cor. RE Tobit*: extends Model II to allow the unobserved individual effects to be correlated with some time variant observed variables through a linear form. But the assumption of independent transitory shocks is maintained.
- *Model IV AR. Cor. RE Tobit*: Extends Model III to allow the transitory shocks to be (autoregressively) correlated over time.

Assumptions imply restrictions in model estimation. The more numerous the assumptions, the more restrictive the model. In this sense Model IV is the most general model since it relies on the least assumptions about the determinants of labour supply. Estimating all four models provides a test for the assumptions embodied in the more restrictive models. The estimation of Models I to III also provide a robust check of the results obtained from the general model.

3.2 Initial condition problem

The dynamic nature of the model implies that current working hours depend on the hours worked in the previous period. In this formulation consistent estimates of the coefficient parameters rely on the assumption that the unobserved error v_{it} is independent across individuals and over time for the same individual. This assumption is only maintained for Model I.

When unobserved individual effects (either random or correlated random effects) are allowed (Models II to IV), the composite error term v_{it} becomes correlated over time for the same individual. Consequently, the lagged dependent variable is correlated with the error term and thus becomes endogenous (Hsiao 2003). One solution, originally suggested by Heckman (1981c), is to approximate the unknown initial conditions (working hours in the first wave) with a static equation that utilises information from the first wave of panel data, and then jointly estimate the dynamic model with the initial condition equation.

Following Heckman (1981c), when random unobserved individual effects are assumed, the static equation for the initial value of the latent dependent variable can be specified as:

$$y_{i0}^{*} = z_{i0}^{'}\lambda + \gamma\eta_i + \varepsilon_{i0}, \text{ with } y_{i0} = \begin{cases} y_{i0}^{*} & \text{if } y_{i0}^{*} > 0 \\ 0 & \text{if } y_{i0}^{*} <= 0 \end{cases} \quad (5)$$

where z_{i0} is a vector of exogenous variables including x_{i0}; and ε_{i0} has the same distribution as ε_{it}.[5]

When correlated random unobserved individual effects are assumed, the initial condition equation takes the form:

$$y_{i0}^{*} = z_{i0}^{'}\lambda + \bar{x}_{i}^{'}\pi + \gamma\mu_{i} + \varepsilon_{i0}, \text{ with } y_{i0} = \begin{cases} y_{i0}^{*} & \text{if } y_{i0}^{*} > 0 \\ 0 & \text{if } y_{i0}^{*} <= 0 \end{cases} \quad (5').$$

3.3 Estimation strategies

In the four models, estimates of the coefficients of the parameter for the observed characteristics of married women, (α and β in equation (1)) are of primary interest as they provide insights into what drives inter-temporal labour supply decisions. The auxiliary parameters associated with the error components and the initial condition equation also need to be estimated. For ease of exposition, θ is used to represent the vector of all parameters to be estimated.[6]

A maximum likelihood estimator (the appropriate estimator for Tobit models) is used estimate these parameters. This requires the formulation of the likelihood function for the observed sample.

First, the likelihood function is formulated for the pooled Tobit model (Model I). Assuming that v_{it} follows the normal distribution and is independent across time for the same individual i, the conditional (on the observed variables) probability of observing a sequence of y_{it} (for *t*=1,…,T) is:

$$L_{i}^{1}(\theta) = \prod_{t=1}^{T} [\sigma_{v}^{-1}\phi(\Delta_{it}^{1})]^{D(y_{it}=0)} [\Phi(\Delta_{it}^{1})]^{D(y_{it}>0)}, \quad (6)$$

where $\phi(.)$ and $\Phi(.)$ refer to the probability density and cumulative probability functions of the standard normal distribution respectively with $\Delta_{it}^{1} = [y_{it} - (\alpha y_{it-1} + x_{it}^{'}\beta)] / \sigma_{v}$, and $D(.)$ representing an indicator function equal to 1 if the condition in the bracket holds, and zero otherwise.

5 The initial condition equation includes the proportion of time employed; the proportion of time unemployed since an individual first left full-time study; and their mother's occupation as additional identification variables (see table 4.1 in chapter 4).

6 Note that the elements of θ vary depending the model to be estimated.

When unobserved individual effects are introduced and are assumed to be random (Model II), the probability of observing a sequence of y_{it} (for t=1,…,T) can be written in a similar way to equation (6), but with the addition of being conditional on the unobserved individual effects η_i:

$$L_i^2(\theta \mid \eta_i) = \prod_{t=1}^{T} [\sigma_\varepsilon^{-1}\phi(\Delta_{it}^2)]^{D(y_{it}=0)} [\Phi(\Delta_{it}^2)]^{D(y_{it}>0)}, \tag{7}$$

where $\Delta_{it}^2 = [y_{it} - (\alpha y_{it-1} + x_{it}^{'}\beta + \eta_i)] / \sigma_\varepsilon$.

To account for the initial condition problem, the likelihood function $L_i^2(\theta \mid \eta_i)$ needs to be combined with the probability of observing the initial working hours of individual i (the first line on the right hand side of equation (7′)) to form,

$$\begin{aligned} L_i^{2'}(\theta \mid \eta_i) = &\{[\sigma_\varepsilon^{-1}\phi(\Delta_{i0}^2)]^{D(y_{i0}=0)} [\Phi(\Delta_{i0}^2)]^{D(y_{i0}>0)}\} \\ &\times \prod_{t=1}^{T} [\sigma_\varepsilon^{-1}\phi(\Delta_{it}^2)]^{D(y_{it}=0)} [\Phi(\Delta_{it}^2)]^{D(y_{it}>0)}, \end{aligned} \tag{7′}$$

where $\Delta_{i0}^2 = [y_{i0} - (z_{i0}^{'}\lambda + \gamma\eta_i)] / \sigma_\varepsilon$

The likelihood function of the correlated random effects model (Model III) is essentially the same as in equation (7′) except that the function is now conditional on μ_i (instead of η_i) and η_i is replaced by $\bar{x}_i^{'}\pi + \mu_i$.

The likelihood function of the model with serially correlated transitory errors is a bit more involved. Conditioning on the random effects μ_i, for a given sequence of the transitory errors $\tilde{\varepsilon}_i = \{\tilde{\varepsilon}_{i0}, \tilde{\varepsilon}_{i1}, ..., \tilde{\varepsilon}_{iT}\}$, the probability of observing a sequence of y_{it} (for t=0,…,T) can be written as:

$$\begin{aligned} L_i^3(\theta \mid \mu_i; \tilde{\varepsilon}_i) = &\{[\sigma_\varepsilon^{-1}\phi(\Delta_{i0}^3)]^{D(y_{i0}=0)} [\Phi(\Delta_{i0}^3)]^{D(y_{i0}>0)}\} \\ &\times \prod_{t=1}^{T} [\sigma_\zeta^{-1}\phi(\Delta_{it}^3)]^{D(y_{it}=0)} [\Phi(\Delta_{it}^3)]^{D(y_{it}>0)}, \end{aligned} \tag{8}$$

where: $\Delta_{i0}^3 = [y_{i0} - (z_{i0}^{'}\lambda + \bar{x}_i^{'}\pi + \gamma\mu_i + \tilde{\varepsilon}_{i0})] / \sigma_\varepsilon$, and

$\Delta_{it}^3 = [y_{it} - (\alpha y_{it-1} + x_{it}^{'}\beta + \bar{x}_i^{'}\pi + \mu_i + \rho\tilde{\varepsilon}_{i,t-1})] / \sigma_\zeta$ (for t=1,…,T).

When working hours are positive ($y_{i,t-1} > 0$), $\tilde{\varepsilon}_{i,t-1}$ can be calculated as $\tilde{\varepsilon}_{i,t-1} = y_{i,t-1} - (\alpha y_{i,t-2} + x_{i,t-1}^{'}\gamma + \bar{x}_i^{'}\pi + \mu_i)$. When working hours are zero ($y_{i,t-1} = 0$), $\tilde{\varepsilon}_{i,t-1}$ need to be simulated as:

$$\tilde{\varepsilon}_{i,t-1} = \sigma_{\varepsilon}\Phi^{-1}[\xi\Phi(\Delta^{3}_{i,t-1})], \quad (9)$$

where ξ is a random draw from a uniform distribution. Fifty Halton sequence draws were used to simulate the likelihood function where required.

In order to estimate the parameter coefficients, the unobserved individual effects in the likelihood equations (7), (7′) and (8) above need to be integrated out. This was carried out using the Gaussian-Hermite quadrature method with the assumption that the unobserved individual effects follow the normal distribution.

The sample likelihood function, which is maximised with respect to the parameters, is obtained by taking the product of the individual likelihood function. All the model estimations are implemented using the Gauss package with code written by the author.