F Deriving the Ramsey equation[1]

As set out in Appendix H, $p = E(mx)$ is the basic consumption based pricing equation, where p is the price of the asset, x is its payoff next period and $m = \gamma \frac{u'(c_{j+1})}{u'(c_j)}$ is the stochastic discount factor. This is an equation that holds for each individual, for each asset to which he has access and for any two periods of a multi–period model. If the equation does not hold, then the investor would buy more or less of the asset (and change current and future consumption) until it does hold. It comes from the first order conditions for a consumer with the utility function $U(c_j, c_{j+1}) = u(c_j) + \gamma E_j[u(c_{j+1})]$. The assumption that the consumer maximises time and date separable expected utility could be dropped. The basic consumption pricing equation would hold for any general non-separable utility function. The relation between the discount factor and real variables would, however, be more complicated. It would be a function of many variables, not just the discount on future utility and present and future consumption.

For a risk free asset the basic pricing equation becomes $1 = E(mR^f) = R^f E(m)$ where R^f is the gross risk-free return. This gives $R^f = 1/E(m)$

If the consumer has a power utility function $u(c_j) = \frac{(c_j^{1-\eta} - 1)}{(1-\eta)}$ with $\eta > 0$, which gives $u(c_j) = ln(c_j)$ for $\eta = 1$. Power utility has $u'(c) = c^{-\eta}$ and so a constant elasticity of marginal utility (or co-efficient of relative risk aversion) of $-\eta$. Then the basic pricing equation for a risk free asset is: $R^f = \frac{1}{\gamma}\left(\frac{c_{j+1}}{c_j}\right)^{\eta}$

Taking logs gives the Ramsey equation: $r_t^f = \theta + \eta g$, where $r_t^f = \ln R_t^f$ is the continuously compounded risk free rate (see Appendix A for the relationship between continuously compounded rates and their discrete per period counterparts; they are approximately equal), $\gamma = e^{-\theta}$ and $g = ln(c_{j+1}/c_j) = lnc_{j+1} - lnc_j$ is the continuously compounded growth rate of consumption.

1 Cochrane (2005, pp. 10-11, 35-36).

The power utility assumption is a convenient simplification that makes the elasticity of marginal utility independent of consumption. It gives the simple form of the Ramsey equation, where the risk free rate depends on consumption growth. It is a first approximation to the more general case.