Australian Government
Productivity Commission

The Armington General Equilibrium Model: Properties, Implications and Alternatives

Staff Working Paper

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February 2008

ISBN 978-1-74037-244-2

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An appropriate citation for this paper is:

Zhang, X.G. 2008, *The Armington General Equilibrium Model: Properties, Implications and Alternatives*, Productivity Commission Staff Working Paper, Melbourne, February.

JEL code: F11; F13; F15; Keywords: Armington assumptions; Armington models; the Heckscher-Ohlin model; General equilibrium model; Product differentiation; Comparative advantage; International trade.

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Preface

The Productivity Commission has a strong research program into the development and application of analytical frameworks in support of its commissioned and supporting research. Part of this program has been dedicated to examining and clarifying the role of the Armington assumption commonly used in trade policy analysis. The Armington assumption differentiates products by country of origin and is often used in applied general equilibrium model of world trade.

The purpose of this paper is to contrast an Armington-based model with the traditional Heckscher-Ohlin framework familiar to textbook trade theory. The models concentrate on different aspects of the gains from trade, and both have deficiencies. The paper argues that by combining both frameworks, the hybrid Armington-Heckscher-Ohlin model inherits the strengths of both models.

In publishing research in this area, the Commission hopes to improve the quality and usefulness of quantitative analysis of global economic issues within economywide frameworks.

Acknowledgments

The author wishes to acknowledge helpful comments made by Drusilla K. Brown, Peter Cribbett, Rod Falvey, Patrick Jomini and Jonathan Pincus and editorial assistance from Peter Garrick. Thanks are also due to Peter Lloyd for useful discussions following the publication in 2006 of the Staff Working Paper on *The Armington Model*. However, the author is solely responsible for any remaining errors and omissions.

The views expressed in this paper are those of the author and do not necessarily reflect those of the Productivity Commission.

Abbreviations

A-H-O	Armington-Heckscher-Ohlin
CES	Constant elasticity of substitution
CET	Constant elasticity of transformation
CGE	Computable general equilibrium
CRS	Constant returns to scale
DMR	Dervis, de Melo & Robinson model
FPE	Factor-Price-Equilisation
GTAP	Global Trade Analysis Project
H-O	Heckscher-Ohlin
LOP	Law of One Price
LoV	Love of Variety
NTG	Non-traded goods
PEP	Price expansion path
PPF	Production Possibility Frontier
S-D-S	Spence-Dixit-Stiglitz model
S-S	Stolper-Samuelson

OVERVIEW

Overview

Many of the large general equilibrium models that have been used in trade and industry policy analysis adopt the Armington assumption of product differentiation by country of origin. This is because most trade databases are aggregated and exhibit ‘cross-hauling’, that is, a country appears to be importing and exporting the ‘same’ good. By differentiating products by country of origin, the Armington assumption provides a rationale for the existence of ‘cross-hauling’ data and offers a basis for modelling intra-industry trade.

However, the Armington assumption results in a model whose properties and behaviour are fundamentally different from the well-known Heckscher-Ohlin (H-O) model. More importantly for our purposes, these properties make some results from popular models, which are based on the Armington assumption, difficult to use in policy analysis.

What are the drawbacks?

There are two well-known features of Armington models:

1. larger than expected changes in inter-country relative prices, which result in excessive terms of trade effects, especially for small countries
2. smaller-than-expected changes in inter-industry relative prices and, therefore, in national outputs, leading to an underestimate of possible reallocation efficiency gains from trade liberalisation (sections 2.2-2.3)

These general equilibrium properties of an Armington model are controlled by users’ preferences over differentiated goods, especially the value of the Armington elasticities of substitution. For model builders, the choice of elasticities involves unfortunate tradeoffs.

An elasticity below unity implies that consumers regard nationally differentiated goods as gross complements, which causes peculiar behaviours in the model.

For Armington models to function normally, therefore, the value of Armington elasticities needs to be set well above unity. However, high elasticity values are incompatible with highly dissimilar products. Yet, applied models all use data that aggregate commodities into industry groups, and this aggregation increases the dissimilarity for national products.

Furthermore, the unique structure of the Armington model also implies that some of the most popular trade theorems (the Heckscher-Ohlin theorem, the Factor Price Equalisation theorem, the Stolper-Samuelson theorem and the Rybczynski theorem), which are derived from the H-O model, can no longer be used for interpreting the Armington model's results (section 2.4).

How do gains from trade arise?

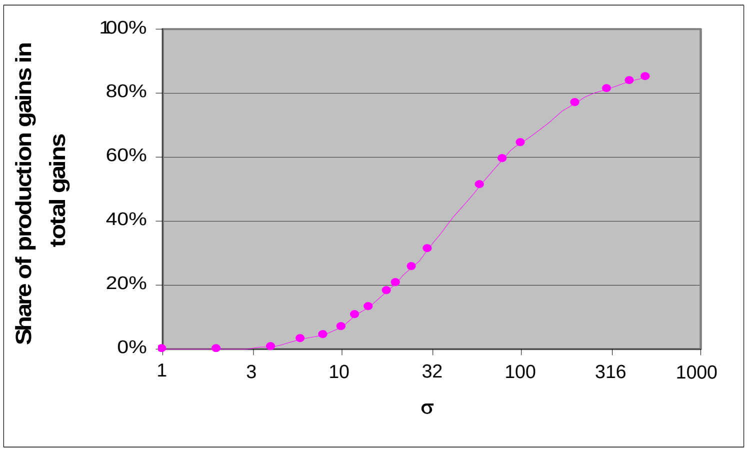
The gains from trade in an Armington model are determined by the value of the Armington elasticities (σ) (section 3.1). The results from a simple numerical model show the following.

1. When $\sigma < 1$, imports and domestically produced goods are gross complements, because income effects dominate substitution effects. As a result, introducing a tariff on an import does not increase domestic production
2. Unless σ is set well above unity, exchange gains dominate:
   (a) As shown in figure 1, when σ reaches 10, exchange gains still account for 90 percent of the gains from trade.
   (b) When σ reaches 100, production gains account for the majority of gains from trade.
   (c) When σ reaches 500, the share of production gains is close to 90 per cent, the level that is observed in a similarly defined H-O model.

Although these results are obtained with small stylised models, they also hold in applied models with large databases and more complex structures. In the stylised models, consumers are assumed to prefer domestic goods and their import counterparts equally. In applied models, however, imports often account only for a small share in total consumption. This home bias can significantly reduce the share of production gains from trade. As a result, consumption gains, that is, distributional gains across countries, can dominate results in an applied model even at very high elasticities. As most applied models set Armington values below 10, the gains from trade in those models come mainly from the re-distribution of consumption of a given level of differentiated outputs. Opening trade gives rise to very little output expansion.

This is in stark contrast with the sources of gains from trade in the H-O model, which arise from a reallocation of production across the world according to comparative advantage, and give rise to an increase in world output.

Figure 1 **Share of production gains in total gains from trade**[a]
Armington model, σ = 1 to 500,



[a] Gains from trade that arise from a country moving from autarky to free trade in a simple two-country model.

Another difference between the gains from trade in the Armington model and in the H-O model is that free trade does not exhaust gains from resource allocation in the Armington model:

- Whereas, in the H-O model, trade in goods replaces trade in factors,
- In the Armington model, however, opening trade does not equalize goods and factor prices across countries. Even in free trade, further gains are possible by moving resources across countries to satisfy preferences across the world. Moving resource increases world output – and wellbeing (section 3.2).

Alternatives to the Armington model

There are two types of alternatives to the Armington model: models assuming homogenous goods, and models using other ways of representing differentiated goods. Neither are ideal.

The H-O model represents the former. Despite its popularity in trade theory, the H-O model has its limitations as an applied model due to serious problems such as multiple equilibria and hypersensitivity to changes in trade costs – that is, overspecialisation (section 4.1).

Love of Variety (LoV) models offer an alternative way to represent differentiated goods. Unlike Armington models, LoV models incorporate assumptions about monopolistic competition and scale economy, thus providing a production side explanation for product differentiation and intra-industry trade. However, consumer preferences for country-specific varieties give monopoly power to each country, which leads to large terms of trade effects, similar to what is observed in the Armington model. In this regard, for policy analysis, LoV models are not fundamentally different from the Armington model.

Alternatives to the Armington model either suffer from drawbacks similar to those of the Armington model (eg, the LoV models) or are inconsistent with databases that incorporate cross-hauling (eg, neoclassical models) (section 4.1).

Toward a hybrid model

Gains from trade arise from comparative advantage and from access to different varieties that best satisfy customer demands. Different models emphasise different possible sources of gains. For example, homogenous goods models capture inter-industry trade and thus emphasise gains from comparative advantage, which arise from countries' different resource or technological endowments. On the other hand, differentiated goods models capture intra-industry trade. They emphasise the benefits to consumers and firms of the greater variety that can only be obtained through trade. Models of inter-industry trade and of intra-industry trade each emphasise one of these gains. A hybrid model offers the prospect of combining the rationale for both types of gains (section 5.1).

One attractive way of creating a hybrid model is to introduce homogenous goods into an Armington model. This offers the possibility of integrating the strengths of both types of models and of reducing some of the problems associated with each type. An 'Armington-Heckscher-Ohlin' (A-H-O) model can capture production gains from comparative advantage and consumption gains from product differentiation. At the same time, it can avoid the dimensionality problem that makes it difficult to solve H-O models. It can also reduce to plausible levels the terms of trade effects that are the hallmark of models with product differentiation.

The main challenge in building an A-H-O model from an existing model such as the GTAP model and database is to identify some homogenous products and split them from the current commodity aggregates. This non-trivial task requires a good theory on which to base the splits (section 5.2).

1 Introduction

Computable general equilibrium (CGE) models are powerful tools for applied trade and economic policy analysis. Most of today's applied CGE models incorporate the Armington assumption of product differentiation by country of origin (Armington 1969). The Armington assumption provides a convenient interpretation for analysts and allows them to calibrate models to trade statistics in which 'cross-hauling'[1] is widespread. However, incorporating a preference for differentiated products within a conventional neoclassical production structure creates a general equilibrium model that has properties and behavioural features that are very different from well-known neoclassical models, such as the Heckscher-Ohlin (H-O) model. These properties have important implications for policy analysis based on results from these models.

1.1 Understanding the Armington Model

Most theoretical knowledge on the behaviour of international trade and the welfare implications of trade policy is based on the neoclassical H-O model. Although CGE models have been used increasingly in trade policy analysis, analysts are often unaware that many conventional trade theorems no longer apply in an Armington model.

Some features of the Armington model have been discussed in the literature over the years.[2] Some of this discussion is based on single country models. For multi-country trade models, knowledge of the behaviour of Armington models comes mainly from applied models.[3] In practice, the Armington parameters of these models have been calibrated to specific databases. As each model has a different database, the calibrated parameters are unlikely to be the same even for models with

[1] Cross-hauling refers to the situation where there is two-way trade of a product between countries. Cross-hauling can be an artefact of aggregation. If the aggregate was disaggregated into separate products, which are not themselves aggregates; the 'cross-hauling' may disappear.

[2] See, for example, de Melo and Robinson (1981 and 1989), Brown (1987), Alston et al. (1990), Zhang (2006) and Lloyd and Zhang (2006).

[3] An exception is Brown (1987), which uses a pure exchange theoretical model of n countries/ commodities to analyse the terms of trade effect of a tariff.

the same theoretical structure. This obscures comparison of different models' behaviours.

1.2 Purpose

Although applied Armington models have become increasingly popular, the Armington model has seldom been treated as a theoretical model in the same way as the H-O model.[4] For this reason, until recently, the general equilibrium properties of the Armington model have not been thoroughly investigated and the differences between it and the well-known H-O model have not been systematically analysed.

This paper, in part, seeks to fill this gap. The Armington model is treated as a theoretical model, in the same way the H-O model is treated in a standard trade textbook. Some assumptions are made to simplify the structure of the model and make it comparable with a similarly defined H-O model. This is designed to highlight the fundamental differences between the Armington model and the H-O model and to illustrate how these differences shape the behaviour of the Armington model.

1.3 Scope

Both the Armington model and the H-O model used in this paper are numerical. Instead of being calibrated to a specific database, the model parameters are given by assumption so that alternative settings for some parameters could be used to test the sensitivity of the model's properties. The general equilibrium properties of the models are then illustrated using diagrams rather than numerical tables. This is because a diagram is capable of handling much more information than a table. Different variations of a general equilibrium pattern can be readily plotted and described in the same diagram. Also, a diagram is better able to provide the flavour of some of the qualitative results.

The models used in this paper are multicountry general equilibrium models. This is to avoid arbitrary external closures that are necessary in single country models. For ease of diagrammatic exposition, the dimension of the models is kept at a 2-country 2-good 2-factor (2×2×2) level. As shown later, this limited dimension retains the essential properties of the Armington model.

[4] For a recent paper on the general equilibrium properties of the Armington model, see Lloyd and Zhang (2006).

Most applied Armington models adopt a two-level substitution structure. The top level defines a substitution relationship between a domestically produced good and an imported composite; the composite is an aggregate of imperfectly substitutable goods imported from different countries (the second level). This structure requires at least three countries and two goods. That said, the essence of the Armington model can be captured by a simpler 2×2×2 model which contains only the top level substitution relationship. This simple model is readily comparable with a standard textbook version of the H-O model with the same dimension. The general equilibrium properties of both models can then be readily explained using familiar two-dimensional diagrams.

Using this diagrammatic comparison, the Armington model is found to be fundamentally different from the H-O model. The behaviour of the Armington model is influenced largely by changes in intra-industry relative prices or term of trade.[5] As a result, gains from trade in the Armington model are mainly from exchange or redistribution of given outputs between trading countries rather than more efficient allocation of resources and output expansion within each country.

As trade models, the Armington and the H-O models both have strengths and weaknesses. This is because each model focuses exclusively on one aspect of international trade. The Armington model explains international trade as entirely intra-industry trade in differentiated products, [6] while the H-O model explains it as entirely inter-industry trade in homogenous products.[7]

This paper proposes an alternative model, one that mixes the features of the Armington model and the H-O model, and which incorporates both intra- and inter-industry trade in differentiated and homogenous products. As will be shown in chapter 4, the 'Armington-Heckscher-Ohlin' (A-H-O) model not only has the strengths of both models but also, importantly, avoids the weaknesses of both.

[5] This is consistent with the findings of previous studies. See, for example, Petri (1984), Brown (1987) and Brown and Stern (1992).

[6] It may be argued that inter-industry trade appears in the Armington model as net trade, that is, the difference between intra-industry exports and imports. However, inter-industry trade here is not captured directly, but as a residual.

[7] It has been known in the literature that product differentiation and comparative advantage are complementary explanations of trade. As Victor D. Norman indicates, "[P]roduct differentiation explains intra-industry trade but has no intersectoral repercussions; comparative advantage explains intersectoral specification and trade, but provides no explanation of intra-industry phenomena." (1990, p.726)

1.4 Outline

The remainder of the paper is organised as follows. The properties of the Armington model are outlined in chapter 2, using a simple numerical model. That chapter also provides comparisons between the Armington model and the H-O model. Chapter 3 describes the implications of these model properties for policy analysis. This chapter includes an analysis of the welfare implications of a tariff in both models. In chapter 4, existing alternative model structures to the Armington model are discussed. In chapter 5, a model that combines the features of both the H-O model and Armington model is introduced as a preferred alternative. The properties of the A-H-O model are illustrated and discussed along with some indication of work required to apply it on a large scale. Chapter 6 provides some concluding remarks.

2 General equilibrium properties of the Armington model

This chapter begins by developing the structure of the general equilibrium Armington and Heckscher-Ohlin (H-O) models. Next, these models' properties, in autarkic, integrated and free trade equilibria, are compared and discussed.

To keep differences between the models to a minimum, the Armington model shares the same configuration on the supply side as the H-O model. The only difference between them is on the demand side and concerns differences in household preferences over the goods produced by the two countries.

2.1 Comparison of model structure

Let the two countries (r) be A (home) and B (foreign). Assume that each country is endowed with given quantities of two factors, K_r (capital) and L_r (labour). Each country produces two goods, good 1 and 2, using the same constant returns to scale (CRS) technologies. Each country has a representative household that maximises utility by choosing, purchasing and consuming an optimal bundle of the two goods subject to a budget constraint.

The producer in country r of a given quantity X_{rj} of good j minimises the total cost G_{rj} of factor services used

$$G_{rj} = r_r K_{rj} + w_r L_{rj}, \qquad (r = A, B; j = 1, 2) \qquad (1)$$

subject to a production function

$$X_{rj} = X_{rj} (K_{rj}, L_{rj}) \qquad (r = A, B; j = 1, 2) \qquad (2)$$

where r_r and w_r denote the rental price of capital and the wage rate for labour, and K_{rj} and L_{rj} denote the demand for capital and labour to produce the good.

The cost per unit of output c_{rj} of good j in country r is

$$c_{rj} = r_r k_{rj}(r_r, w_r) + w_r l_{rj}(r_r, w_r) \quad (r = A, B; j = 1, 2) \qquad (3)$$

where k_{rj} and l_{rj} are the cost minimizing requirements of capital and labour per unit of output.

CRS ensures zero pure profits in a competitive equilibrium for both industries in both countries, so each firm's unit costs equal the price of the good it produces

$$c_{rj} = P_j \qquad (r = A, B; j = 1, 2) \qquad (4)$$

In equilibrium, factor markets clear in both countries

$$K_r = k_{r1} X_{r1} + k_{r2} X_{r2} \qquad (r = A, B) \qquad (5)$$

$$L_r = l_{r1} X_{r1} + l_{r2} X_{r2} \qquad (r = A, B) \qquad (6)$$

where K_r and L_r are the endowments of capital and labour in country r.

Depending on the assumption made about the substitutability between good j produced in one country and good j produced in the other, the model can be either a traditional H-O trade model (perfect substitution) or an Armington model (imperfect substitution).

The H-O model

In an H-O model, the household in each country purchases the combination of the two goods that maximises utility

$$U_r = U_r (C_{r1}, C_{r2}) \qquad (r = A, B) \qquad (7)$$

using income from factor services

$$Y_r = G_{r1} + G_{r2} \qquad (r = A, B) \qquad (8)$$

where C_{rj} represents consumption of good j in country r. An assumption, also required for some of the results later, is that the utility functions are identical and homothetic.[1]

Worldwide, goods markets clear

[1] If the demand functions for goods are derived using a homothetic utility function subject to a budget constraint, one important result is that the ratios of the quantities of goods demanded depend only on relative prices, and not on income. And if the utility functions are identical and homothetic, if both countries face the same prices, then world demand for each good depends only on those relative prices and is not dependent on how that income is distributed between countries. In this situation both countries consume quantities of goods in the same ratio.

$$X_{Aj} + X_{Bj} = C_{Aj} + C_{Bj} \qquad (j = 1, 2) \qquad (9)$$

The system of equations and variables for this model can be reduced to ten unknowns: P_j, X_{rj}, w_r and r_r. They can be solved from the ten equations that define the general equilibrium conditions for this model — two goods market clearing conditions (Eq 9), four zero pure profit conditions (Eq 4) and four factor market clearing conditions (Eqs 5 and 6).

As this model assumes that the two goods produced in both countries are homogenous, the exports of good i for country r can be expressed as

$$E_{rj} = X_{rj} - C_{rj} \qquad (r = A, B; j = 1, 2) \qquad (10)$$

where a negative E_{rj} indicates imports.

This completes the basic structure of a 2×2×2 general equilibrium H-O trade model.

The Armington model

In the Armington model, households treat goods produced by the same industry but in different countries as though they are different. There is a two-stage utility maximisation process. At the top level, there are two composite goods, each composed of both imperfectly substitutable goods. Households choose between these two composite goods to maximise their utility subject to a budget constraint. Once the demands for the composites are determined and a budget allocated for each, households minimise expenditure on each composite by choosing the cost-minimising mix of the domestically produced and foreign-produced good. More formally, the top utility function can take the same form as before

$$U_r = U_r (C_{r1}, C_{r2}) \qquad (r = A, B) \qquad (11)$$

However, C_{rj} is no longer an actual product but a composite of two differentiated products. A sub-utility function, which introduces the Armington assumption, is

$$C_{rj} = C_{rj} (D_{rj}, M_{rj}) \qquad (r = A, B) \qquad (12)$$

where D_{rj} and M_{rj} are the quantities demanded of good j produced by country r and imported into country r, respectively. It is this nested utility function that turns an H-O model into an Armington model.

In this model, the structure of production is identical to that used in the H-O model. Each country still produces two goods using the same technologies.[2] However, the households no longer treat goods produced by the same industries, but in different countries, as perfect substitutes. Each country's household now demands four goods — two domestically produced and two imported goods. The trade now becomes a two-way exchange in each industry and so-called 'cross-hauling' emerges.

Worth noting, and not widely recognised, is that, compared with the H-O model, the system of equations and variables can be reduced to only four unknowns, w_r and r_r. These can be solved using the four factor market clearing conditions (Eqs 5 and 6). The other six variables can now be defined by other variables and equations. Due to product differentiation, the previous two world prices now become four, P_{rj}. These are now determined directly from unit costs and each country's industry is now a price maker, not a price taker. As a result, the two goods market clearing conditions (Eq 9) become redundant and can be dropped. Finally, the four equilibrium outputs, X_{rj}, can now be defined as the sum of domestic demand and foreign (export) demand

$$X_{Aj} = D_{Aj} + M_{Bj} \qquad (j = 1, 2) \qquad (13)$$

$$X_{Bj} = D_{Bj} + M_{Aj} \qquad (j = 1, 2) \qquad (14)$$

This reduction in the number of unknowns and equations makes the Armington model structurally similar to an autarky model, that is, a closed economy model without trade. This is because production in different countries is disjoint, with each country's industries becoming the sole producer of their products, just like in an autarky economy where each industry is the sole producer of its own product and there is no corresponding foreign producers. This structure makes the model easier to solve and quicker to reach an equilibrium. Moreover, unlike the H-O model, the Armington model suffers no dimensionality problem. It can be extended to include any number of factors and goods without suffering from the problem of multiple equilibria.[3]

As outlined in the next three sections, introduction of the convenient Armington assumption completely alters the nature of the traditional H-O trade model. To illustrate this, the properties of three different equilibria are examined and compared in both models — an autarkic, an integrated and a free trade equilibrium.

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[2] It is not necessary to assume that differentiated goods are produced using the same technology. The assumption is introduced for comparative purposes only. With this assumption, the only difference between the H-O model and the Armington model is on the demand side. This allows the effects of introducing the Armington assumption of product differentiation by country of origin into the traditional H-O trade model to be highlighted.

[3] This point is elaborated in chapter 4.

Without loss of generality, production technology is assumed to be more labour intensive for good 1. Also assumed is that both countries use the same technologies in production and have identical homothetic preferences in consumption. The only difference between countries lies in their factor endowments, with country A being more labour abundant than B.

Given specific functional forms for utility and production, a unique equilibrium can be found for this model. Following a common practice, the nested utility function is assumed to take the form of a constant elasticity of substitution (CES) function. In a CES function, there are two parameters that influence the choices of a household — the Armington elasticity, σ, and the share parameters, α for domestic goods and $(1-\alpha)$ for imported goods. For ease of exposition, the Armington elasticities are assumed to be equal for all goods in all countries as are the share parameters in the nested CES utility functions (with $\alpha = 0.5$). At the top level of the utility function, households allocate their budget equally between the two composite goods, while in the two sub-utility functions households also allocate their expenditure equally on the domestically produced good and its imported counterpart.

The CES function's elasticity parameter can take any value between 0 and infinity, except 1. When σ approaches 1, the CES function approaches, in the limit, the Cobb-Douglas function.

Every setting for the Armington elasticity creates a distinct model, with a unique free trade equilibrium. Armington models fall into two groups around the Cobb-Douglas benchmark, that is, according to whether σ is greater or smaller than 1. Models in each group have distinctive features. To begin with, an Armington model with a unitary elasticity[4] is analysed to enable later comparison with other Armington models having different elasticities.

2.2 Free trade equilibrium with unitary Armington elasticities

The model with unitary Armington elasticity has an important property, namely intra-industry trade is balanced: that is, exports and imports of goods produced by the same industry are equal.

Also important is that the inter-industry relative price of a country's good 1 to good 2 remains the same as in autarky in the model with unitary Armington elasticity. Without inter-industry price change, factors are not reallocated across

[4] The CES elasticity σ in this model is set at a value close to 1.

industries, and production patterns in both countries remain the same as in autarky. The free trade equilibrium of this Armington model can be illustrated using two diagrams (see figures 2.1 and 2.2).

Goods market equilibrium

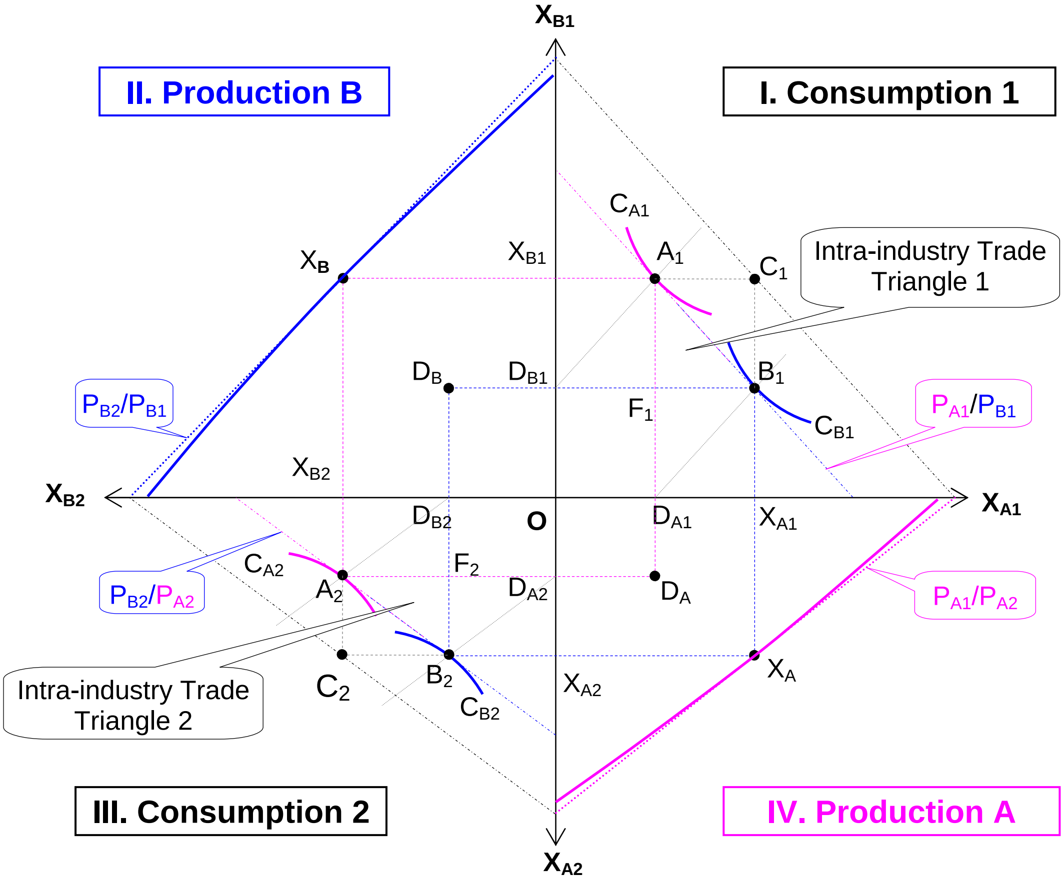
The properties of the goods market equilibrium in this Armington model are illustrated in figure 2.1. The figure combines, in a single diagram, production of both products in both countries with the consumption of all four differentiated products. The quadrant IV contains country A's Production Possibility Frontier (PPF) while the quadrant II contains country B's PPF. Because country A is relatively small compared with B, A's PPF is smaller than B's. Country A produces two goods X_{A1} and X_{A2} while B produces X_{B1} and X_{B2}. The two good 1s produced by the two countries, X_{A1} and X_{B1}, are imperfectly substitutable in consumption. This is shown in the quadrant I. A similar story for the two good 2s is shown in the quadrant III.

In equilibrium, country A produces OX_{A1} of good 1 and OX_{A2} of good 2 at point X_A on its PPF in quadrant IV. Of the total output, OD_{A1} and OD_{A2} are consumed domestically and $X_{A1}D_{A1}$ and $X_{A2}D_{A2}$ are exported. D_A divides output into domestic sales or exports. A similar story holds for country B in quadrant II.

Demand for both pairs of differentiated goods produced by the industries in each country is shown in quadrants I and III. C_1 represents the total demand of the good 1s while C_2 total demand of good 2s. These four goods are distributed between the countries. Country A consumes the two good 1s at point A_1 with respect to its origin at D_{B1} in quadrant I, that is, OD_{A1} of domestic good 1 and $D_{B1}X_{B1}$ of imported good 1. The domestically produced good 1 and the imported good 1 form a composite, whose quantity is represented by the indifference curve C_{A1}, tangent to the relative price line P_{A1}/P_{B1} at point A_1. Similarly, country B consumes the two good 1s at point B_1 with respect to its origin at D_{A1} in quadrant I, that is, OD_{B1} of domestic good 1 and $D_{A1}X_{A1}$ of imported good 1. Again, these two goods form a composite good indicated by indifference curve C_{B1}, tangent to the same relative price line at point B_1. Similarly, points A_2 and B_2 in quadrant III represent consumption for the two countries of the differentiated good 2s.

Figure 2.1 **Free trade equilibrium in goods markets: the Armington model**

A 2-country 2-sector 2-factor model with σ = 0.99



In contrast with the H-O model, there are four relative goods prices in this Armington model. In quadrant IV, the slope of the relative price line (P_{A1}/P_{A2}) represents the inter-industry relative price of the good 1 to good 2, both produced in country A. Similarly, quadrant II contains the inter-industry relative price line for B, (P_{B2}/P_{B1}). In quadrant I, the slope of the price line represents the intra-industry relative price (P_{A1}/P_{B1}) of the differentiated good 1s produced by each country. This also represents the sectoral terms of trade in good 1s for A. A similar story is represented in quadrant III, which contains the intra-industry relative price line (P_{B2}/P_{A2}) for the differentiated good 2s. This represents B's sectoral terms of trade in good 2s. The inter-industry relative prices can be found in the H-O model, while the intra-industry relative prices are unique to the Armington model.

The Armington elasticity determines the curvature of the indifference curves in quadrants I and III. By controlling the responsiveness of consumers to price changes, the Armington elasticity directly influences the consumptions of differentiated goods and intra-industry relative prices, which, in turn, affect the general equilibrium properties of the model.

With $\sigma = 1$, free trade equilibrium outputs and inter-industry relative prices are identical to those in autarky.[5] This is because the unitary elasticity results in a set of intra-industry relative prices that equalise the value of each of the four outputs produced. This is illustrated in figure 2.1 where the intra-industry relative price lines in quadrants I and III coincide with the inter-industry relative price lines in quadrants IV and II on the horizontal and vertical axes. This implies that, in equilibrium, the values of the four goods are equal. With the choice of parameter values the two countries allocate their budgets equally to all four goods. To ensure balanced trade in each market, the prices of four goods must be so adjusted to equate the values of exports and imports in each of the two goods market. This can be seen in quadrant I and III.

The triangle $A_1C_1B_1$ in quadrant I represents intra-industry trade, between the countries, in industry 1 (goods 1). Similarly, the triangle $A_2C_2B_2$ in quadrant III is intra-industry trade in industry 2. The general equilibrium solution is reached when overall trade is balanced, that is, when the value of a country's total exports equals the value of its total imports. With unitary elasticities, as in the figure, pair-wise trade in each industry is equalised, also. That is, country A's exports of its good 1 equal, in value, its imports of good 1, based on the intra-industry relative price. Geometrically, this implies that the two consumption points A_1 and B_1 in quadrant I are linked by an intra-industry relative price line P_{A1}/P_{B1}. Similarly, trade is also balanced in industry 2 in quadrant III.

Despite the disparity in size and initial endowments between countries A and B, output values equalise due to the relatively high equilibrium prices for all goods produced in country A. The unitary elasticity gives the smaller country relatively high market power, because their unique products are in short supply compared with the goods from the larger country. As can be seen in the following, in free trade relatively low Armington elasticities always give the smaller country favourable terms of trade at the expense of the larger country. This is one of the unique features of the Armington model. This differs from the H-O model, in which terms of trade effects do not heavily influence the general equilibrium results.[6] This

[5] The autarky equilibrium of an Armington model may not be defined if a nested CES utility function is used. However, such a situation can be approximated by an equilibrium in which all imports are reduced close to zero by a prohibitive tariff. It can be shown that, in the limit, the output patterns and inter-industry relative prices in such an equilibrium approach those in an H-O model with the same production and endowment settings. For a comparison with the autarky equilibrium of such an H-O model, see figure A.2 in the Appendix.

[6] It should be noted that in an H-O model the small country gains more from trade than the large country, as well. This is because the equilibrium terms of trade are close to the autarky relative prices of the large country. However, the difference between the H-O model and the Armington model is that in the former all countries face the same prices while in the latter the small country

is because, in the H-O model, the same goods can be produced in different countries and the terms of trade are influenced more by endogenous production specialisation across countries, rather than household preferences about some particular country's products.

The unitary Armington elasticity creates a particular set of intra-industry relative prices that preserve the autarky difference in inter-industry relative prices between the two countries, that is $P_{A1}/P_{A2} < P_{B1}/P_{B2}$. The relative price of a labour-intensive product is lower in a labour-abundant country than in a capital-abundant country. The inter-industry relative price differentials reflect the relative factor endowments and production technologies. If preferences over the goods are not biased,[7] the autarky production pattern can be determined together with its inter-industry relative price. In the H-O model, free trade equalises the prices of goods and factors and reallocates resources according to each country's comparative advantage. As a result, output patterns change in all countries. In the Armington model, however, whether or not, or to what extent, free trade will change a country's inter-industry relative price and output pattern depends on the substitutability between nationally differentiated products.

Factor market equilibrium

The free trade equilibrium in factor markets, for the simple Armington model, can be shown in a familiar box diagram. The box in figure 2.2 represents the world economy's resources. Country A's endowment is measured from the lower left corner O_A while B's endowment is measured from O_B. Point E is the joint endowment point, dividing the world endowment of capital and labour into two parts, country A's and B's endowment. Country A is assumed to be labour-abundant relative to B.

In isolation (autarky), country A has a lower wage-rental ratio than B ($w_A/r_A<w_B/r_B$). As these factor prices are not equal, the capital–labour ratios, in the same industry across countries, are not equal. This is represented by the two parallelograms, $O_AX_{A1}EX_{A2}O_A$ and $O_BX_{B1}EX_{B2}O_B$, which are not congruent. In equilibrium, country A devotes resources O_AX_{A1} to production of good 1 and O_AX_{A2} to good 2. Likewise, B devotes resources O_BX_{B1} to good 1 and O_BX_{B2} to good 2. Due to country A's lower wage-rental ratio, more labour and less capital is used in both industries, compared with B. In equilibrium, both countries use all their resources to produce both their country-specific goods. The endowment point E

faces lower prices than the large country. As a result, the small country enjoys more favourable terms of trade.

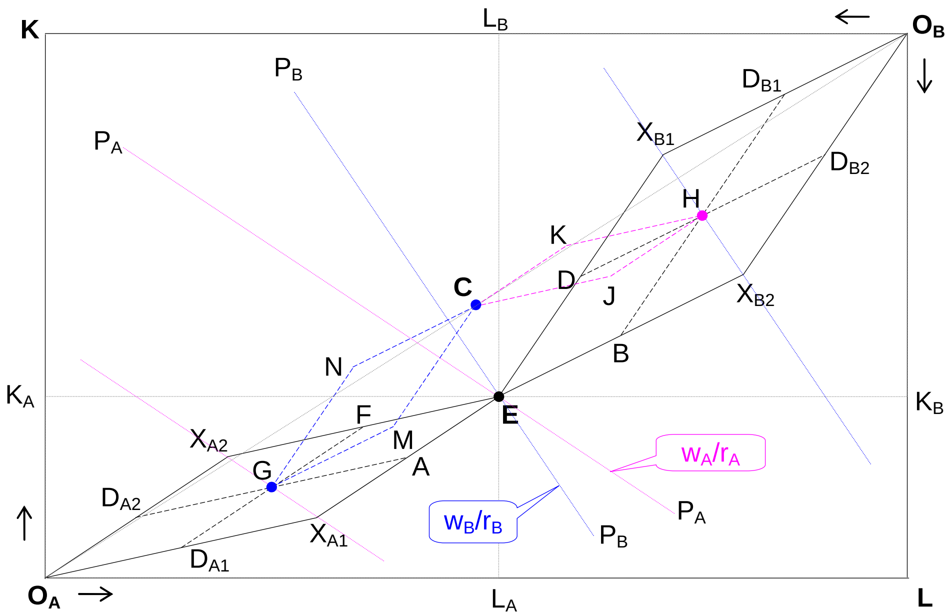
[7] That is, preferences are homothetic and identical for both countries.

coincides with equilibrium consumption C. This means that all of the resources endowed in each country are used to produce their two goods, which are consumed domestically.

As mentioned earlier, with σ close to 1, free trade does not alter the countries' inter-industry price ratios and output patterns. In factor markets, this implies that the autarky wage-rental ratios and the capital–labour ratios for the two industries remain unchanged, in both countries. This is shown in figure 2.2. The rays O_AX_{A1} and O_AX_{A2} are country A's factor requirements for producing its good 1 and 2. Likewise, the rays O_BX_{B1} and O_BX_{B2} are B's factor requirements for producing its good 1 and 2. The parallelograms $O_AX_{A2}EX_{A1}O_A$ and $O_BX_{B2}EX_{B1}O_B$, therefore, represent full employment of factors in country A and B. These levels of factor usage are identical to those prevailing in autarky.

Figure 2.2 **Free trade equilibrium in factor markets: the Armington model**

A 2-country 2-sector 2-factor model with σ = 0.99



In contrast with autarky, free trade allows both countries to exchange the goods they produce. In country A, the O_AD_{A1} component of O_AX_{A1} represents the factor requirement for producing the share of good 1 consumed domestically. Similarly, for good 2, the O_AD_{A2} part of O_AX_{A2} is required to supply domestic consumption. Hence, the parallelogram $O_AD_{A2}GD_{A1}O_A$ represents country A's factor requirement for producing its domestically consumed goods. This is referred to as the 'domestic parallelogram'. It follows that parallelogram GFEAG represents the factor requirement for producing exported goods — referred to as the 'export parallelogram'. Similarly, for country B, $O_BD_{B2}HD_{B1}O_B$ and HDEBH are its domestic and export parallelograms.

In contrast with the H-O model, trade in the Armington model is a two-way exchange by industry — country A exports both goods in exchange for both of the goods produced in country B. Illustrating this, note that country B's export parallelogram HDEBH can be turned into A's import parallelogram GNCMG. Similarly, country A's export parallelogram GFEAG can be turned into B's import parallelogram CKHJC. The two export and import parallelograms are joined at the point C, which represents the total factor requirements for producing the four goods consumed by each country in a free trade equilibrium. Note that C is located at the centre of the box diagram. This confirms that the two countries share equally all the resources used in production as well as consuming equally all the output produced, even though their endowments were not identical. The smaller country is the greater beneficiary of the gains from trade.[8]

Recall that in the goods market equilibrium shown in figure 2.1, in both countries neither country has net exports of any product. Nevertheless, the factor market equilibrium reveals net exchanges of factor services between the countries. This is indicated by the location of consumption point C being northwest of endowment point E. This implies that goods consumed by country A contain more capital and less labour than the goods it produces. The opposite is true for B. As a result, the relatively labour-abundant A is a net exporter of labour services, embodied in the goods it trades, while the relative capital-abundant B is a net exporter of capital services. Vanek's theory concerning the factor content of trade (Vanek 1968) holds in this Armington model with σ close to 1, even though the Heckscher-Ohlin theorem fails. This will be discussed in more detail in section 2.4.

Compared with autarky, free trade allows both countries to consume at point C, beyond their endowment point E (see figure 2.2). Consumption is a Pareto optimum in the sense that no redistribution of output between the countries could make one better off without making the other worse off.[9]

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[8] Nevertheless, both countries gain from the intra-industry trade.

[9] In figure 2.2, the consumption point is on the diagonal. However, it should be noted that, unlike in the H-O model, whether the equilibrium consumption point is on the diagonal or not is not an indicator of (Pareto) optimality. As we assume in this model that both countries' households allocate their budgets equally between domestically produced and imported goods, the equilibrium consumption point C happens to be on the diagonal. It can be shown that if a household's preferences are biased toward domestically produced goods, the equilibrium consumption point C will be below the diagonal. Likewise, with preferences biased toward imported goods, C will be above the diagonal.

2.3 Free trade equilibrium with other Armington elasticities

The general equilibrium properties of an Armington model are very sensitive to the magnitude of the Armington elasticities.[10] As demonstrated in the last section, when σ is close to 1, free trade does not close the gaps in inter-industry relative prices between the countries as it does in the H-O model. However, when σ is not equal to 1, free trade does alter inter-industry relative prices.

The Armington elasticity determines the properties of an Armington model through its influence on intra-industry relative prices, or the terms of trade. If the elasticity is not equal to 1, the intra-industry price ratios will not equate the value of the two countries' outputs. As a result, intra-industry trade will no longer be balanced, individually, in each good's own market. Net sectoral exports will occur and, in equilibrium, the surplus from one market will offset the deficit in the other. The imbalance in trade within each industry will trigger inter-industry relative prices to change, leading to resource reallocation and output changes in both countries.

That said, depending on whether σ is below or above 1, the general equilibrium properties of an Armington model differ sharply. These two cases are discussed in the following sections.

Small Armington elasticities (σ < 1)

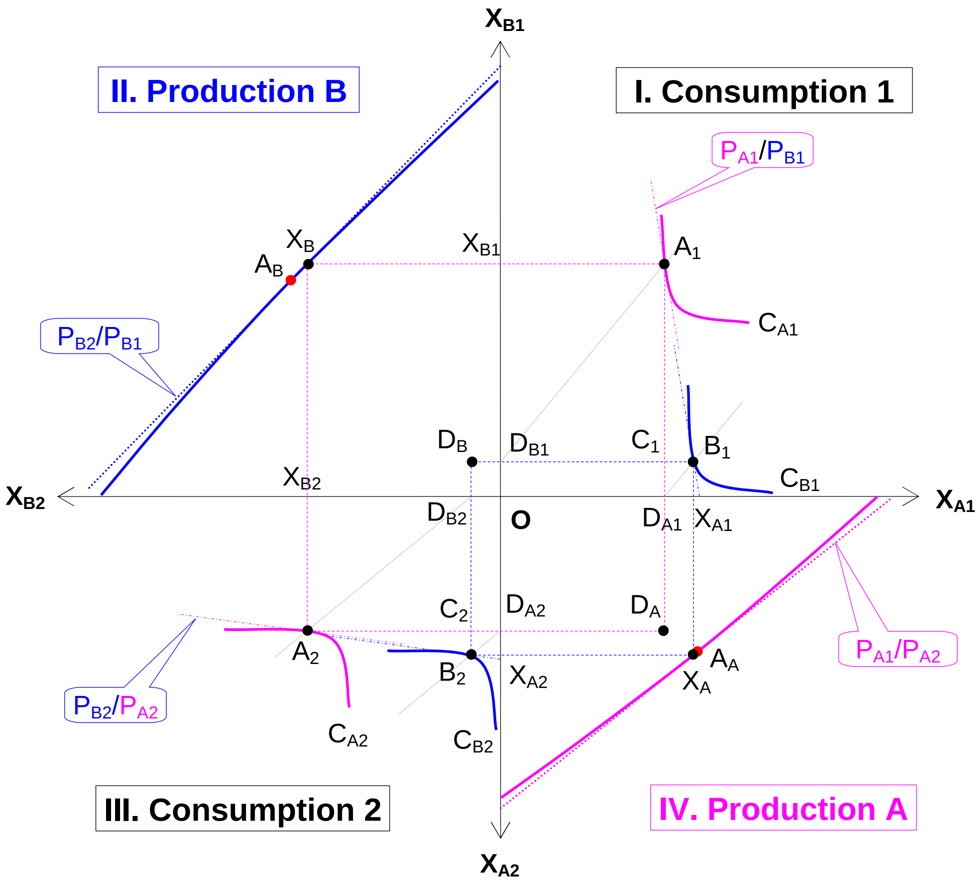
If the Armington elasticities are below 1, the relative demand for domestic and imported goods is very unresponsive to changes in relative prices. This implies that households are willing to pay a high price for an import if its supply is constrained. This gives the smaller country stronger market power, as the supplies of its goods are relatively small. As a result, it receives high prices for all its goods in a free trade equilibrium. The goods market equilibrium with a low Armington elasticity is shown in figure 2.3 where country A is assumed to be small, The price line for both good 1s in quadrant I is steeper than in figure 2.1, and for both good 2s in quadrant III, flatter. The terms of trade conditions for country A are more favourable than when σ is close to 1. This encourages country A to produce more

[10] It is not always clear to CGE model users that the general equilibrium properties of the Armington model depend to such an extent on the elasticity parameter. This is because most CGE models are calibrated to a given database. In the process of calibration, Armington elasticities are introduced into the model as given parameters, and are given virtually any value. The share parameters have to be calibrated to accommodate these artificially chosen Armington elasticities. This creates an impression that the choice of Armington elasticities does not change the database, which, in turn, obscures how sensitive the model's general equilibrium properties are to the particular Armington elasticity values that have been chosen.

good 2 and country B more good 1, as these goods are relatively scarce compared with the other goods. A's production point X_A moves down slightly from its autarky point A_A, and B's X_B up from its autarky point A_B, along their respective PPFs in quadrants IV and II.

Figure 2.3 **Free trade equilibrium in goods markets: the Armington model**

A 2-country 2-sector 2-factor model with σ = 0.1

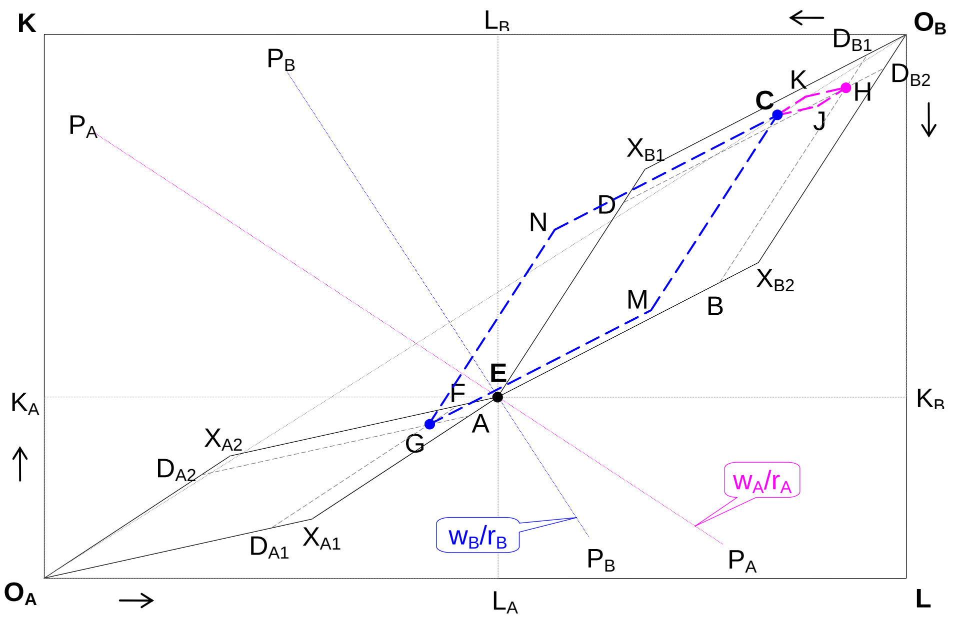


More importantly, the income generated by the favourable terms of trade conditions allows A to import large quantities of both goods from B, reciprocating with constant, or even decreasing, exports. Graphically, this implies that A's consumption point D_A moves closer to X_A in quadrant IV. This forces B's consumption point D_B closer to origin O. Country A tends to be a net exporter of good 2, and B a net exporter of good 1, because each country is a net exporter of the good that uses intensively its relatively scarce factor.

In factor markets (figure 2.4), the equilibrium consumption point C is located to the northeast of endowment point E on the diagonal, implying that A becomes a net importer of both factor services[11] and B, a net exporter of both factor services. In equilibrium, the biased terms of trade conditions allow A to consume a large share of the world output of all four goods. When σ is close to zero, the terms of trade condition becomes so favourable that A consumes almost all the output of the world while B consumes negligible quantities of each of the four goods.[12]

Figure 11 **Free trade equilibrium in factor markets: the Armington model**

A 2-country 2-sector 2-factor model with σ = 0.1



Large Armington elasticities (σ > 1)

If the Armington elasticities are greater than 1, price differences, within industries, across the countries are mitigated, so the intra-industry price ratios move toward 1.[13] Also, with intra-industry goods being better substitutes, the magnitude of the terms of trade effects tends to be smaller.

In figure 2.5, as σ increases to 500, the intra-industry price ratios in quadrants I and III are close to unity. Households switch their demand toward the cheaper differentiated product — the one that uses intensively the country's relatively abundant factor. The demand for A's good 1 and B's good 2 is greater than for their respective substitutes. This increases their inter-industry relative prices. In response,

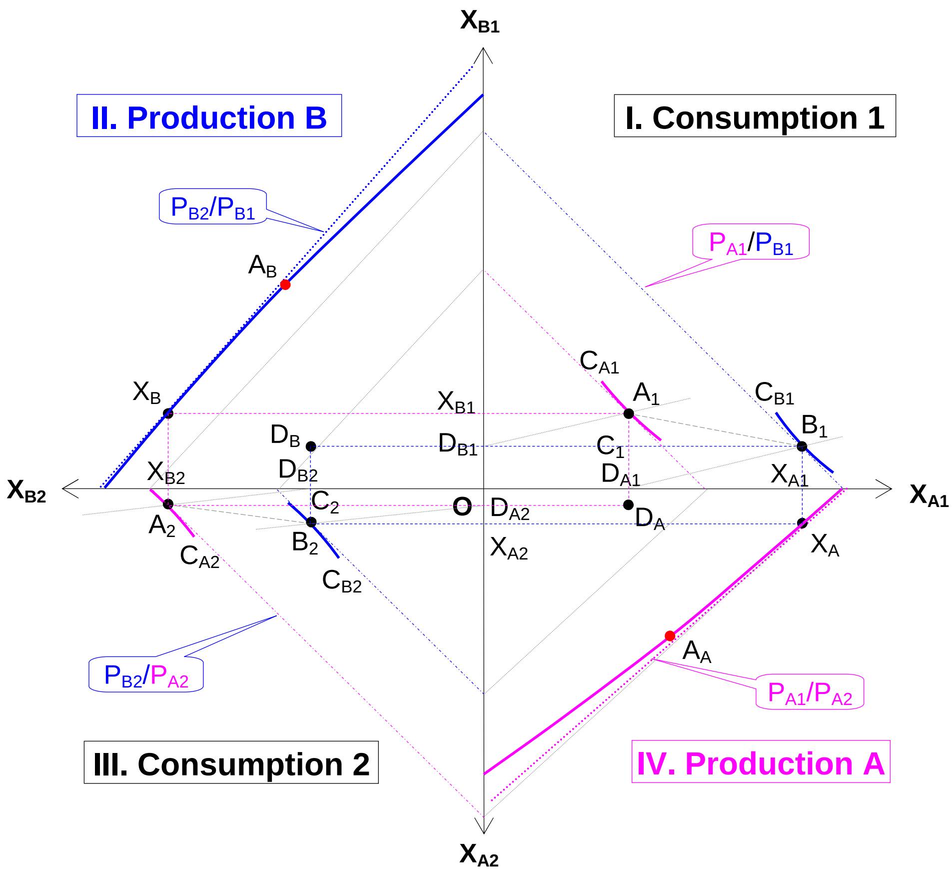
[11] Note that net imports of capital services are still greater than net imports of labour services.

[12] Despite consuming such small quantities, country B is better off for having traded with country A because, through trade, it obtains two extra, uniquely different and highly valued, goods.

[13] With a price ratio of 1, achieved under perfect substitution, the limit as σ approaches ∞.

resources are reallocated away from the production of other goods to increase the output of these two goods. Graphically, A (B)'s production point X_A (X_B) moves up (down) from its autarky point A_A (A_B) along its PPF in quadrants IV (II) in figure 2.5. As all goods are demanded equally in equilibrium, A tends to produce more of its good 1 than it is willing to consume, while B produces more good 2. As a result, A is a net exporter of good 1 and B a net exporter of 2. The production patterns of both countries display a mixture of both intra-industry and inter-industry specialisation. With such a large value of σ, inter-industry trade dominates in both countries.

Figure 12 **Free trade equilibrium in goods markets: the Armington model**
A 2-country 2-sector 2-factor model with σ = 500



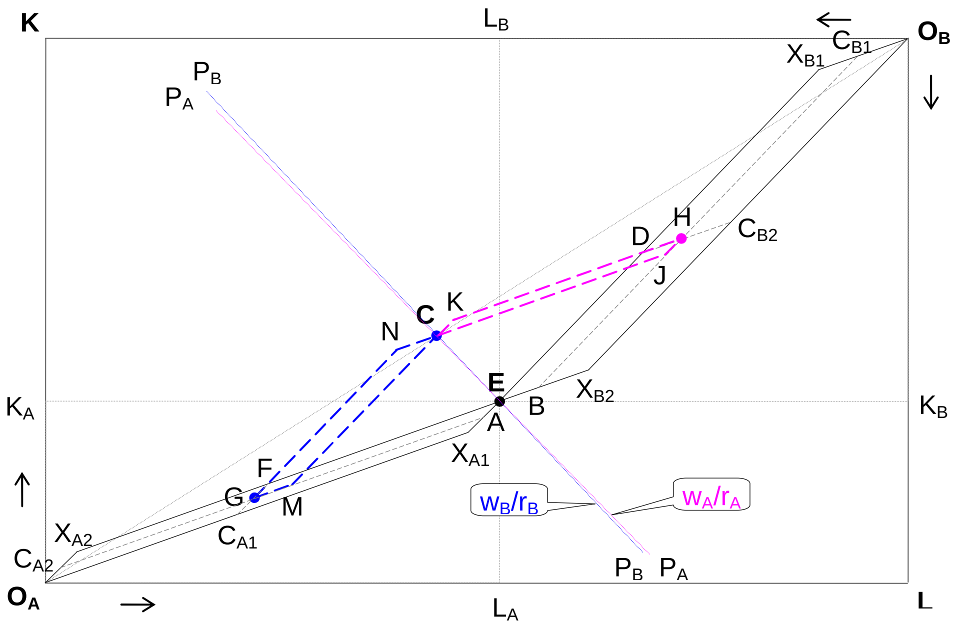
This figure also shows how both goods and factor markets clear in equilibrium. Goods market clearance can be seen from the trade balance condition. Take country A as an example. In equilibrium, A has a surplus in intra-industry trade in good 1 and a deficit in good 2. In quadrant I the intersections of the two price lines on the horizontal axis give the net export of good 1. The net import of good 2 can be measured on the vertical axis in a similar fashion in quadrant III. For trade to balance, the values of net exports and imports should be equal. This is confirmed in quadrant IV, in which two parallel budget lines link net exports and imports.

Moreover, this goods market clearing coincides with factor market clearing because the inter-industry relative price line is tangent at the equilibrium production point X_A on A's PPF. In this two-country model, country A's equilibrium also implies country B's equilibrium.

The changes in output and trade can also be seen in factor markets (figure 2.6). With a very large σ, consumption point C is located further to the southeast along the diagonal, implying that the factor content of imports differs significantly from that of exports. Because the goods from the same industries, across the countries, are better substitutes, both countries are better able to augment their initially scarce factor, by importing goods in which that factor is used more intensively. This can happen without an adverse terms of trade raising the cost of imports significantly, which would limit the quantities traded.

Figure 13 **Free trade equilibrium in factor markets: the Armington model**

A 2-country 2-sector 2-factor model with σ = 500



As domestic and imported goods become close substitutes, their price differences will diminish. More equal goods prices lead to more equal factor prices. As a result, the two countries tend to use similar capital–labour ratios in each industry and produce and export more of those goods that use their relatively abundant factors intensively. This implies more inter-industry trade and less intra-industry trade.

It should be noted that this general equilibrium production pattern is very similar to that of the H-O model (figures A.3 and A.4). In fact, this is expected because, as defined in section 2.1, this Armington model has the same specification on the supply side as the H-O model. The only difference between the two models is on the demand side: the consumer preference over the goods produced. It is expected that, when domestic and imported goods become close substitutes, the Armington model converges to the production pattern of a similarly defined H-O model. However, as the demand side specification is still dominated by the Armington assumption, consumers continue to allocate their income between domestic and imported goods according to the CES share parameters, despite their very close substitutability; consumption of each country-differentiated product is never zero. As a result, intra-industry trade in both sectors remains. Increasing elasticity parameters does not alter the properties of an Armington model. This demonstrates that the H-O model is not a special case of the Armington model.

CES elasticity and consumer preference

The previous section showed that when the value of Armington elasticity turns from greater to smaller than unity, the general equilibrium properties and the behaviour of the Armington model change completely. In this section, two cases are used to show that a small elasticity value does not just make domestic and imported goods less substitutable, as most might expect. In fact, it can actually turn consumer perceptions of domestic and imported goods from gross substitutes to gross complements. Goods in a CES function are *defined* as net substitutes. However, net substitutability does not prevent goods from being gross complements at lower elasticity values. It is this gross complementarity that causes the peculiar properties of the Armington models with low elasticities discussed above.

Case I: aggregate domestic and imported goods

The first case is an aggregated one. If the prices of all imports vary in the same proportion, the two-tier nested CES utility system can be treated as a single tier CES utility function, as if all imports and all domestic goods are aggregated into a single import and a single domestic good respectively. The aggregate utility function may have the following form,

$$U = [\alpha D^{\frac{\sigma-1}{\sigma}} + (1-\alpha) M^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \qquad (15)$$

Solving a consumer's utility maximisation problem yields uncompensated demands for the aggregated domestic and imported goods. To determine whether domestic and imported goods are substitutes or complements, one needs to sign the own- and cross-price elasticities of demands. If the cross-price elasticity of demand is positive, the domestic and imported goods are gross substitutes. Otherwise, they are gross complements.

The elasticity of the uncompensated demand for domestic good D with respect to the price of imported good P^m, or the cross-price elasticity, can be derived as follows

$$\varepsilon^{md} = \frac{\partial D}{\partial P^m}\frac{P^m}{D} = -(1-\sigma)\frac{(1-\alpha)^{\sigma} P^{m\ 1-\sigma}}{\alpha^{\sigma} P^{d\ 1-\sigma} + (1-\alpha)^{\sigma} P^{m\ 1-\sigma}} \qquad (16)$$

It can be seen that if $\sigma > 1$, then $\varepsilon^{md} > 0$, and both goods are gross substitutes. If $\sigma < 1$, domestic and imported goods become gross complements. If σ is close to 1, the two goods are neither gross substitutes nor gross complements.

This can also be shown in a diagram. In the first panel of figure 2.7 with $\sigma > 1$, D and M are gross substitutes: when P^m falls, the demand for M declines and the demand for D increases. The price expansion path (PEP)[14] is downward slopping. The distance between A and B captures the substitution effect, while the distance between B and S captures the income effect. The income effect allows the consumer to jump to the higher indifference curve C′.

In the second panel with $\sigma < 1$, D and M are gross complements: the demand for D responds in the same way as the demand for M does to a fall in P^m. The PEP is therefore upward slopping. The income effect more than offsets the substitution effect.[15]

Case II: disaggregate domestic and imported goods

In the second case, only the price of one imported good, say P_1^m, varies. We will determine under what condition D_1 and M_1 are substitutes. In this case, both tiers of the nested CES utility functions determine the response of consumers. In the upper tier, the utility function is defined over composite goods

$$U = (\beta_1 C_1^{\frac{\mu-1}{\mu}} + \beta_2 C_2^{\frac{\mu-1}{\mu}})^{\frac{\mu}{\mu-1}} \qquad (17)$$

Solving the consumer's utility maximisation problem yields an uncompensated demand for composite goods C_i.

14 Also known as the offer curve.

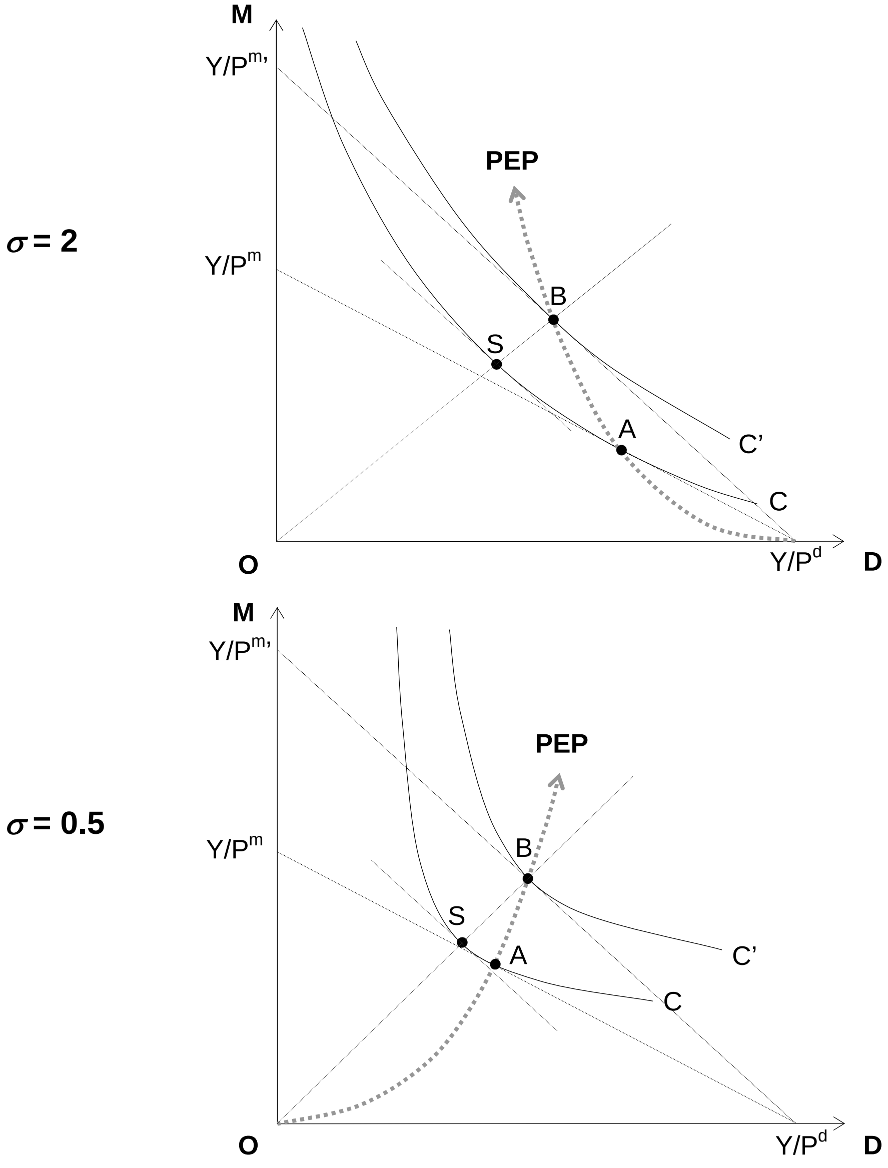
15 It can be shown that when σ is close to 1, the income effect of a fall in P^m just offsets the substitution effect. PEP becomes a vertical line, implying that the two goods are independent; neither gross substitutes nor gross complements.

In the lower tier, the sub-utility function is defined over a domestic good and an imported good,

$$C_i = [\alpha_i D_i^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - \alpha_i) M_i^{\frac{\sigma_i - 1}{\sigma_i}}]^{\frac{\sigma_i}{\sigma_i - 1}} \qquad i = 1, 2 \qquad (18)$$

For a given C_i, solving the expenditure minimisation problem yields compensated demands for domestic good *i* and imported good *i*.

Figure 14 **CES elasticity and consumer preference: Case I**



To determine if domestic and imported goods in sector *i* are gross substitutes or complements, one needs to check the value of the cross-price elasticity of demand for D_i with respect to P_i^m. The cross-price elasticity of the compensated demand for D_1 can easily be derived from the lower tier CES utility function as follows,

$$\varepsilon_1^{dm} = \frac{\partial D_1(C_1, P_1^d, P_1^m)}{\partial P_1^m} \frac{P_1^m}{D_1} = \sigma_1 \frac{(1-\alpha_1)^{\sigma} P_1^{m\ 1-\sigma}}{\alpha_1^{\ \sigma} P_1^{d\ 1-\sigma} + (1-\alpha_1)^{\sigma} P_1^{m\ 1-\sigma}} \tag{19}$$

It shows that if $\sigma_1 > 0$, $\varepsilon_1^{dm} > 0$, confirming that good 1s are net substitutes. However, this cross-price elasticity is derived from a compensated demand function, which itself is derived from the lower tier sub-utility function. As the composite good C_1 is given, the consumer can only respond to a rise in P_1^m by moving along the indifference curve. D_1 and M_1 are always net substitutes so long as the curve is convex, that is, $\sigma_1 > 0$. However, this calculation fails to capture the impact that the upper-tier optimisation may have on the consumer's demand for domestic and imported good 1s. In the upper tier the consumer maximises utility subject to a budget constraint. The demands for composite goods are uncompensated. A price change will cause not only a substitution effect but also an income effect, which affects the lower tier substitution.

As C_1 is a function of income Y and composite prices P_1^c and P_2^c, which are CES averages of P_i^d and P_i^m, the uncompensated cross-price elasticity from a nested demand function for D_1 is

$$\varepsilon_1^{dm} = \frac{\partial D_1(Y, P_1^d, P_1^m, P_2^d, P_2^m)}{\partial P_1^m} \frac{P_1^m}{D_1} = (\sigma_1 - \mu)\frac{P_1^m M_1}{P_1^c C_1} - (1-\mu)\frac{P_1^m M_1}{Y} \tag{20}$$

where $\frac{P_1^m M_1}{P_1^c C_1}$ and $\frac{P_1^m M_1}{Y}$ are the shares of import 1 in the expenditure on composite good 1 and in total budget, respectively. For D_1 and M_1 to be gross substitutes, that is, $\varepsilon_1^{dm} > 0$, the CES elasticity σ_1 has to satisfy the following condition

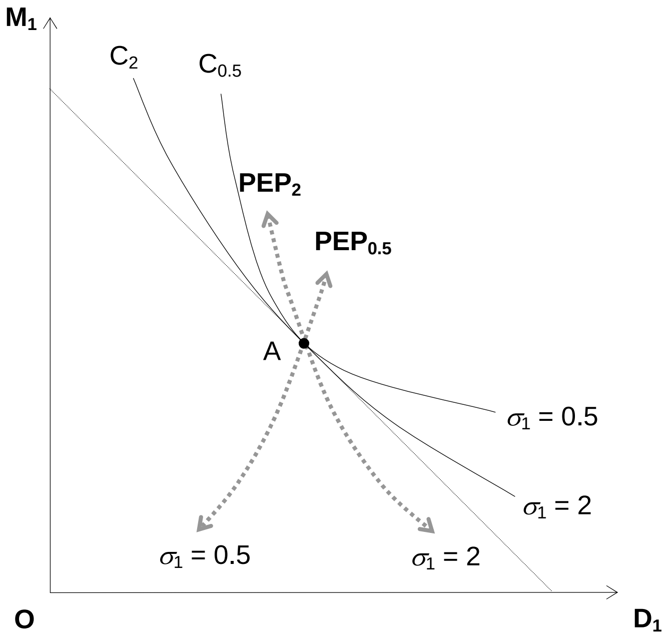
$$\sigma_1 > \frac{P_1^c C_1}{Y} + \mu \frac{P_2^c C_2}{Y} \tag{21}$$

where $\frac{P_i^c C_i}{Y}$ is the budget share for composite good i. Given μ and the budget shares the values of σ_1 for positive ε_1^{dm} can be specified. For example, if μ closely approximates unity (a Cobb-Douglas function), σ_1 must be greater than 1. Otherwise, the two good 1s become gross complements even though they are defined as net substitutes.

Figure 2.8 plots two PEPs for σ_1 = 0.5 and σ_1 = 2. In the former case the PEP is upward slopping, indicating that D_1 and M_1 are gross complements. In the latter case the PEP is downward slopping, implying that D_1 and M_1 are gross substitutes.

The above analysis suggests that there are additional constraints on the setting of CES elasticity in an Armington model. For differentiated goods to be gross substitutes, as a rule of thumb, the CES elasticity parameter for domestic and imported goods in a nested Armington demand system needs to be set above unity. Otherwise, domestic and imported goods will be seen by consumers as gross complements.

Figure 15 **CES elasticity and consumer preference: Case II**



Implications for Armington models

The previous section showed how the choice of Armington elasticity alters the general equilibrium properties of an Armington model, especially when σ crosses the critical value of unity: it switches consumers' perceptions. If domestic and imported goods are complements, a change in their relative price will cause the demands for both goods to move in the same direction. This has a profound impact on terms of trade conditions, or intra-industry relative prices.

Armington models with domestic and imported goods being gross complements tend to have peculiar properties. In these models the terms of trade conditions are extremely unbalanced between countries and, more specifically, biased in favour of smaller countries. This tends to create a set of output, consumption and trade patterns based almost entirely on relative market power between countries, rather than on relative endowments of productive factors.

Although most CGE models use above-unity Armington elasticities for most industries, low Armington elasticities can still be found in certain industries. There are reasons to believe that the Armington elasticities used in current CGE models may be inconsistent with their database. This is because the magnitude of Armington elasticities is likely to be affected by the level of aggregation of an industry.[16] It is likely that aggregation reduces the substitutability between differentiated goods because it reduces the similarity between domestics and imports. Moreover, empirical evidence seems to indicate that differences in the composition of imported and domestic products may increase over time, due to intra-industry specialisation brought about by free trade and globalisation. A study using a highly aggregated 2-digit data set for 11 French industries suggests that, although the average elasticity was about 0.84 over the period 1976-81, it declined almost continuously to 0.21 during 1982-97 (Welsch 2006). For such highly aggregated databases and low Armington elasticity estimates, one should consider alternative model structures to avoid the peculiar behaviour and results that arise from low elasticities.

In order for Armington models to behave reasonably, domestic and imported goods have to be gross substitutes in consumption. Net substitutability is not sufficient to prevent the peculiar behaviour discussed above. This requires Armington elasticities to be reasonably large, often well above unity. Armington models are probably not suitable for highly aggregated databases because the implied low CES elasticities often lead to gross complementarity and unusual responses to changes in trade policies

2.4 The Armington model and basic trade theorems

The Heckscher-Ohlin theorem, the Factor-Price-Equalisation theorem, the Stolper-Samuelson theorem and the Rybczynski theorem underpin the theory of international trade. These trade theorems are often used to interpret Armington model results. However, they are all based on the neoclassical trade model or the H-O model. The introduction of Armington preferences and the existence of intra-industry trade alters the properties of a neoclassical trade model to the extent that the theorems often do not hold.

[16] Also, if two goods are close substitutes, the tariff quickly becomes prohibitive and no imports would be recorded in the database. See also p38.

The Heckscher-Ohlin theorem in the Armington model

The Heckscher-Ohlin (H-O) theorem is one of the most influential theorems in the theory of international trade. It states that countries tend to export products that use more intensively those factors in which they have a relative abundance.

This theorem is derived from an H-O model, in which homogenous goods are produced in different countries, so a direct link can be established between factor intensities in the goods a country exports (or imports) and the relative factor abundance of that country's endowment. In the Armington model, however, because trade is intra-industry in nature and traded goods are differentiated by country of origin, a labour-abundant country can export and import, simultaneously, both labour- and capital-intensive goods. Consequently, the link between the factor intensity of a country's exports (or imports) and the relative abundance of factors within that country becomes tenuous or even lost.

In the Armington model the pattern of gross trade cannot be predicted simply from the production technologies and relative factor endowments of each country. Might the H-O theorem still predict the pattern of net trade? The answer from earlier sections is that the applicability of the H-O theorem to net trade depends on the value of the Armington elasticities.

If the Armington elasticities are below 1, the H-O theorem does not hold even for net trade. In such cases, there can even be a reversal of the H-O theorem result. A labour-abundant country (country A) can become a net exporter of capital-intensive goods while a capital-abundant country (country B) becomes a net exporter of labour-intensive goods. This failure of the H-O theorem in the Armington model can be explained by the imbalance in the terms of trade between the two countries, which is in turn the result of gross complementarity. Only if the Armington elasticities are above 1, which implies that all goods are gross substitutes, can a labour-abundant country have a surplus in its intra-industry trade in labour-intensive goods and a capital-abundant country have a surplus in its intra-industry trade in capital-intensive goods. In these cases, the H-O theorem holds in net trade.

The Factor Price Equalisation theorem in the Armington model

The Factor-Price-Equalisation (FPE) theorem is another important result derived from the H-O trade model. In the H-O model, with no transport costs, in free trade, the 'Law of One Price' (LOP) holds, so the price of any good is equalised across all countries. This, and assuming identical technologies across countries, leads to factor price equalisation. The FPE theorem embodies the principle of comparative advantage. The costs of production are directly comparable between countries because the same goods are produced in all countries. In free trade, competition between firms producing the same good in different countries leads to each country concentrating on producing the goods in which it has a relative cost advantage, a comparative advantage. With resources allocated efficiently in each country, world output of all goods can be maximised. Comparative advantage is at the core of trade theory, but this explanation for trade is absent in the Armington model.

Armington models assume that domestic and imported goods are imperfect substitutes in consumption. Since the prices of differentiated goods do not equalise,[17] there is no factor price equalisation. Therefore, the costs of differentiated goods from different countries are not directly comparable, even if they are produced with the same technologies. If every country produces a unique product, there will be no international market where different countries' firms compete with each other. The concept of comparative cost advantage becomes irrelevant in the Armington model.

In the Armington model, free trade does not necessarily reduce differences in goods and factor prices. Nevertheless, depending on the Armington elasticity used, free trade can affect the differences in good and factor prices between countries. With $\sigma < 1$, implying gross complementarity, free trade actually increases inter-industry price differentials in the Armington model. With $\sigma > 1$, implying gross substitutability, free trade reduces price differentials. In the simple Armington model shown above, good and factor prices equalise only if both countries have identical factor endowments. In other words, the factor price equalisation (FPE) 'cone',[18] which is used in the H-O model, collapses to a point in the centre of the box diagram (figure 2.2). Also noteworthy is that, in contrast with the H-O model, relative price equalisation in an Armington model does not necessarily imply equalisation in the levels of the prices involved. Under certain configurations of household preferences, the relative prices of goods and factors could be made equal. However, the underlying price levels are not equal because of the imbalanced terms of trade condition.[19] In the H-O model, free trade leads to price equalisation by supply-side competition. The Armington model does not have this mechanism for price equalisation. When free trade in the Armington model does result in price

[17] The prices of composite goods may equalise in a 2×2×2 Armington model. This is because of symmetry in domestic and import prices, that is, one country's domestic price is equal to the other country's import price. However, they do not equalise in higher dimension Armington models with a two-tier substitution structure, because this price symmetry no longer exists.

[18] The FPE cone is the set of all possible factor endowments that lead to a factor price equalisation equilibrium. Outside this cone, not all countries produce all goods, hence, it is no longer meaningful to talk of the price of such a good having equalised across all countries.

[19] In this regard, the box diagram such as figure 2.2 may not be sufficient to demonstrate factor price equalisation in an Armington model because it shows only relative prices, not price levels.

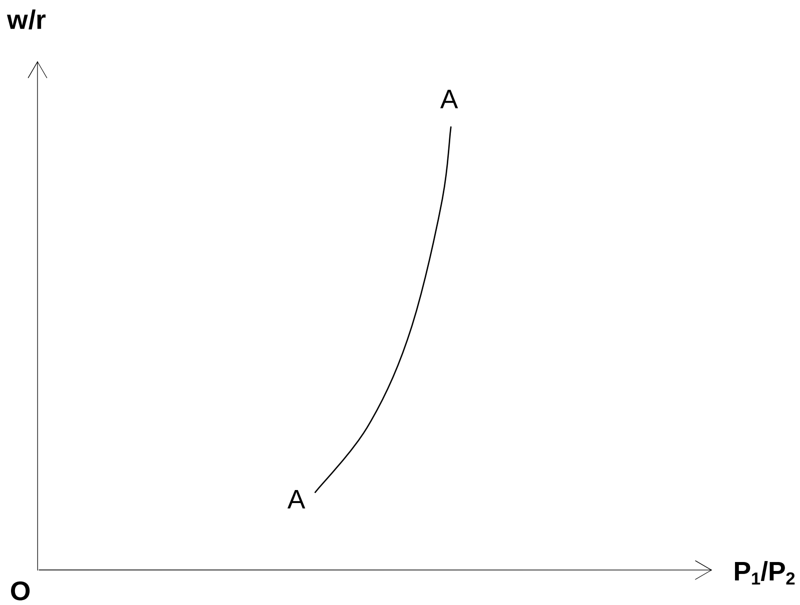
equalisation, it does so by coincidence, and only with a specific configuration of the model's parameters and endowments, and not through head-to-head competition in the same market between firms of different countries.

The Stolper-Samuelson theorem in the Armington model

The Stolper-Samuelson theorem refers to the link between factor prices and goods prices in an economy. In a two-good two-factor H-O model, the Stolper-Samuelson (S-S) theorem states that an increase in the price of a good raises real returns to the factor that is used intensively in its production and lowers real returns to the other factor.

The link between factor and goods prices can be illustrated in figure 2.9. The horizontal axis measures the price of good 1 relative to good 2 and the vertical axis, the wage rate relative to the rental rate of capital. Using our previous two-country two-good model assumptions, good 1 is labour-intensive while good 2 is capital-intensive. According to the S-S theorem, an increase in the relative price of the labour-intensive good always causes a greater increase in the wage-rental ratio, so that both labour benefits and capital owners lose in real terms. Graphically, the curve AA traces the changes in relative factor prices due to changes in relative goods prices. The curve is upward slopping with a slope greater than 1: the relative returns to labour rise faster than the relative price of the labour-intensive good.

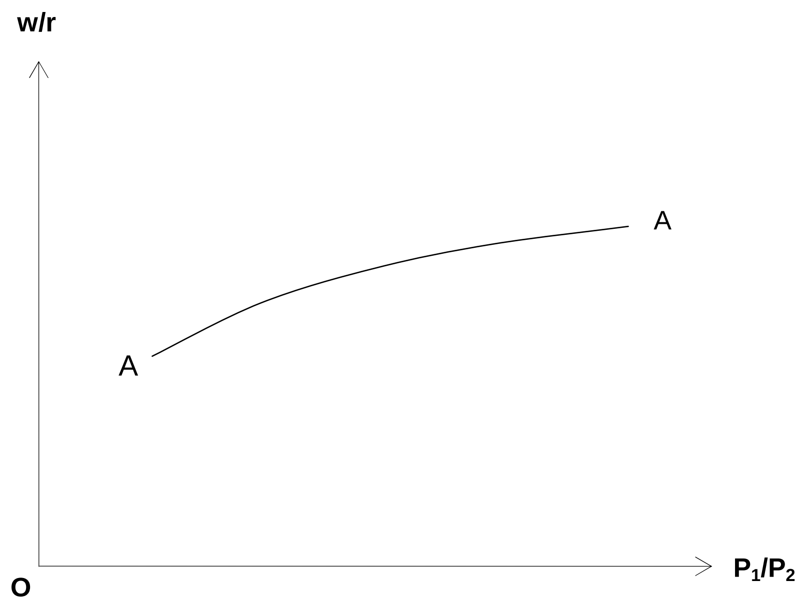
Figure 16 **The Stolper-Samuelson theorem in the H-O model**



The S-S theorem holds in the Armington model when the wage-rental ratio is compared with the price ratio for a country's own domestic goods. It does not hold when compared with the price ratio for the composite goods. This is because Armington preferences alter the link between goods prices and factor prices that prevails in the H-O model. In the Armington model, an imported labour-intensive product is combined with its domestically produced counterpart to form a composite product. Since only part of the composite is produced domestically, the effect of a change in the composite price on domestic factor prices is reduced.

This relationship in an Armington model, between the relative prices of factors and the relative prices of composite goods, can be seen in figure 2.10. The curve AA is much flatter than that in figure 2.9, indicating that a large rise in the relative price of a composite good is associated with a small rise in the relative price for the factor used intensively in producing that good. It is the composite good, rather than the original goods, that enter the top level of the utility function. The welfare and distributional implications of the S-S theorem will therefore be altered in the Armington model. For example, a rise in the relative price of good 1 may not benefit the owner of labour because the rise in wage rate may be offset by a greater rise in goods prices.

Figure 17 **The Stolper-Samuelson theorem in the Armington model**

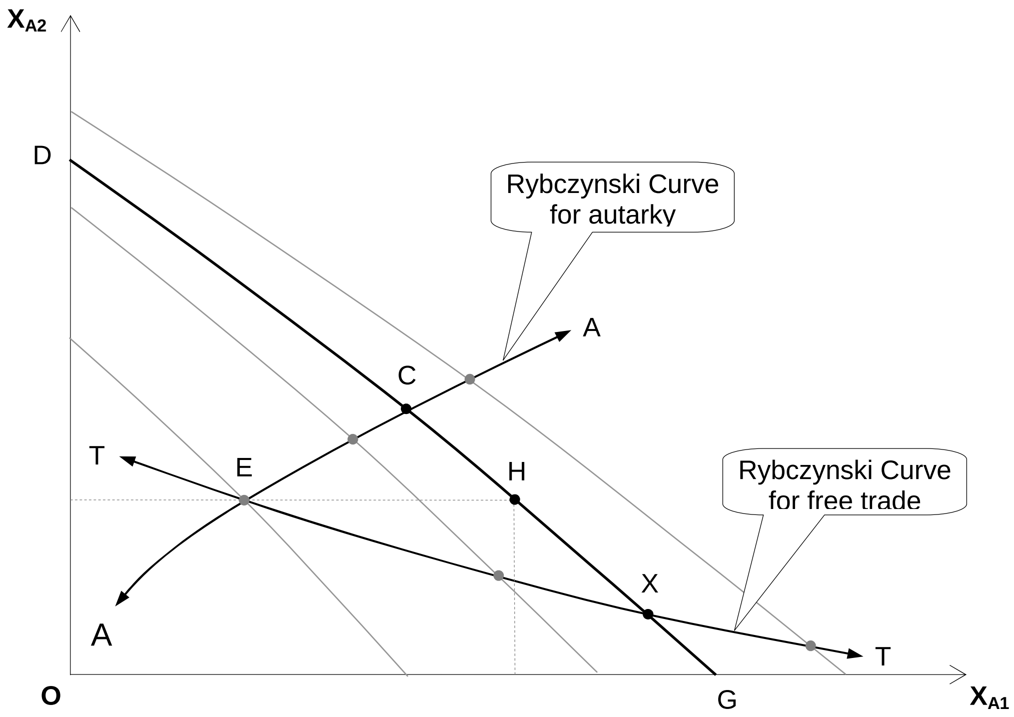


The Rybczynski theorem in the Armington model

The Rybczynski theorem states that increasing a factor endowment leads to an increase in the output of the good that uses that factor intensively and a decrease in the output of the other good, provided that goods prices are constant. The Rybczynski theorem has been applied to the case of international trade, in which the economy is 'small' so that product prices can be regarded as fixed. In the Armington model, however, there is no 'small country'. It is worthwhile to explore whether the Rybczynski conclusions still hold in such a world.

The Rybczynski conclusions can be illustrated in a two-good two-factor model. Figure 2.11 shows the production possibility frontier (PPF) for two goods, borrowed from country A's PPF in figure 2.1. Good 1 is labour-intensive. The initial PPF is shown as curve DG. With its initial endowment of factors, country A can produce more good 1 than good 2.

Figure 18 **Autarky and free trade Rybczynski Curves in the H-O model**



If the supply of labour increases, the PPF will shift out — the output of labour-intensive good 1 expands more rapidly than that of capital-intensive good 2. Similarly, if the supply of labour decreases, the PPF will contract. Many different PPFs could be drawn in the figure, each associated with a particular factor endowment.

Whether the Rybczynski conclusions hold depends on what happens to the relative price of the two goods. If the relative price of good 1 to good 2 is fixed, constant returns to scale technologies ensure that the Rybczynski conclusions must hold. Graphically, this means that the locus of all tangent points of a constant price line on a series of expanding and contracting PPFs is a downward-sloping straight line (not shown). In other words, the so-called 'Rybczynski curve' is linear.

In a general equilibrium model, however, the supply of a factor will affect factor prices, which, in turn, affect goods prices. If labour supply increases, for example, the wage rate will decline relative to the rental rate of capital. As a result, the price of labour-intensive goods will fall relative to capital-intensive goods. Graphically, the relative price of good 1 to good 2 will become flatter, which reduces the Rybczynski effect on output. If these price changes are too strong, the Rybczynski conclusions may not hold.

With an H-O model, the Rybczynski conclusions hold in free trade. This is because free trade allows a country to reallocate factors so as to increase the output of its comparative advantage goods. As a result, free trade eases the downward pressure that an increase in a factor puts on its own price, which ensures that the Rybczynski effect holds. The free trade Rybczynski curve is plotted in figure 2.11 as the downward sloping curve, TT.

In autarky, however, an increased supply of a factor has to be absorbed by domestic industries through large adjustments in their prices. As a result, the Rybczynski effect on outputs may not prevail. Such a situation is shown in figure 2.11. The upward sloping Rybczynski curve, AA, indicates that the output of both labour- and capital-intensive goods increases as labour supply increases. This results from a large fall in the wage-rental ratio and in the relative price of the labour-intensive good 1. In this case, the Rybczynski conclusions do not hold entirely, but neither are they reversed.[20]

As we discussed earlier, except with very high elasticities, the production pattern of an Armington trade model resembles autarky in an H-O model.[21] Graphically, this implies that the free trade equilibrium output point is not far below point C on the initial PPF as shown in figure 2.11. As a result, the Rybczynski conclusions do not hold in most Armington models. They only hold when the Armington elasticity used is great enough for the initial equilibrium output point to be located below point H on the initial PPF. Point H is determined by the intersection of the two Rybczynski curves at point E on a specific PPF, which results from equal supplies of labour and capital. In other words, the Rybczynski conclusions are conditional on the substitution elasticity used in the Armington model.

20 Under some unusual circumstances, the Rybczynski conclusions can be reversed. See, for example, Sonnenschein (2005).

21 In fact, the similarity between an Armington model and a closed economy model is also to be found in the general equilibrium conditions required to solve these models. See sections 2.1 and A.1 (appendix A).

Since the general equilibrium properties of the Armington model differ significantly from the H-O model, it comes as no surprise that familiar trade theorems derived from the H-O model lose their explanatory power in Armington models. For that reason, these theorems can be misleading when used to interpret Armington model results.

3 Implications for policy analysis

The last chapter outlined the distinctive features of the Armington model in a free trade equilibrium. This chapter reveals that introducing Armington preferences also alters the familiar behaviour of a neoclassical trade model, in particular its response to a policy change. To keep the exposition simple, an *ad valorem* import tariff change is used as an example of a policy change. The Armington model from the previous chapter is used to explain how different countries might respond to this policy change and how the model reaches a tariff-distorted equilibrium. The welfare implications of the policy change for different countries are outlined and these results are shown to be significantly different from the familiar Heckscher-Ohlin story.

3.1 Properties of tariff-distorted equilibria

Trade is intra-industry in the Armington model, in contrast with the H-O model where trade is inter-industry. As goods are imported in every industry, a tariff can be imposed on the imports of any industry. Consequently, the responses of an Armington model to different import tariffs can vary considerably, and need to be discussed on a case-by-case basis. Two types of import tariffs are discussed below, a uniform tariff and a discriminatory tariff.

A uniform tariff

A uniform tariff is imposed on all imports equally. If both countries share identical preferences over domestically produced and imported goods, then the effects of a uniform tariff on each country depend on the size of Armington elasticities. Three cases are discussed, when σ is approximately equal to 1, smaller or greater than 1.

Case 1: Unitary elasticity ($\sigma \approx 1$)

The free trade equilibrium with a unitary elasticity Armington model was discussed in chapter 2 (see figures 2.1 and 2.2). If country A imposes the same tariff on all its imports, the domestic prices of imports rise equally. Due to the unitary elasticity, expenditure shares for the domestically produced and the imported goods remain unchanged. The tariff improves the terms of trade for A, that is, intra-industry relative prices rise in both goods markets, and the prices of A's exports increase relative to B's. With the increased income, A can purchase the same quantity of imports while reducing its exports. Industry trade remains balanced in individual markets so both countries' domestic inter-industry relative prices as well as output of the two goods remain unchanged. The only effect of the uniform tariff under a unitary elasticity is to allow A to reduce its exports. A still buys the same volume of imports because of its improved terms of trade. At high tariff rates, the domestic demand point D_A approaches output point X_A in quadrant I of figure 2.1. Country A exports negligible quantities of both goods, yet is still able to import the same volume of both goods from B, due to the extremely favourable terms of trade effect.

Case 2: Small elasticities ($\sigma < 1$)

With Armington elasticities less than 1, the domestic and imported goods become gross complements. As the income effect dominates the substitution effect, the terms of trade effect of a tariff is very large. A tariff introduced by the smaller country (country A) will reinforce its market power. The terms of trade improvement is so large that the large increase in its income allows it to increase its imports of both goods while reducing its exports. As the tariff increases, the extreme terms of trade effect eventually allows A to buy almost all of B's output. Graphically, not only does D_A converge to X_A in quadrant I of figure 2.1, but D_B converges to the origin O in quadrant III.

Case 3: Large elasticities ($\sigma > 1$)

With Armington elasticities greater than 1, the domestic and imported goods are seen by consumers as gross substitutes. As the substitution effect dominates, the terms of trade effect of a uniform tariff tends to be smaller than in the previous case. For a given Armington elasticity, if the tariff rate is sufficiently high, output in each country starts to resemble its autarky pattern. That is, a country in the Armington model can be forced back to its autarky position through a prohibitive tariff.[1]

The peculiar features of the Armington model's response to a tariff can also be illustrated using an offer curve diagram. As Zhang (2006) demonstrates in a simple pure exchange Armington model, with $\sigma > 1$, the Armington model's offer curves have the same shape as in the H-O model. When σ approximates 1, the offer curve becomes a vertical line. With $\sigma < 1$, the shape of the offer curve is reversed. This

[1] When the elasticity is less than 1, no tariffs, however high, can force the country's output back to its autarky levels. Instead, a high tariff forces a country further away from autarky. This is different from what is expected in an H-O model.

unusual result is due to the extreme terms of trade effects created by small Armington elasticities.

A discriminatory tariff

In the Armington model, a discriminatory tariff can also be imposed on selected imports. Its effects are difficult to generalise. Effects depend not only on the model's configuration but also on which industry's imports are tariffed. In the H-O model, a tariff on some imports will always reduce the demand for those imports and increase the domestic production of import-competing goods. In the Armington model, this need not be the case.

For example, with σ close to 1, a tariff on an import will not change the output of its domestically produced substitute. The foreign country bears the full burden of output adjustment. If $\sigma < 1$, a discriminatory tariff on one import will even *reduce* the demand for, and the supply of, the domestic substitute. It will also increase the demand for, and the supply of, the other good. *Both* countries will produce and consume more of the goods that are not tariffed. This unusual response is yet again caused by the extreme terms of trade effects, which allow the tariffing country to import a large volume of the non-tariffed good, in exchange for a reduced volume of exports.

If $\sigma > 1$, a discriminatory tariff in an Armington model will generate the same response as that expected in an H-O model. That is, both the demand for, and the supply of, the domestic goods increase while demand for the tariffed import declines. In this situation, a finite prohibitive tariff rate exists. At this rate, imports of the tariffed good fall to zero and the output of its domestic counterpart increases to a maximum level.

Tariffs in Armington models with large elasticities

It is commonly believed that large substitution elasticities make an Armington model behave like an H-O model. However, this belief is unfounded. It is true that, in free trade, inter-industry trade dominates intra-industry trade. However, high elasticities do not eliminate all intra-industry trade in free trade. This is because consumers still allocate their incomes to domestic and imported goods according to the CES shares, no matter how substitutable they are. This distinguishes the Armington model from the H-O model.

Moreover, as σ increases, the general equilibrium solution for an Armington model becomes very sensitive to any price change. Even a small uniform tariff can eliminate the imports of the goods that use intensively a country's relatively scarce factor. As a result, there is only one-way trade in each industry: each country exports only the good that uses intensively its abundant factor. Inter-industry trade dominates only when a tariff is in place. This response is also different from that of an H-O model.

In an applied Armington model, as section 2.3 shows, high substitution elasticities may be inconsistent with the underlying database. Applied models embody two-way trade flows in their databases. This is why the effect of a tariff in an Armington model is always different from what might be anticipated in an H-O model, no matter how large the elasticities used. In fact, the magnitude of Armington elasticities is constrained by the tariffs and imports embodied in the database. For example, a large elasticity is inconsistent with imports that have large tariffs. This is because importing under a high tariff implies a low substitutability between the import and its domestic counterpart. If two goods are close substitutes, the tariff quickly becomes prohibitive and no imports would be recorded in the database.

The magnitude of parameters needs to be consistent with the model's theory and database. Unless the theoretical structure and the database are altered, the behaviour of a general equilibrium model will not change just by changing parameter values .

3.2 Implications for welfare measurement

The unique properties of the Armington model have significant implications for policy analysis based on their use. Introducing intra-industry trade in differentiated products means that the Armington model includes sources of gains from trade that are different from those explained by the H-O model. In the H-O model, gains from trade are due to an optimal reallocation of resources with an expansion in the total output of all industries. These are real increases in physical outputs. In the Armington model, however, gains from trade are due almost entirely to the redistribution of the outputs of differentiated goods between countries. Depending on how substitutable these goods are, moving from autarky to free trade in an Armington model need not result in an expansion of the world output of all industries (chapter 2). Whether world output of an industry increases or decreases largely loses its importance as a welfare indicator. As shown earlier, without output changes, households still gain from intra-industry trade in differentiated goods.

Furthermore, unlike in the H-O model, free trade is not a substitute for factor mobility. There are additional gains from factor mobility in an Armington model. These gains arise from an optimal reallocation of resources *across countries*. In this section, a simple Armington model is used to explain where these gains actually come from.

Welfare gains from trade

The effect of removing a tariff can be decomposed into exchange gains and production gains. Exchange gains arise from redistributing given outputs across countries to satisfy household preferences. Production gains arise from the reallocation of resources across industries within each country, consistent with its comparative advantage. In contrast to exchange gains that are obtained using given outputs, production gains are obtained by increasing total output. Before attempting to outline the welfare implications of a tariff in an Armington model, the welfare effects in an H-O model are outlined for comparison.

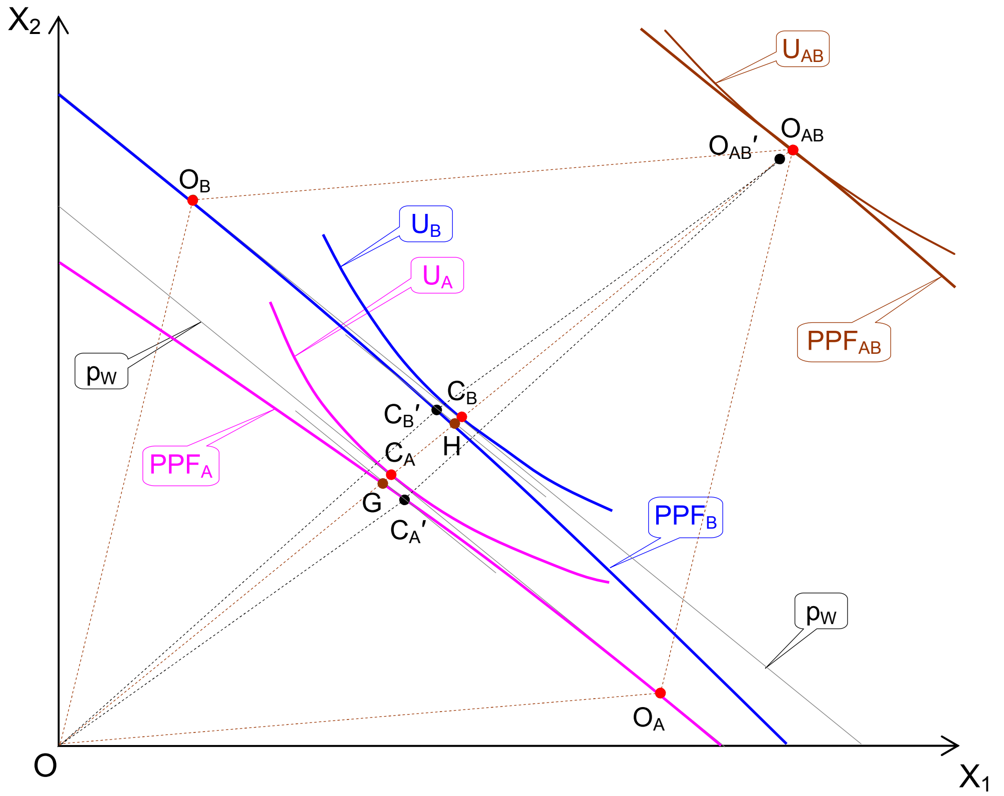
Welfare effects in an H-O model

Again, using a two-country two-good two-factor H-O model, the welfare effects of a tariff are depicted in figure 3.1. In free trade, both A and B produce at points O_A and O_B on their production possibility frontiers, PPF_A and PPF_B, and consume at points C_A and C_B on their own indifference curves, U_A and U_B. For each country, the world equilibrium relative price line (p_W) is tangent to the PPF and the indifference curve is at the equilibrium production and consumption points. In free trade, country A (B) exports good 1 (2) and imports good 2 (1). As both countries use identical technologies in production and have identical preferences in consumption,[2] an aggregated PPF and an aggregated indifference curve (PPF_{AB} and U_{AB}) are tangent at point O_{AB}. Output and welfare for the world as a whole is maximised at O_{AB}. Utility is maximised by maximising output in the H-O model.

When A imposes a tariff on its imports of good 2, the domestic price of good 2 increases and the demand for domestic good 2 falls. In response, A reduces its imports of good 2 and increases its own production of good 2. Country A’s production point moves closer toward C_A', its autarky equilibrium point. As the tariff also distorts the domestic price for households, A’s consumption point also moves toward C_A'. A similar process occurs in country B. Both B’s production and consumption points move toward its autarky equilibrium point C_B'. As the consumption points move, each country’s indifference curve shifts backward, indicating a decline in utility. When A’s tariff reaches a prohibitive level, both countries’ indifference curves become tangent to their own PPFs at points C_A' and C_B'.

[2] Assume also that each country’s utility function is homothetic.

Figure 3.1 **Welfare effect of a tariff on country A's imports in an H-O model**
Goods markets



Looking at the world as a whole, total output of both goods is reduced from O_{AB} to O_{AB}', which is the point created by adding the two autarky production vectors, together. Total world consumption of both goods is equal to total world output, which has declined, so total utility (or welfare) for the world has been reduced by the tariff.

In the H-O model, a change in utility is attributable not only to a change in consumption, but more importantly, to a change in total output. Changes in total output are due to a reallocation of resources. When a tariff distorts domestic prices, a country's resources are reallocated against its comparative advantage. This leads to production inefficiencies, which are manifested as a decline in worldwide output levels and contribute to an overall decline in utility in the H-O model. As demonstrated later in this section, this is a fundamental difference between the H-O model and the Armington model.

An individual country's gains can be decomposed into an exchange gain and a production gain. Take country A, for example. The total gains from trade are represented by the difference in utility between C_A and C_A'. If both countries are allowed to trade using their autarky outputs, their utilities will increase. The new indifference curves (not drawn) should be tangent to a world price line, through their respective autarky production points C_A' and C_B', at points G and H. The difference between these indifference curves and the ones that are tangent to the autarky points C_A' and C_B' provide a measure of the *exchange gains* from trade. The remaining gains, measured by the distance between C_A (C_B) and G (H) for country A (B), are production gains. In the H-O model shown in figure 3.1, production gains account for most of the total gains from trade and the exchange gains are negligible.

Welfare effects in an Armington model

Compared with the H-O model, the welfare implications of a tariff in the Armington model are more complex. This is because the Armington model has a nested utility function. At the top of the nesting structure, the utility function is defined over two composite goods. In the H-O model, some industries import and others export. In the Armington model, all industries have imports and exports. As a result, a uniform tariff in the Armington model tends to impact more on a country's general price level, relative to that of its trading partners, than on its inter-industry relative prices. The tariff distorts the intra-industry relative price or terms of trade immediately. It only affects the inter-industry relative price indirectly.

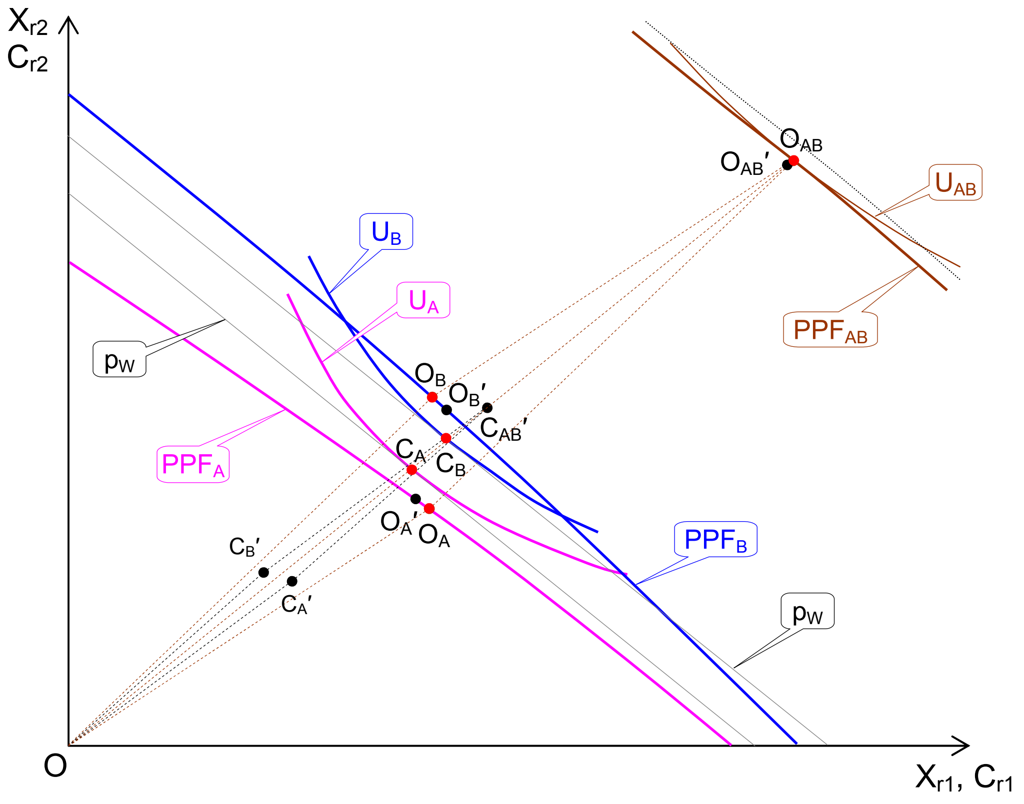
The secondary inter-industry relative price changes induce resource reallocation in this model. The extent to which a tariff alters a country's inter-industry relative prices depends on the elasticity of substitution between domestic and imported goods. As demonstrated in section 2.3, only with $\sigma > 1$, will a tariff on all imports alter inter-industry relative prices and induce resource reallocation in the same direction as in the H-O model. We use an Armington model with $\sigma = 2$ to illustrate the gains from trade.

Top level utility functions and composite goods

To illustrate the welfare effects of a uniform tariff in this Armington model, a diagram similar to figure 3.1 can be used to show the changes in the top level utility function for each country (see figure 3.2). Note that there are some fundamental differences between the Armington model and the H-O model:

- The top level utility in the Armington model is defined over composite goods, not individual goods as in the H-O model.
- Moreover, the goods produced by the same industry in different countries are not identical, and hence are not perfect substitutes, as they are in the H-O model.

Figure 3.2 **Welfare effect of a tariff on A's imports in an Armington model**
Markets for goods and composites



Therefore, the indifference curves and the PPFs, which appear in the same diagram, are measured in different units and should be seen as overlapping. The dimensions of the diagram cover both the supplies of differentiated goods X_{ri} and the demands for composites C_{ri} (r = A, B; i = 1, 2). Including goods and composites in the same space shows how goods produced in different countries are aggregated into composites, and how the composites are distributed among consumers. The diagram allows comparison with results from the H-O model, because both models share the same structure on the supply side. Hence, the PPFs are the same in both models.

Free trade equilibrium

Figure 3.2 shows an Armington model with σ = 2 and the same assumptions used in chapter 2. In the free trade equilibrium, countries A and B produce their own differentiated goods at points O_A and O_B, on their respective PPFs. These differentiated goods are then aggregated to form two composite outputs, indicated by point O_{AB} on the aggregated production possibility frontier, PPF_{AB}. Note that this PPF is lower than that in the H-O model (figure 3.1) because in the Armington model, each country has limited specialisation in production.

On the demand side, two countries consume composite goods at points C_A and C_B on indifference curves U_A and U_B, respectively. As both countries share identical preferences, the equilibrium relative prices for the composites are equal in free trade,[3,4] as indicated by the parallel world price lines p_W tangent to both indifference curves at the consumption points. As a result, both countries' consumption patterns are the same. Graphically, both consumption points C_A and C_B lie on the same ray OO_{AB}. In equilibrium, total consumption of the four composites equals the total production of the four goods produced by the countries, that is, O_{AB} is on the same ray and equal to $C_A + C_B$, where PPF_{AB} is tangent to the aggregated indifference curve U_{AB}.

An important feature of the Armington model is illustrated by this figure — the separation of consumption and production. In the H-O model, because a country produces and consumes the same goods, changes in a country's utility are directly related to its production. In the Armington model, however, a country produces two goods but consumes composites of four goods, two of its own and two foreign ones. Domestic production and consumption are not directly linked. As figure 3.2 demonstrates, A's free trade indifference curve is above its PPF while B's indifference curve is below its PPF. Favourable terms of trade allow country A to:

- purchase a larger portion of total output than B can
- raise its utility accordingly.

Uniform tariff

A uniform tariff on A's imports moves both countries' production points toward their respective autarky points O_A' and O_B'. As shown in figure 3.2, in contrast with the H-O model, output patterns are not significantly affected because of the small degree of substitution between the differentiated goods. Even under a prohibitive tariff, total world output of the two composites is only reduced marginally, from O_{AB} and O_{AB}'. However, the prohibitive tariff does have an extreme adverse impact on the welfare of both countries.

Unlike in the H-O model, where the worst a prohibitive tariff can do is to shift a country's consumption point back onto its PPF, in the Armington model a

[3] With identical preferences and technologies, composite price equalisation occurs only in two-country Armington models. This is due to price symmetry: one country's domestic price is the other country's import price. As a result, the price indices for each composite become identical in free trade. However, in models with more than two countries, price symmetry does not exist, and composite prices do not equalise.

[4] Note that the inter-industry relative prices are not equal in free trade in the Armington model. This is confirmed from the differences between the slopes of the price lines (not drawn) tangent to the two countries' PPFs at O_A and O_B. See also the discussion in chapter 2.

prohibitive tariff can push a country's consumption point to a much lower level, not constrained by the PPF. As figure 3.2 shows, under a prohibitive tariff, A and B's consumption points are at C_A' and C_B' respectively. The aggregate consumption point for the world is reduced from O_{AB} to C_{AB}'. These large welfare losses occur without a large change in total output. Country A's output point shifts only from O_A to O_A' and B's output point from O_B to O_B'. The total world supply of composite goods is only reduced marginally from O_{AB} to O_{AB}'.

This example shows that, in the Armington model, welfare changes are not directly associated with changes in output quantities. Instead, they are affected by the distribution of a relatively constant output between the countries. This distribution occurs through intra-industry trade and terms of trade changes. This explains why Armington models always generate less output change and more terms of trade change than commonly expected from an H-O model.

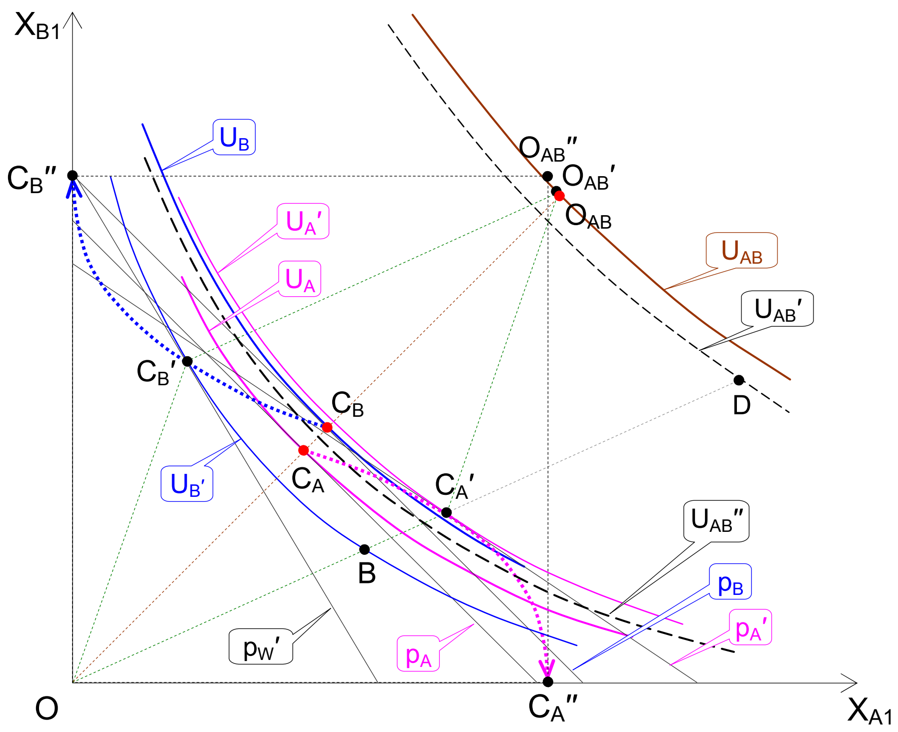
In this example, the total welfare gains from trade can be measured by the distance between C_{AB}' and O_{AB}. Of the total gains, the production gains are tiny, measured by $O_{AB}'O_{AB}$. Almost all of the gains from trade in this Armington model are from pure exchange or redistribution of the relatively constant output ($C_{AB}'O_{AB}'$). This is the opposite of what was observed in the H-O model shown earlier, where production gains dominated.

Sub-utility functions and differentiated goods

To discover how a tariff redistributes a given output between countries in this Armington model requires an understanding of how substitution between the two differentiated goods takes place. This is defined in the sub-utility function. This is where substitution between the domestically produced good and its imported counterpart takes place. Adopting the technique used in the last section, a similar diagram can be drawn showing the consumption of the two imperfect substitutes.

Figure 3.3 shows the effects of a uniform tariff introduced by country A on the market for good 1. As both goods are equally preferred by consumers, the effects of the uniform tariff on both goods markets are similar. Therefore, only the effects in the market for good 1 are shown. Figure 3.3 is a modification of quadrant II of figure 2.1 and the goods market equilibrium diagrams shown earlier in this chapter. In figure 3.3, the horizontal and vertical axes measure the quantities of the two differentiated good 1s. In contrast with quadrant II of figure 2.1, each country's indifference curve is drawn with respect to the same origin. Each country exports some of its own good 1 and imports some of the good 1 produced by the other country. For country A, the horizontal axis represents its domestic output and the vertical axis represents its imports, with the opposite for B.

Figure 3.3 **Welfare effect of a uniform tariff on the market for good 1**



The welfare effects are measured with the movement of the countries' indifference curves. Starting first with free trade, countries A and B consume at C_A and C_B on indifference curves U_A and U_B. The free trade equilibrium world price of A's good 1 relative to B's good 1 is equal in both countries ($p_A = p_B$). The relative price lines are tangent to the two indifference curves at C_A and C_B. As the preferences for the two goods are identical (and homothetic) in both countries, the two equilibrium consumption points lie on the same ray from the origin, indicating that both countries have the same optimal consumption ratio of the two goods because they face the same world prices. The total output of the two differentiated goods can be obtained by adding the distances between the origin and the two consumption points together. The total output point is O_{AB}, representing equilibrium production for both countries.[5] Note that O_{AB} is also on the aggregated indifference curve U_{AB}, indicating optimality.

When A places a tariff on all its imports, the domestic price of good 1 increases, and the relative price line becomes flatter (p_A'). As A reduces its demand for imported goods, the world price of goods produced in B will fall relative to the prices of A's goods. The world relative price line becomes steeper (p_W'), indicating a terms of trade improvement for country A. Initially, a tariff will shift A's consumption point to the right and B's consumption point to the left. Country A's

5 See the two countries' PPFs in quadrants I and III in figure 2.1, for details.

consumption point C_A' is on its highest indifference curve, U_A'. The tariff rate that results in this outcome is A's optimum tariff. The same tariff will shift B's consumption point to C_B', which is on a lower indifference curve, U_B'. Country A is better off while country B is worse off.

Total output can be measured by drawing a parallelogram through the origin and the two consumption points, C_A' and C_B'. As the figure shows, the tariff distorted total output point, O_{AB}', is marginally different from the free trade output O_{AB}. Country B's output increases while A's output decreases slightly. This decrease implies a small reallocation of resources in the two countries. This limited effect from the tariff on output is due to the relatively small Armington elasticity. Despite these small changes in output, the tariff can alter total welfare significantly. The aggregate tariff-distorted indifference curve is U_{AB}'.[6] Not only is it lower than U_{AB} but it is also lower than the tariff distorted output O_{AB}'. This implies that the welfare loss in B is greater than the welfare gain in A. The world as a whole will lose from A imposing a tariff.

If the tariff increases toward a prohibitive level, A's consumption point will approach C_A'' on the horizontal axis, while B's consumption point will approach C_B'' on the vertical axis. Both countries' welfare falls significantly, even though country A still enjoys a terms of trade improvement. In the extreme case, the output parallelogram becomes a rectangle, with a corner at O_{AB}'', the autarky total output point. As the figure shows, the autarky production point is not far from the free trade point, which implies a slight shift in the production of good 1 from A to B. However, despite a small change in output, the aggregate utility level for the world as a whole drops dramatically to U_{AB}'', which is about one half of that implied by U_{AB} under free trade. This is because when approaching autarky, each country can only consume one of the two differentiated goods. This reduces by one half the total utility that is derived from consuming both goods under free trade. Welfare in the Armington model can change dramatically, even without a change in output or resource reallocation. In this respect, the Armington trade model with production behaves more like a pure exchange model than a production model.

The role of Armington values

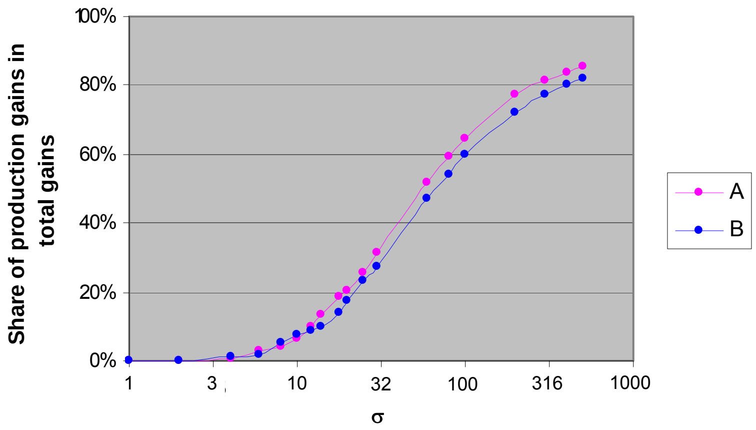
The model in the previous section has an Armington elasticity of 2. Figure 3.4 shows the share of production gains in total gains from trade in Armington models with σ ranging from 1 to 500. When σ is close to 1, there is no production gain

[6] The aggregated utility level for the two countries can be derived by adding the two indifference curves along the same rays. For example, adding OB (B is on country B's indifference curve U_B') to OC_A' gives point B, which is on the aggregated indifference curve U_{AB}'.

from trade. This is because, as shown in section 2.2, output patterns remain the same as in autarky. When σ reaches 10, production gains still account for no more than 10 per cent of the total gains from trade in both countries. Not until σ = 60 can production gains overtake consumption gains as the main source of the gains from trade. At an extremely large value of σ = 500, the proportion of production gains in total gains from trade is close to matching what is observed in a similarly defined H-O model. This relationship between production gains from trade and Armington elasticity is robust to changes in model size and can be observed in much larger applied models as well.

Figure 3.4 **Share of production gains in total gains from trade** [a]

Armington model, σ = 1 to 500, countries A and B



[a] Gains from trade that arise from both countries moving from autarky to free trade.

As most applied models set Armington elasticities below 10, more than 90 per cent of the gains from trade in Armington models are likely to be generated by distributional consumption changes (figure 3.4). This is in contrast with the H-O model, in which the opposite is the case — more than 90 per cent of the gains arise from production expansion. Moreover, it is assumed in the models used in this study that consumers prefer domestically produced goods and their import counterparts equally. In applied models, however, imports often account for a small proportion of total consumption or use. This home bias moves the curves in figure 3.4 further to the right, implying that the share of production gains is low even at high elasticity values.

In the Armington model, a country's gains or losses from a policy change are not due to increases or decreases in world output. Instead, it is due to the policy change allowing one country to *claim more of a given output at the expense of the other*. This is shown in figure 3.3. The welfare changes between the countries are caused, almost entirely, by changes in the intra-industry relative price of the two goods produced in both countries, or the terms of intra-industry trade for each good. The gains from trade in the Armington model are primarily determined by terms of trade changes, instead of efficiency gains through resource reallocation. As shown in figure 3.3, the terms of trade effects of a tariff are directly affected by the curvature of indifference curves, or the Armington elasticity.[7]

Gains from economic integration

Another important difference between the two models is that free trade in the Armington model cannot exhaust all the gains possible through optimal resource reallocation. In the H-O model, Mundell (1957) has shown that free trade and factor mobility can substitute for each other. This means that an integrated equilibrium, where factors are perfectly mobile, is identical to a free trade equilibrium in terms of output patterns, goods and factor prices, and overall utility levels. As a consequence, there are no additional gains from economic integration in the H-O model.

In the Armington model, however, the integrated equilibrium is different from the free trade equilibrium. This is because, even under free trade, goods and factor prices are not equalised. With factor prices unequal, there is an incentive for primary factors to migrate between countries even under free trade. Such a cross-country migration of factors will equate factor and goods prices, and alter each country's output and output for the world as a whole. In the integrated equilibrium, total output will increase and the combined utility of two countries will also increase. This is because factors move to the country offering a higher return. This result is equivalent to comparative advantage gains from trade in an H-O model. Factor movements stop eventually when their prices equalise across countries. Under the current assumptions, in integrated equilibrium, both countries end up with the same endowment of factors and produce the same amounts of the two goods. This integrated Armington economy shares some of the properties of a comparable H-O model (figure A.5).

The additional gain from economic integration in the Armington model is a production gain. In the Armington model, only factor mobility can achieve the optimal reallocation of resources that maximises world output. This gain is missing in the free trade equilibrium of an Armington model.

[7] Depending on the Armington elasticity, the framework of figure 3.3 can be used to show other patterns of output distribution. Each has different welfare implications for the two countries. For example, when σ approaches 1, the two dotted curves, which show the locus of equilibrium consumption points under different prices, become horizontal. Total outputs of the differentiated goods do not change so the gains from trade come only from redistribution. Due to space limitations, however, these different patterns are not presented here.

4 Existing alternatives to the Armington model

When compared with other general equilibrium models, the Armington model has some advantages. Most notably, it can readily accommodate trade data that contain cross-hauling. In addition, the model tends to converge quickly to a unique solution, making it an especially attractive structure for large scale models.

However, as the discussion in chapter 2 and 3 revealed, the Armington model tends to produce results that emphasise the exchange gains from trade and underestimate possible production gains from trade. The explanation for trade being entirely intra-industry in nationally differentiated goods makes the Armington model incapable of capturing the gains from resource reallocation or comparative advantage. In the Armington model, these gains can only be obtained through economic integration.

This failure to capture some important sources of the gains from trade damages the credibility of the Armington model as a suitable tool for trade policy analysis. So the question arises, are there workable alternatives to the Armington model?

Attempts have been made over the years to improve the behaviour of applied models, particularly Armington models. Some researchers have concentrated on improvements in parameter estimation and others on alternative model structures. Two general directions may be identified in these attempts to fix the shortcomings of Armington models. One is to return to the Heckscher-Ohlin (H-O) model and the other is to attempt to find a more appropriate differentiated products model structure. The former strategy abandons the model of intra-industry trade in differentiated goods completely, by returning to inter-industry trade (or net trade) in homogenous goods. The latter strategy attempts to provide the intra-industry trade model with a firmer theoretical foundation to explain better why differentiated goods are produced and how consumers value them. This would fill an important gap in the Armington model's underlying rationale, which provides no answer to these questions. However, as general equilibrium trade models each of these alternatives has its own problems.

4.1 Inter-industry trade models

The H-O model, a remarkable theoretical tool, lays the foundation for our understanding of international trade. It explains not only the sources of international comparative advantage, but also the effects that trade will have on patterns of specialisation and on real returns to factors of production. In the simple two-country two-good two-factor (2×2×2) textbook version of the H-O model, trade, production, and factor prices all respond nicely and plausibly to the opening up of trade and to changes in any impediments to trade. Nevertheless, the H-O model has not been used widely in applied trade policy modelling. This is because it has a number of serious problems that prevent it from serving as a basis for applied general equilibrium models.

The most serious problem with the H-O model is a dimensional one. This problem has long been known (see Samuelson (1953) and Melvin (1968)). In a theoretical H-O model that assumes all countries are identical in every aspect except for factor endowments, if the number of goods produced by each and every country is greater than the number of factors used to produce these goods, the H-O model no longer produces a unique solution. Production and trade patterns in this situation are indeterminate.

When goods and factor prices equalise in free trade, more than one configuration of equilibrium production and trade patterns is possible. Introducing some additional features, which effectively preclude factor price equalisation, into the model may remove this indeterminacy. These include trade costs, non-constant returns to scale technologies, or industry specific factors. However, in a model of homogenous goods and transport costs, for example, free trade will result in a high degree of production and trade specialisation. That is, only a limited number of goods will be produced and traded by each country.

This is also the case for applied models that adopt the H-O assumptions. In applied models, the production technologies and household preferences are determined by calibrating model parameters to a given database. Typically, different countries then have different preferences and use different technologies to produce identical goods. In this case, precluding factor price equalisation does give an H-O model a unique general equilibrium solution. However, as only goods prices are equalised in free trade with factor prices remaining unequal due to the use of different technologies, an H-O model tends to predict a high level of specialisation in many industries.

> It can be shown that as the number of countries increases, the fraction of all possible bilateral trade flows that can be positive in equilibrium becomes very small, asymptotically approaching zero. (Deardorff, 2005)

Moreover, in such a model, trade flows among countries become extremely sensitive to trade costs. This hypersensitivity seems to undermine the ability of the standard H-O model to describe reality and destroys its credibility as a base for empirical trade models.[1]

4.2 Other intra-industry trade models

Introduction of the Armington assumption was, in part, an attempt to 'fix' the problems of the H-O model. The intention was to circumvent the H-O model's problems with indeterminacy, complete specialisation and hypersensitivity to trade costs. The Armington model fixes these problems by abandoning inter-industry trade in homogenous goods and by treating, instead, all trade flows as intra-industry trade in goods differentiated by their country of origin.

Initially, adoption of the Armington assumption in world trade models was a convenient choice to accommodate the widespread 'cross-hauling' found in international trade statistics. Intra-industry trade is introduced as a simple assumption about household preferences, with no reason provided why products are differentiated by their country of origin. One peculiarity is that, as a differentiated goods model, the Armington model does not have a variable to account for the variety of differentiated products. Indeed, the number of differentiated goods is fixed by the *number of countries trading* and is not affected by free trade or by any barrier to trade, eliminating any possibility for increases in variety to be a source of gains from trade. Moreover, product differentiation is modelled as a purely demand side phenomenon, a particular preference of households. On the supply side, as shown in chapter 2, the Armington model can have a perfect competition neoclassical production structure identical to an H-O model.

Motivated to eliminate these undesirable features, researchers have attempted to improve on the Armington model by developing other differentiated goods models. One group of alternatives to the Armington model has attempted to redefine the source of product differentiation and to give intra-industry trade models a supply-side explanation. Two of these models have attracted the most attention in the literature — the Love-of-Variety (LoV) model and the non-traded goods (NTG) model. A particular version of this latter type was developed by Dervis, de Melo and Robinson (1982) and de Melo and Robinson (1989), and is referred to as the DMR model.

[1] See Deardorff (2005) for a comprehensive discussion of the features and flaws of the H-O model.

The DMR model

De Melo and Robinson (1989) provided a simple single country version of this type of model. The ideas can be readily introduced into multicountry Armington models. The model is in the Salter-Swan tradition of traded and non-traded goods models. Adding to Armington substitution on the import side, it introduces transformation between domestically consumed and exported goods. Product differentiation is introduced on the export side for the same reason that product differentiation was introduced on the import side – that is, multisector models are not sufficiently disaggregated. Introducing product differentiation on both sides can disaggregate a model further. This reduces the danger of a possible underestimation, due to aggregation, of the costs of protection (de Melo and Robinson, 1989, pp. 49-50).

The simplest version of the DMR model consists of two goods produced by two countries. One good is produced for domestic use and the other for exporting. The domestic good is non-traded, while the other is traded. Unlike the Salter-Swan model, in which there is clear division between traded and non-traded goods, in the DMR model, the two goods are 'imperfectly transformable' in production. Supply of the two goods is determined by a constant elasticity of transformation (CET) function. A country's supply of an exportable good responds to changes in its price relative to the domestic good. On the demand side, however, households choose between the domestic good and an imported good, which are imperfect substitutes. This is a familiar Armington structure. The degree of tradability of goods depends on the elasticity of transformation. If domestic and exportable goods are perfectly transformable, the model becomes a standard Armington model.

It is easily recognised that the DMR model does not alter the basic properties of the Armington model in any significant way. The model just adds product differentiation to the supply side, as a supplement to the Armington substitution on the demand side. As a result, a country's output from each industry becomes a composite good, composed of a part designated for the domestic market and another for foreign market. The two components are sold at different prices. Introduction of imperfectly tradable goods can help reduce the terms of trade effect of a tariff if the elasticity of transformation is small. It also provides some additional insights into the standard Armington model. For example, a real exchange rate can be introduced as the price ratio of traded and non-tradable goods, which is a useful indicator for trade policy analysis. However, as the Armington substitution remains dominant on the demand side, all the distinctive features of the Armington model, described in chapters 2 and 3, remain in this model. More importantly, introduction of a CET structure on the supply side severs the link between production and consumption in an Armington model, with a consequence that the inter-industry relative price and the equilibrium output pattern cannot be affected by a tariff change. A change in the import tariff will only affect intra-industry relative prices and the distribution (for consumption) of the given outputs between the two countries. In this regard, introduction of the CET on the supply side makes the DMR model even more rigid than the Armington model — it shields production from consumption and therefore can completely eliminate production gains from trade.

The LoV model

Another popular intra-industry trade model is the Love of Variety (LoV) model. There are two versions of the LoV model. One is based on the work of Lancaster (1980) and the other on the work of Spence (1976) and Dixit and Stiglitz (1977). The latter model is also referred to as the Spence-Dixit-Stiglitz (S-D-S) model. As both types of models are based on similar ideas, in this section, the SDS model is used to represent the LoV model. The equilibrium properties of the LoV model have been discussed in Helpman and Krugman (1985) and are only summarised here.

The major difference between the LoV model and the Armington model is that the LoV model has a plausible theory explaining intra-industry trade and the production of differentiated products. This also distinguishes the LoV model from other differentiated goods models. The LoV model is based on the 'new trade theory', which incorporates, in an integrated framework, increasing returns, imperfect competition and intra-industry trade.

In the LoV model, industries are characterised by Chamberlain's model of monopolistic competition. That is, there are many firms in an industry, each a little monopolist producing a product differentiated from others and using a technology that exhibits internal economies of scale. Free entry into the industry can drive each firm's pure profits to zero. In the industry, each firm's product can be regarded as a distinctive variety of the industry's total output. On the demand side, it is assumed that households love variety so that an increase in the number of varieties available increases welfare.

In autarky, the size of the domestic market determines only a limited number of varieties; and each is produced with relatively high average costs because of firms' relatively small scale. Free trade results in foreign firms entering into a market previously dominated by domestic firms and this increases the number of varieties available in a given industry. Moreover, the intensified competition will force firms to reduce costs by expanding output to take advantage of economies of scale. In equilibrium, production of all of the varieties available in an industry is shared between countries, while each household consumes all of the varieties available. This results in an exchange of varieties within an industry or intra-industry trade in different varieties between countries. Households benefit from free trade through the increase in the number of varieties and the increased competition within each industry, which results in lower prices.

The LoV model offers a supply-side theory for intra-industry trade in differentiated goods. More importantly, it also preserves all the attractive properties of the H-O model. It can be shown that, in a 2×2×2 framework, if household preferences and production technologies are identical for all countries and all industries, and if varieties from the same industries are assumed to be similarly differentiated, with equal prices, free trade will result in goods and factor price equalisation in a LoV model, just as it does in an H-O model. As a consequence, the general equilibrium properties of a LoV model resemble those of an H-O model. In contrast with the Armington model, the major trade theorems hold in a LoV model. Moreover, there is another parallel with the H-O model. In an H-O model, free trade and factor mobility can be substitutes for each other. These results are also obtainable in the LoV model. The general equilibria of free trade and of an integrated world economy in the LoV model are identical.

The version of the LoV model that generates the above results is based on an important assumption: all firms produce similar varieties in an industry. In other words, varieties are firm specific not country-specific. As a result, all firms in an industry face the same industry demand curve and choose their output supply on the basis of a common demand. This ensures that all varieties have the same price under free trade, regardless of their country of origin. This makes the model fundamentally different from the Armington model and similar to the H-O model.

In the Armington model firms in different countries do not 'compete' directly in the sense emphasised by Ethier (1982), Krugman (1980) and Helpman and Krugman (1985). By assuming firm specific variety, the LoV model reintroduces supply-side competition into the model. Combined with the symmetry and equal price assumptions, this effectively breaks the direct linkage between the substitution elasticities in households' preferences and the market power of national firms inherent to the Armington model. In the LoV model, the terms of trade conditions are no longer directly determined by household preferences concerning differentiated products. Firm level competition and intra- and inter-industry resource reallocation dampen the terms of trade effect of a policy change.

Although this type of the LoV model shares many desirable properties with the H-O model, it also shares some serious problems. The most serious is the dimensionality problem. In a theoretical model, with identical preferences and technologies for all countries and industries, but with different endowments, if the number of goods produced in each country is greater than the number of factors used to produce these goods, the model does not produce a unique solution under free trade. [2] When free trade equalises factor prices, more than one configuration of equilibrium outputs and trade flows is possible. In an applied setting, where different preferences and technologies, in each country, is the typical result from calibration to a database, free trade production and trade patterns tend to be more specialised than is credible. Very few goods are produced and traded in each country.

In fact, the LoV model has been introduced into applied policy studies since the early 1980s.[3] The applied models do not seem to encounter the problems discussed above. The reason lies in an additional assumption on firms' behaviour. Unlike the theoretical model, most applied models assume that firms from different countries base their supply decisions on the demand for their own varieties, instead of an industry demand. Under such an assumption, if consumers still prefer all varieties equally, the costs of individual varieties will depend on their supply conditions. The supply of a country's variety is constrained by its productive resources. If the demand is the same, the cost of a variety from a country with limited resources tends to be higher, even under free trade. Although the assumption of country-specific firms or variety protects a LoV model from indeterminacy or overspecialisation, it also makes the behaviour of such a model very similar to an Armington model. It replaces 'product' differentiation by country of origin with 'variety' differentiation by country of origin. Here, firms of different countries have monopoly power over the varieties they produce. Just like in the Armington model, the strength of firms' monopoly power depends on the size of each country's productive resources. The firms from the smallest country tend to have the largest market power, which can be measured by the country's terms of trade with the rest of the world.[4]

Another way of avoiding indeterminacy or complete specialisation in a LoV model is to reintroduce a form of Armington preferences.[5] That is, placing an Armington substitution preference between the variety groups produced by different countries on top of the variety preference structure. This means that the varieties produced by

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[2] Note that in a model with more than two countries, with a two-tier substitution structure, the assumption of identical preference across countries may not hold, even if all countries share the same nested utility function. In such cases, the country-specific preferences will prevent goods (varieties) and factor prices to be equalised in free trade.

[3] The first applied model using the SDS preferences was Harris (1984). Since then there has been an extensive literature on the empirical models with similar features. One of such models with many applications is the Michigan model, see Brown, Deardorff and Stern (1993).

[4] A similar point was made by Francois and Shiells (1994) when they write that the Armington model and the monopolistic competition model are potentially identical. The argument is based on the similarity in the firms' price mark-up rules adopted in the two models (pp.28-30).

[5] An example of this type of model can be found in Francois and Roland-Holst (1997).

the firms in the same country are first aggregated into a composite, which has a composite price. This price is different from that of the variety composites of other countries. This modification to the original SDS specification effectively precludes price equalisation between countries. This approach ensures the uniqueness of a model's solution. However, it turns the model back into an Armington model. This is because firms in the same industry, but from different countries, no longer compete directly with each other. The direct link between substitution elasticities and national market power (or terms of trade) is reinstated. Armington preferences override the effects that intra-industry firm level competition may have on prices or resource reallocation and dominate the welfare properties of the new model, as in a standard Armington model.

One of the major contributions of the LoV model to differentiated products models is that it explicitly defines 'variety' as a possible source of gains from trade. This source of gains is fundamentally different from production gains, which are increases in output that are due to a reallocation of resources.

As a differentiated products model, therefore, the LoV model shares the same limitation with the Armington model: neither model accounts for comparative advantage gains from trade because they do not have supply-side competition. Without competition between national firms, it is difficult for a country to have production gains from trade.[6] Helpman and Krugman (1985) acknowledge that differentiated products models have a strong limitation:

> … [I]n the presence of increasing returns, trade always offers the opportunities for a simultaneous increase in the diversity of products available and in the scale at which each product is produced. If the world in fact takes advantage of this opportunity, there will be gains from trade over and above those from conventional comparative advantage. …
>
> Unfortunately, imperfect competition, even if it takes as sanitized a form as monopolistic competition, does not lead the economy to an optimum. As a result, there is no guarantee that expanding the economy's opportunities, through trade or anything else, necessarily leads to a gain. We cannot prove in general that countries gain from trade in the differentiated products model. [The larger font is in the original.] (Helpman and Krugman, 1985, p.179)

The gains in a differentiated products model are mainly due to an increase in the number of varieties, whether it is implicitly or explicitly defined, which may or may not be associated with an increase in world output:

- In Armington models, as demonstrated in chapters 2 and 3, an increase in utility may be due solely to the redistribution between countries of a given

6 See figure 3.4 in which production gains can increase with σ. But in an applied model, a large σ may still be inconsistent with two-way trade in the underlying data.

configuration of different national goods. In LoV models, gains in utility may be due to an increased availability of foreign varieties for home consumers.

- In Armington models, the gains in household consumption do not necessarily imply that total output has increased. In LoV models, as the entry and exit of firms are explicitly modelled, some production gains from trade can be captured.
- Compared with a homogenous product model such as the H-O model, neither model is capable of capturing in full the most well known gains from trade — the gains from comparative advantage. These gains are embodied in the expansion of output which results from direct competition between firms in the same market.

Differentiated products models attempt to capture supply-side competition through peculiar specifications of consumers' preferences. In these models, each firm operates in its own market and does not compete directly with others. Firms interact with others solely because their products are perceived by consumers as imperfect substitutes. As a result, output gains from trade, if any, are primarily due to demand side shifts in consumer preferences, not supply side competition. Comparative advantage gains can only be captured by supply side competition, which is embodied in homogenous products models.

5 A hybrid model: the A-H-O model

The last chapter outlined problems with both intra-industry and inter-industry trade models. Each is capable of capturing some, but not all, of the different types of potential gains from trade.

To address this deficiency, this chapter argues the need for a hybrid model combining both intra- and inter-industry trade. A model of this type is then developed — the Armington-Heckscher-Ohlin (A-H-O) model.

5.1 The need for a hybrid model

Preceding chapters have outlined the properties of two types of models:

- Homogenous product models capture inter-industry trade and emphasise comparative advantage gains from trade.
- Differentiated product models capture intra-industry trade and emphasise various other gains, for example, gains from consuming a greater variety of products, or from lower prices due to scale economies.

The two types of models are mutually exclusive; each excludes the other's gains.[1] This is because their narrow characterisation of international trade focuses exclusively on trade in one type of product. Individually, either type of model does a commendable job representing the behaviour of one type of trade. That said, neither type of model can capture comprehensively the nature of international trade, which involves both homogenous and heterogeneous products. A possible solution to this problem would appear to be the creation of a mixed model, which captures the features of both homogenous and differentiated product models.

In constructing this mixed model, the Heckscher-Ohlin (H-O) model can be used to represent homogenous product trade. For differentiated products, the behaviours

[1] For Armington models, as shown in chapter 6, production gains from trade become prominent only when substitution elasticities reach extremely large values. This might be possible in highly disaggregated commodity trade. However, as applied models are based only on aggregated commodity groups or industries, it is unlikely for the Armington elasticities used in those models to be very large. As a result, consumption, or exchange, gains from trade dominate in most Armington models.

embodied in the Love of Variety (LoV) model and in the Armington model are fundamentally similar, despite differences in their specification of consumer preferences and market structures. Compared with the LoV model, the Armington model does have an additional advantage as an applied model: it requires fewer parameters. This makes the model mechanism more transparent and easier to build and calibrate. This chapter shows that, to capture intra- and inter-industry trade, a mixed model, which makes appropriate use of the Armington assumption, is flexible enough to explain both types of gains from trade.

Because the Armington model is so widely used, the easiest way of constructing a mixed model is to incorporate trade in homogenous products into an existing Armington model. Although applied Armington models assume only intra-industry trade, this assumption is not necessarily based on a belief that all goods traded in the world are differentiated. Rather, imperfect substitution between domestic and imported goods in an industry is understood to be an artefact created by commodity aggregation and not due to actual consumer differentiation among national products.

It is a fact that homogenous goods are also traded in the world. Homogenous, does not necessarily mean identical. A homogenous good can be defined as a product that is produced using standard technologies and factors of production that can be readily marshalled in the short term. For example, an ordinary T-shirt may be categorised as a homogenous product despite T-shirts produced in many different countries having different colours and styles. For households, the demand for these products is determined almost exclusively by their relative prices. As a result, under free trade, only firms that are able to produce these goods at the lowest unit cost will survive in this market, that is, under free trade the LOP prevails in homogenous goods markets.

With current standard trade and industry classifications, different homogenous products are aggregated with each other and with differentiated products to form a commodity category. In these categories, homogenous products can no longer be identified individually. The aggregated product groups, due to their diverse composition, must be treated in models as imperfect substitutes by households. Almost all national or global general equilibrium models are based on industries, not on firms. To handle composites produced in different countries the Armington assumption of product differentiation by country of origin appears to be a convenient choice. However, doing so effectively rules out homogenous goods trade completely.

As outlined in chapter 3, the response to a tariff on inter-industry trade in homogenous goods is quite different from that on intra-industry trade in differentiated products. This is because trade models are based on industries.

Resources can only be reallocated between industries not within an industry. Therefore, removing a tariff will cause larger allocation efficiency gains for homogenous products than for differentiated products. If, due to classification and aggregation, these potential effects cannot be captured, policy analysis suffers.

Results from Armington models are dominated exclusively by intra-industry trade and preference driven redistribution. Incorporating trade in homogenous products holds the prospect of bringing back some of the well-known properties of basic trade theory into these models, including gains from trade due to comparative advantage and price competition between firms in world markets. At the same time, some of the peculiar behaviour of the Armington assumption, such as excessive terms of trade effects of policy changes, may be moderated.

5.2 An Armington-Heckscher-Ohlin model

This section presents a simplified version of the Armington-Heckscher-Ohlin (A-H-O) model. The simple Armington model presented in chapter 2 is modified to include trade in homogenous goods in addition to trade in differentiated goods.

Model structure

The structure in the A-H-O model is very similar to the familiar Armington model, and hence this model is used as a starting point. Assume that the two goods produced in the two countries are aggregated from many different products. Some of these products are homogenous and others are differentiated. Next, assume that each of the two goods can be split into a homogenous good and a differentiated good. These are still produced with the same technologies as defined in chapter 2. That is, the goods from industry 1 are labour-intensive while the goods from industry 2 are capital-intensive. The differentiated goods are subject to Armington preferences. That is, the domestically produced and imported goods are imperfect substitutes, so households wish to consume all four differentiated goods and have separate demand functions for each of them. The two homogenous goods produced in both countries are the same, regardless of source, so households do not distinguish by country of origin and have only two separate demand functions for homogenous goods.

More formally, the top utility function for country r contains four goods — two composites of differentiated goods C_{rj} and two homogenous goods C_{rj}^H.

$$U_r = f\,(C_{r1}, C_{r2}, C_{r1}^H, C_{r2}^H) \qquad (r = A, B) \qquad (15)$$

The composite good C_{rj} is an aggregation of the domestically produced good j and imported good j, as in the Armington model

$$C_{rj} = f\,(D_{rj}, M_{rj}) \qquad (r = A, B;\ j = 1, 2) \qquad (16)$$

Two more production functions for the homogenous goods need to be added in each country. The firms producing homogenous goods minimise their cost of the production

$$G^H_{rj} = r_r\, K^H_{rj} + w_r\, L^H_{rj} \qquad (r = A, B;\ j = 1, 2) \qquad (17)$$

subject to the same production technology used to produce the differentiated good in the same industry

$$X^H_{rj} = f\,(K^H_{rj}, L^H_{rj}) \qquad (r = A, B;\ j = 1, 2) \qquad (18)$$

There is perfect substitution between the domestically produced and imported homogenous products. The output of homogenous good j in country r, X^H_{rj}, can be determined by the zero profit condition

$$c^H_{rj} = P^H_j, \qquad (r = A, B;\ j = 1, 2) \qquad (19)$$

where the unit cost of good j in country r is

$$c^H_{rj} = r_r\, k^H_{rj} + w_r\, l^H_{rj} \qquad (r = A, B;\ j = 1, 2) \qquad (20)$$

The prices of homogenous goods P^H_j are determined by the their respective market clearing conditions

$$X^H_{Aj} + X^H_{Bj} = C^H_{Aj} + C^H_{Bj} \qquad (j = 1, 2) \qquad (21)$$

The factor market clearing conditions (eqs.5 and 6) become

$$K_r = k_{r1}\, X_{r1} + k_{r2}\, X_{r2} + k^H_{r1}\, X^H_{r1} + k^H_{r2}\, X^H_{r2} \qquad (r = A, B) \qquad (22)$$

$$L_r = l_{r1}\, X_{r1} + l_{r2}\, X_{r2} + l^H_{r1}\, X^H_{r1} + l^H_{r2}\, X^H_{r2} \qquad (r = A, B) \qquad (23)$$

And the total income of country r (eq.8) becomes

$$Y_r = G_{r1} + G_{r2} + G^H_{r1} + G^H_{r2} \qquad (r = A, B) \qquad (24)$$

Overall, the system for this simple mixed model can be reduced to ten unknown variables and ten equations. The ten unknowns are the four factor prices (r_r and w_r, $r = A, B$), the two homogenous goods prices (P^H_j, $j = 1,2$) and the four output quantities of homogenous goods (X^H_{rj}, r = A, B; j = 1,2). These variables can be solved from a ten equation system — the four factor market clearing conditions (eqs. 22 and 23), the two homogenous goods market clearing conditions (eq. 21) and the four zero pure profit conditions for the homogenous goods (eq. 19).[2]

Assume, for this A-H-O model, the same numerical settings as for the Armington model used in chapters 2 and 3. Assume, also, that all households prefer the four goods that appear in the top utility function, equally. The free trade equilibrium solution can now be found.

Free trade equilibrium

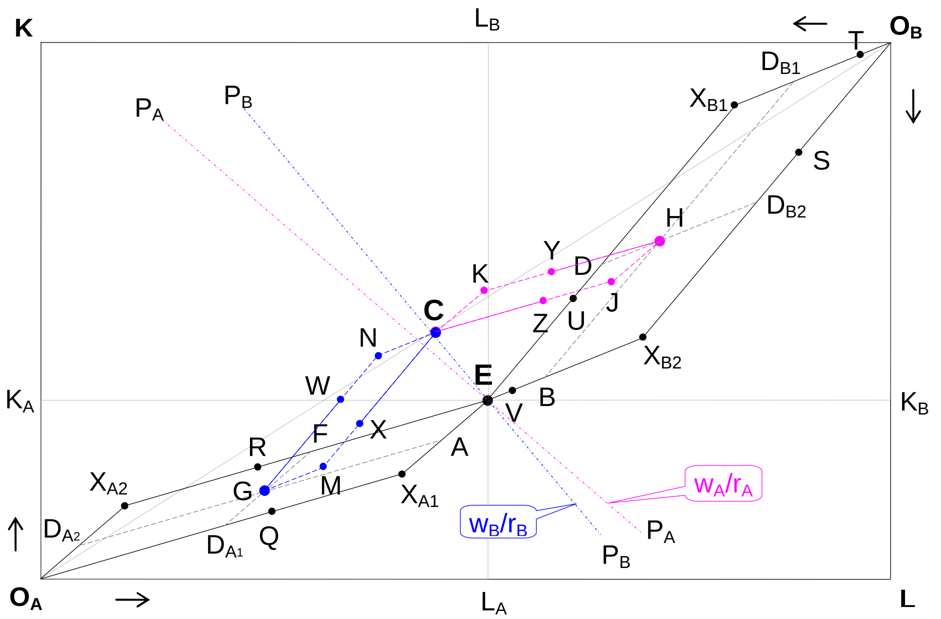
The free trade general equilibrium properties of the model can be characterised by the box diagram for factor markets (figure 5.1). A quick comparison of this figure with figure 2.2 (the Armington model) and figure A.3 (the H-O model) reveals that the pattern of factor use in the A-H-O model is between the patterns in the Armington model and in the H-O model.

The A-H-O model is different from the Armington model in that the free trade factor prices are closer to each other and the industry capital–labour ratios are closer as well. This is because trade in homogenous goods makes the two countries specialise in producing and exporting their comparative advantage goods. These activities draw resources away from the production of differentiated goods and, therefore, reduce the impact of Armington preferences on intra-industry relative prices. The two countries' factor parallelograms are 'thinner' than in the Armington model, implying greater specialisation in each country.

More specifically, country A produces three goods, two differentiated goods and only one homogenous good. Its factor usage for homogenous good 1 is shown by O_AQ; QX_{A1} and $X_{A1}E$ represent production in differentiated goods 1 and 2, respectively. B produces all four goods. Its factor usage for homogenous goods 1 and 2 is shown by O_BT and $X_{B1}U$, and for differentiated goods 1 and 2 is shown by TX_{B1} and UE. The two export parallelograms show the two-way trade in the two differentiated goods and the one-way trade in homogenous goods. Country A imports homogenous good 2, while B imports homogenous good 1 (the solid lines in the import parallelograms). The trade pattern for homogenous goods follows the H-O model's prediction.

2 If one price is used as a numeraire, then the nine remaining unknown variables can be solved from the nine remaining equations.

Figure 5.1 **Free trade equilibrium in factor markets: the A-H-O model**
A 2-country 2-sector 2-factor model with σ = 2



There are marked differences between the A-H-O model and the Armington model. In the A-H-O model, the Law of One Price holds for homogenous products. This reduces the factor price divergence across the two countries. Moreover, including homogenous goods allows production patterns to be shaped by comparative advantage. In free trade, A concentrates on the production of homogenous good 1 while B concentrates on the production of homogenous good 2. As a result, world output of all products increases. These are unambiguous output gains, or efficiency resource reallocation gains, from trade. Because household utility is linked to the output of homogenous goods, the increase in the output of these goods leads to an increase in welfare. This is the link that was missing in the Armington model.

The A-H-O model is also different from the H-O model. This is because intra-industry trade remains part of the story of international trade. The prices of differentiated goods will not equalise in free trade, so that factor prices will not equalise either. As a result, Armington preferences still influence the terms of trade conditions between the trading countries, although this influence is significantly reduced.[3]

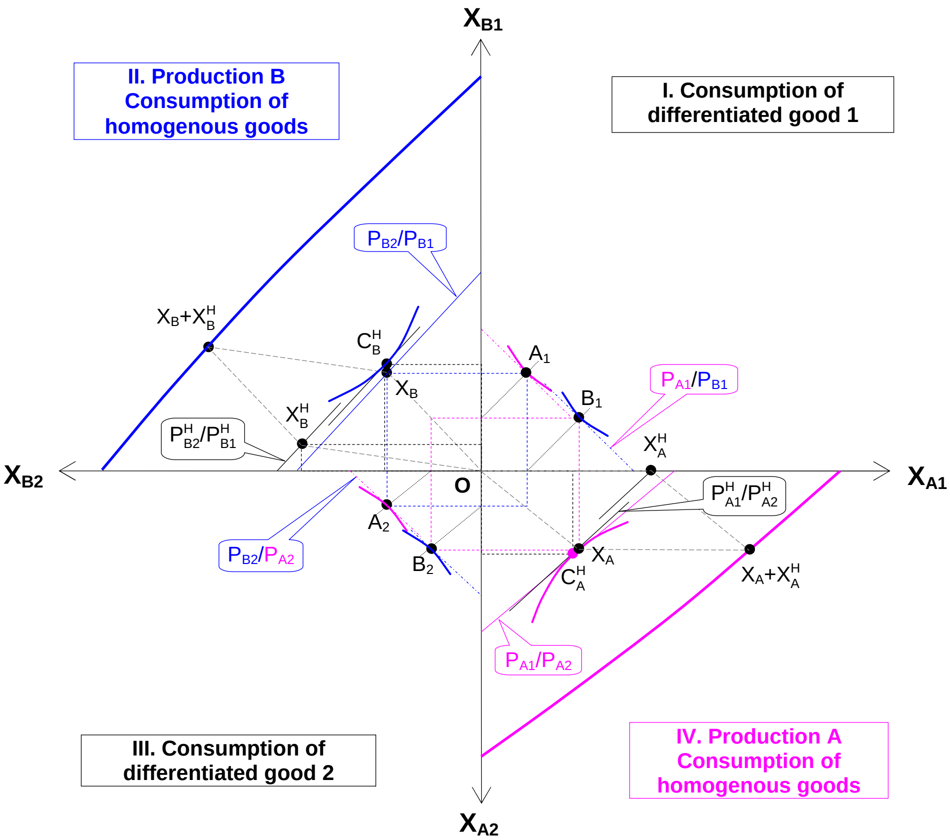
The free trade equilibrium can also be displayed in the two countries' goods markets using the same four quadrant diagram shown in figure 5.2. Although the

[3] An experiment shows that a 10 per cent tariff on country A's imports increases its terms of trade by 1.3 per cent in this A-H-O model, while the same tariff generates an increase in its terms of trade of 4.8 per cent in a comparable Armington model.

number of goods produced in each country has increased from two to four (A produces three in this case), as the homogenous and differentiated goods in each industry use the same technologies all goods can still be included in the same diagram. This diagram is a combination of the diagrams for the Armington model (figure 2.1) and the H-O model (figure A.4). Since the production technologies are identical, separate sub-PPFs can be drawn in each country for the homogenous goods and for the differentiated goods, as if the total supplies of capital and labour are divided between these two types of production. The differentiated goods market equilibrium has a pattern similar to that shown in figure 2.1, while the homogenous goods market equilibrium is similar to figure A.4. The overall equilibrium is a combination of the two sub-market equilibria.

Figure 5.2 **Free trade equilibrium in goods markets: the A-H-O model**

A 2-country 2-sector 2-factor model with σ = 2



In quadrant IV, X_A and X_A^H are country A's production points for differentiated and homogenous goods, respectively, while C_A^H is A's consumption point for the two homogenous goods. Note that X_A^H is on the horizontal axis, indicating A's specialisation in homogenous good 1. Similarly in quadrant II, X_B and X_B^H are B's production points for the two types of goods, respectively, while C_B^H is B's consumption point for the two homogenous goods. In quadrants I and III are two countries' consumptions of all four differentiated goods. It can also be seen in the figure that the prices of homogenous goods are equalised in free trade while the prices of differentiated goods are not.

Dimensionality

In the H-O model, when the number of goods exceeds the number of factors, the general equilibrium solution for the model may not be unique. In the A-H-O model, this so-called dimensionality problem becomes less serious. The possibility of multiple equilibria in the A-H-O model is significantly reduced by the presence of differentiated products. The dimensionality problem is caused by factor price equalisation and the associated incomplete specialisation in production across all countries. In the presence of differentiated products, factor price differences across countries are reduced but not eliminated. Factor price inequality precludes indeterminacy in the model's solution.

This can be explained with a two-country three-industry model. Let an additional industry be added to the above A-H-O model, which uses more capital and less labour than the other two industries to produce a homogenous good and a differentiated good. As before, it is assumed that all countries share identical preferences for all goods and the same technologies in all industries. With a given world endowment of capital and labour, there are many possible ways of dividing the factors between two countries. Each pattern of factor endowment results in a different equilibrium in output and trade. We show this with a familiar box diagram in Figure 5.3.

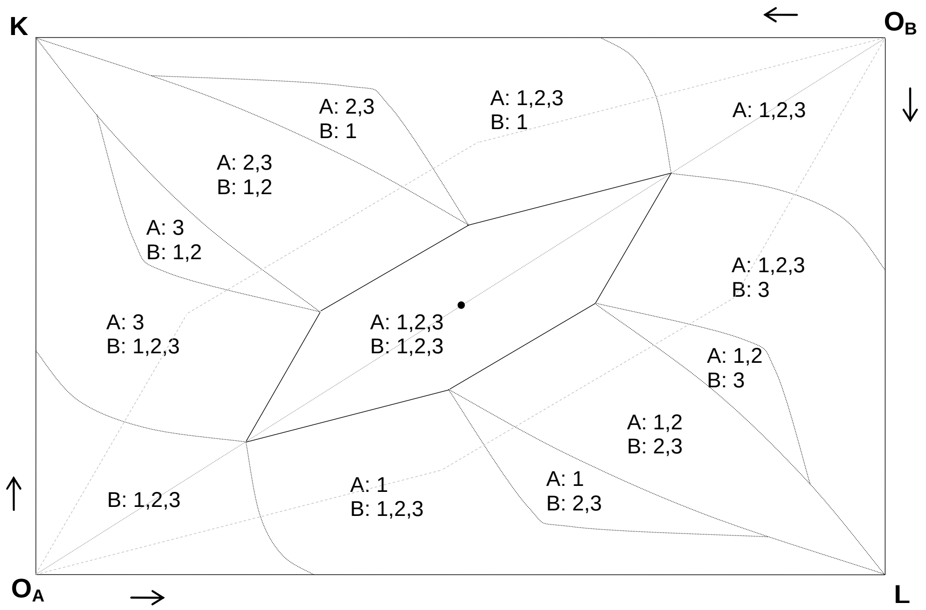
The dimensions of the box measure the world endowment of capital and labour. A point in the box represents a given endowment combination between countries A and B. The box shows the possible production patterns in this A-H-O model.

Before proceeding, we show the general equilibrium solution for the H-O model as a reference. For the H-O model, the large hexagon (dotted lines) represents a factor price equalisation set. Any factor endowment within that zone results in incomplete specialisation, that is, every country produces all three goods. With more goods than factors, however, the solution is not unique and production and trade patterns are indeterminate.

For the A-H-O model, the factor price equalisation set is substantially reduced to the small hexagon (solid lines). If two countries have an endowment falling within the small hexagon, both countries will produce all three goods and, therefore, production and trade patterns are indeterminate. Unless the two countries have similar factor endowments (located in the smaller hexagon), they are unlikely to encounter the problem of indeterminacy. The presence of differentiated products significantly reduces the possibility of indeterminacy in the A-H-O model.

Figure 5.3 **Possible patterns of homogenous production in a 2×3×2 A-H-O model**

A 2-country 3-sector 2-factor model with σ = 2



In an applied global model with countries with diverse endowments, it is likely that the endowment point falls in an area outside the hexagon of factor price equalisation. As the figure shows, any endowment point outside the hexagon will result in production specialisation for at least one homogenous product. The two areas close to the two origins are two extreme cases. In the lower left corner, only country B produces all three homogenous goods (B:1,2,3), while in the upper right corner only A produces all three homogenous goods (A:1,2,3). Between the two extreme cases, the two countries share the production of three homogenous goods but none produces all three. The figure shows all the possible homogenous production patterns for the two countries.

The patterns of production and trade in homogenous goods are based entirely on each country's comparative cost advantage. This is the result of rational behaviour by producers and households under the model's particular configuration. If some of these conditions are altered, incomplete specialisation and production diversification may appear. For example, in applied models, production technologies are not identical across countries so that several countries could produce the same product under the same world price. Moreover, if trade is not completely free and costless, domestic industries could emerge to compete with imports. Trade restrictions, transport costs or geographical distance could assist domestic industries to withstand foreign competition.

In contrast with models that have only one type of trade, specialisation in homogenous goods in an A-H-O model does not cause any practical problem. This is because homogenous production only represents part of an industry. If homogenous good production falls to zero, production does not fall to zero for the entire industry. For example, a clothing industry can be divided into two sub-industries, one producing a homogenous good and the other producing a differentiated good. Without protection, the homogenous sub-industry may not exist. In this case, the country will rely on imports. However, the home clothing industry does not cease to exist. It continues to produce a differentiated good for domestic and foreign markets.

As per capita income and factor endowments are so diverse across countries, no one in the world could competitively produce all homogenous goods. Specialisation in homogenous production is the norm rather than the exception. However, the degree of specialisation is more limited than what the H-O model predicts, because of the existence of trade in differentiated goods. Moreover, international trade is never costless. Even without policy protection, geographical distances between countries could serve as a natural protection, maintaining many domestic industries. In an applied A-H-O model which includes trade costs and transport margins, complete specialisation in the production of homogenous goods will be much less widespread than in a comparable H-O model.

Data issues

The theoretical structure of the A-H-O model is relatively simple — it is a straightforward combination of the Armington and H-O models. As the theoretical structures of both models are well-known, combining the features of both does not create any complications.

The major challenge in building an A-H-O model lies on the data side.

The simplest way of introducing homogenous products into an Armington trade model is to split its existing commodity aggregates (represented as differentiated goods) into sub-aggregates, representing homogenous goods and differentiated goods. However, this is not always straightforward. At the level of aggregation that most computable general equilibrium (CGE) models adopt, it is not always easy to extract all the homogenous components out of an aggregated product group. In most cases, the groups are still too aggregated so that if all the homogenous goods are extracted there will still be two-way trade flows of different homogenous goods between the countries. Two-way trade is inconsistent with trade in homogenous goods. To avoid two-way trade in homogenous goods, therefore, the database needs to be substantially disaggregated. Where that is not possible, only a much smaller number of homogenous goods can be split out.

A further challenge is that additional information is needed to split domestic production and consumption of similar homogenous goods in each country's database. To capture accurately the responses of domestic industries to any policy change, corresponding trade costs and transport margins need to be measured carefully. This is because these costs and margins determine the size or even the existence of domestic industries that produce homogenous goods.

The extent to which an A-H-O model differs from the original Armington model depends on how many homogenous products can be separated from the original database and on the share these goods represent in the whole economy. If a large part of trade in homogenous goods can be separated, the behaviour of the A-H-O model will mean that many resource reallocation responses, missing in the original Armington model, can be recaptured.

6 Concluding remarks

This paper has summarised the general equilibrium properties of the Armington model, including its unique features. As a general equilibrium model, the Armington model is fundamentally different from a textbook neoclassical trade model like the H-O model. Its behaviour is controlled entirely by consumer preferences over nationally differentiated goods, with the Armington elasticity being the main instrument of control. Depending on the value of the elasticities, the Armington model can display a wide range of behaviour, some quite peculiar.

The paper shows that a below unity elasticity implies gross complementarity, which is responsible for all the peculiar behaviour of the Armington model, including very large income effects and terms of trade effects.

To ensure substitutability between nationally differentiated goods, as suggested in the original Armington assumption, the elasticity needs to be set well above unity. Otherwise, goods become complements which is inconsistent with the original Armington assumption and leads to large terms of trade effects and implausible quantity changes.

The unique features of the Armington model have important implications for policy analysis based on the Armington model as a quantitative tool. For example, some well-known trade theorems based on the H-O model do not hold in the Armington model and cannot provide any guidance in interpreting an Armington model's behaviour. Indeed, their inappropriate use is likely to misinform policy analysis.

The gains from trade in the Armington model come mainly from exchange or redistribution of given outputs, rather than from a more efficient reallocation of resources or output expansion, as in the H-O model. As changes in terms of trade play an important role in the equilibrium of the Armington model, gains to some countries are often obtained at the expense of other countries. More importantly, free trade in the Armington model does not result in the optimal reallocation of resources and does not offer the possibility of maximising the welfare of all countries. Additional gains can be obtained from economic integration or factor mobility. Direct international factor mobility in the Armington model plays this resource reallocation role, which is played by free trade in the H-O model.

The behaviour of the Armington model is determined by its unique structure and its exclusive focus on intra-industry trade in nationally differentiated products. Manipulating the degree of substitution does not overcome the limitations of the Armington model.

Existing alternatives to the Armington model have concentrated on either inter-industry trade or intra-industry trade. Each is capable of capturing a specific type of gains from trade but misses other potential gains. The problems associated with these models are closely linked with their characterisation of international trade, emphasising either gains from increased varieties or gains from comparative advantage.

In light of these shortcomings, a logical step forward is to incorporate both inter-industry trade and intra-industry trade, or homogenous goods and differentiated goods, in the same framework. This framework is capable of capturing the different types of gains from trade and of including most adjustments that are likely to occur to production and consumption. The paper proposes a model within this framework, the A-H-O model, which combines features of both the Armington and the H-O models. This mixed model helps moderate the overwhelming influence of household preferences on terms of trade found in the Armington model and, at the same time, avoids the indeterminacy or complete specialisation found in the H-O model. It reintroduces into applied trade model the comparative advantage gains from trade and firms competing on price across countries.

The construction of an A-H-O model involves no theoretical complications. The efforts of developing such a model will concentrate more on the data side. To take full advantage of the model it is necessary to split homogenous goods from the commodity aggregates of a database. This requires additional information on trade flows, production and consumption at a very disaggregate level. The industries that could be split first are those where the relevant data are readily available. Even the use of a partially split database can improve model behaviour significantly, and the rewards for constructing a comprehensively split database are likely to be substantial.

APPENDIXES

A The Heckscher-Ohlin model

This appendix outlines the general equilibrium properties of an H-O model in autarky, free trade and economic integration. These results are used in chapters 2 and 3 to compare properties of the Armington model with the H-O model.

A.1 Autarkic equilibrium

In autarky, both countries consume only the goods they produce. In effect, both countries are isolated from one another.

In this situation, assume that both countries use the same technology in production and have the same preferences in consumption. If the relative factor endowments differ across countries, full employment equilibrium factor prices will also differ. Let country A be relatively more labour-abundant than B. Then in autarky, the wage-rental ratio in country A will be lower than in B. An example of such an autarkic equilibrium in factor markets is shown in figure A.1.

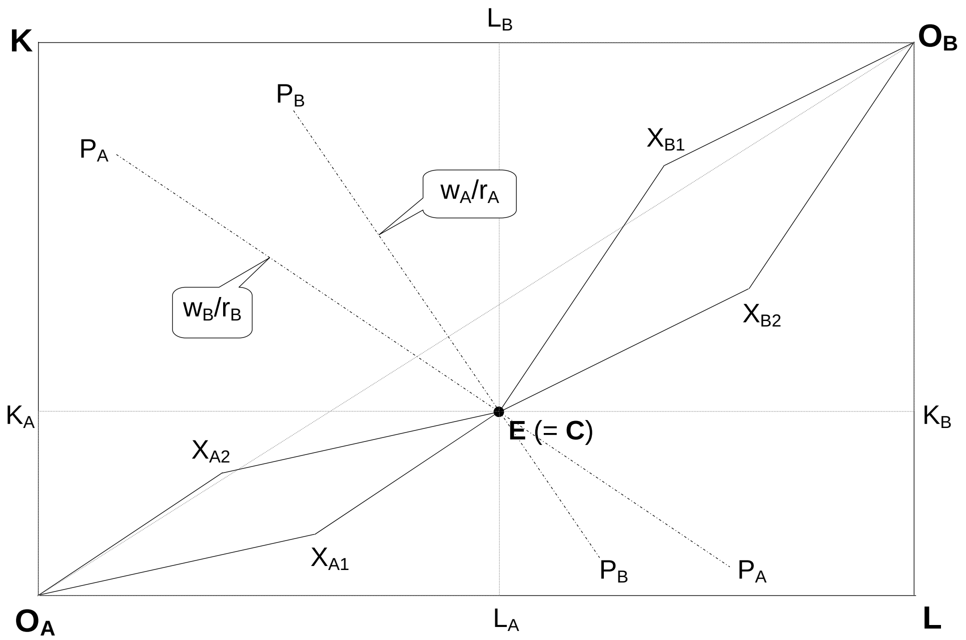
The box in figure A.1 represents the resources of the world economy.[1] Country A's endowment of capital and labour is a vector measured from the lower left corner O_A while B's endowment is measured from the upper right corner O_B. Point E represents the joint endowment point, dividing the world's total endowment of capital and labour into two parts — A's share and B's share. In isolation, full factor employment ensures that country A has a wage-rental ratio lower than B's (that is $w_A/r_A < w_B/r_B$). As factor prices are not equal across the countries, the capital–labour ratios used in the same industry, but in different countries, are not the same.

Usage of factors in the industries is represented by two parallelograms, $O_A X_{A1} E X_{A2} O_A$ and $O_B X_{B1} E X_{B2} O_B$. The sides of the parallelograms represent the vectors of input usage in both industries. For example, in equilibrium, country A devotes resources $O_A X_{A1}$ to production of good 1 and $O_A X_{A2}$ to production of good 2. Because of its lower wage-rental ratio, A uses relatively more labour and less capital in both industries in comparison with B. In equilibrium, both countries utilize all their resources for the production of both goods. The endowment point E

[1] The world is comprised of the two countries, A and B.

coincides with equilibrium consumption (C), because each country consumes only what it produces.

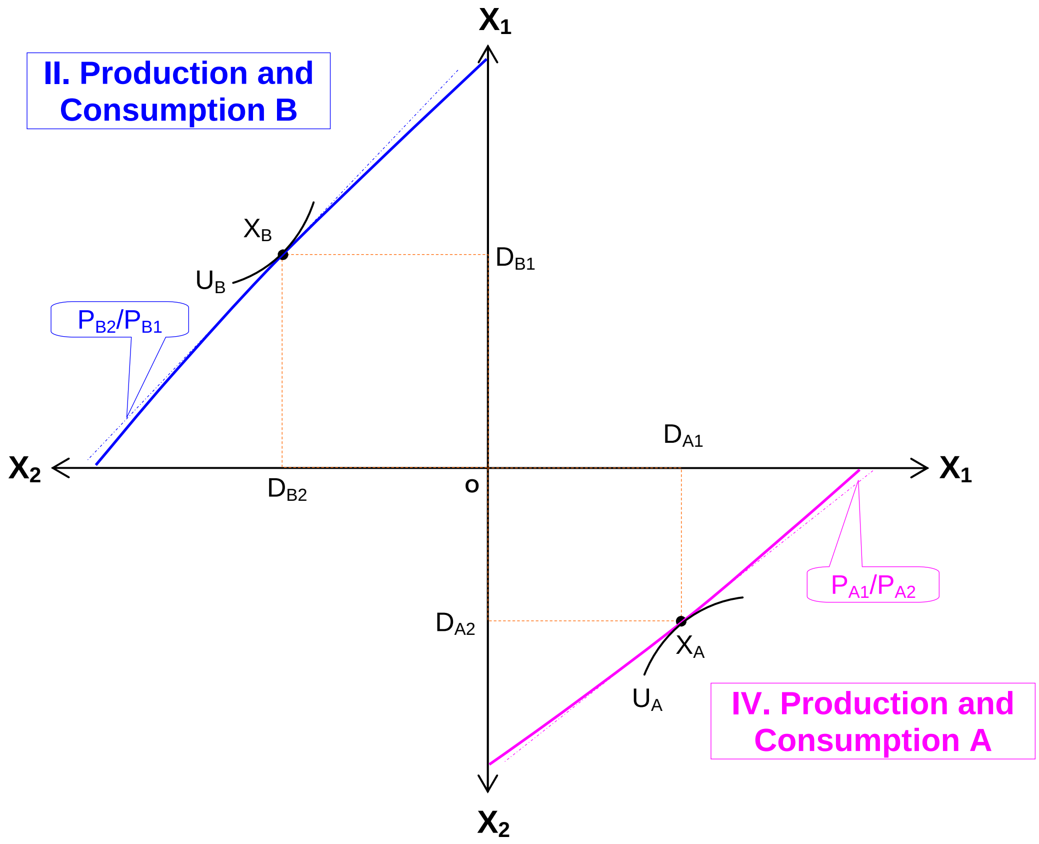
Figure A.1 **Autarkic equilibrium in factor markets: the H-O model**
No trade in goods, no movement of factors between countries



To illustrate the autarkic equilibria in the goods markets for both countries, factor usage in both industries is mapped on to each country's production possibility frontier (PPF). Figure A.2 presents PPFs for both countries. Country A's PPF is in the quadrant IV and B's in quadrant II. In these quadrants, the production and consumption of both goods is presented in the same quadrants.

In autarky, the production and consumption of each country is isolated from the other. The goods' market equilibria of both countries are represented as X_A and X_B. The slopes of the relative price lines show the ratios of the two prices, P_{A1}/P_{A2} in quadrant IV and P_{B2}/P_{B1} in quadrant II. Because relative factor endowments differ, the price ratios of the pairs of goods also differ across countries. The capital-intensive good 2 is relatively more expensive in country A. Likewise, the labour-intensive good 1 is relatively more expensive in B. As preferences are the same (and homothetic) across countries, the two countries produce relatively similar proportions of both goods to meet domestic household demand.

Figure A.2 **Autarkic equilibrium in goods markets: the H-O model**
No trade in goods, no movement of factors between countries



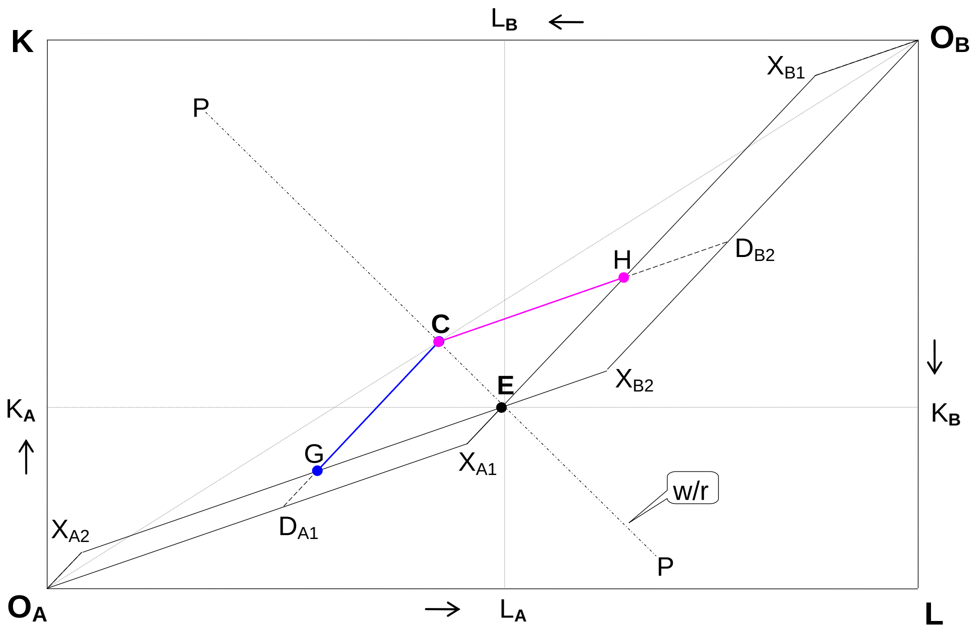
A.2 Free trade equilibrium

In the free trade H-O model, both goods can be costlessly transported between countries so firms producing the same good in different countries compete with each other head-to-head in the world market. Consequently, each good is exchanged internationally at the same price, so the 'Law of One Price' (LOP) prevails. International competition also forces each country to concentrate relatively more of its resources in producing the good in which it has a comparative cost advantage.

Under the pressure of competition, firms in each country choose combinations of factors that minimise their unit costs of production. When goods prices equalise across countries, factor prices are also equalised. A free trade equilibrium in factor markets for both countries is shown in figure A.3.

Figure A.3 **Free trade equilibrium in factor markets: the H-O model**

Free trade in goods across countries, factors cannot move between countries



As shown in this figure, the shape of the two parallelograms, which represent each country's factor services used to produce the two goods, have been changed by trade. To exploit its comparative advantage, country A reduces its use of factors in the production of good 2 and increases its use of factors for good 1, while B does the reverse. Both countries concentrate on producing the good that embodies their respective comparative advantage. This is the good that uses more intensively the factor in which they have a relative abundance. As a result, their respective parallelograms become 'thinner'. As the world resources are used more efficiently, total world output of both goods increases. This represents an allocative efficiency gain from trade.

Geometrically, the parallelogram $O_AD_{A1}GX_{A2}O_A$ represents the factor services embodied in A's consumption of both domestically produced goods. Similarly, $O_BD_{B2}HX_{B1}O_B$ represents the embodiment in B's consumption of its domestically produced goods.

With increased output of both goods, households in each country can maximise their utility by consuming more of both goods. This is achieved by trade between the two countries — country A exports good 1 to B and B exports good 2 to A. The line GE represents A's exports of factor services embodied in good 1, which equals B's corresponding imports, CH (= GE). Similarly, B exports good 2 to A, EH (= CG).

This trade pattern follows the Heckscher-Ohlin theorem.[2] Point C represents equilibrium consumption, representing the factor services embodied in the goods consumed by both countries. At this point, country A is a net exporter of labour services while country B is net exporter of capital services. Vanek's factor content theory (1968) also holds.[3]

Factor prices are equal as shown by the slope of the line PP. Consequently, the capital–labour ratios in the same industry, but different countries, must be equal as well. This is confirmed by noticing that the corresponding sets representing factor use in each industry ($O_AX_{A1}EX_{A2}O_A$ and $O_BX_{B1}EX_{B2}O_B$) are parallel. As both countries share the same homothetic preferences, the equilibrium consumption point C is on the diagonal O_AO_B, implying a Pareto optimum.[4]

The properties of the goods market equilibrium in this H-O model can be shown in figure A.4, which uses the same framework used in figure A.2. As mentioned earlier, quadrant IV and II show country A and B's production and consumption. When trade is introduced, the firms producing the same goods in both countries confront each other in the world market. The substitutability between goods produced in both countries can be seen in quadrants I and III. In this H-O model, country A and B produce two homogenous goods X_1 and X_2. Good 1, produced by both countries, is identical and is therefore a perfect substitute in consumption. This is shown in quadrant I.

Compared with autarky, country A's production moves along its corresponding PPF toward good 1. Country A produces OX_{A1} of good 1 and OX_{A2} of good 2 at point X_A in quadrant IV. Of the total outputs, OD_{A1} and OX_{A2} are consumed domestically and $X_{A1}D_{A1}$ is exported. Similarly, B produces more of good 2 and less of good 1 at point X_B, in quadrant II.

Trade in both goods is shown in quadrants I and III. As the trade is one-way in this H-O model, quadrant I shows only trade in good 1 and quadrant III shows trade in good 2. For country A, domestically produced good 2 and imported good 2 are perfect substitutes. In equilibrium, A chooses to import $X_{A2}D_{A2}$ from B. The

2 This theorem states that countries tend to export products that use more intensively those factors in which they have a relative abundance.

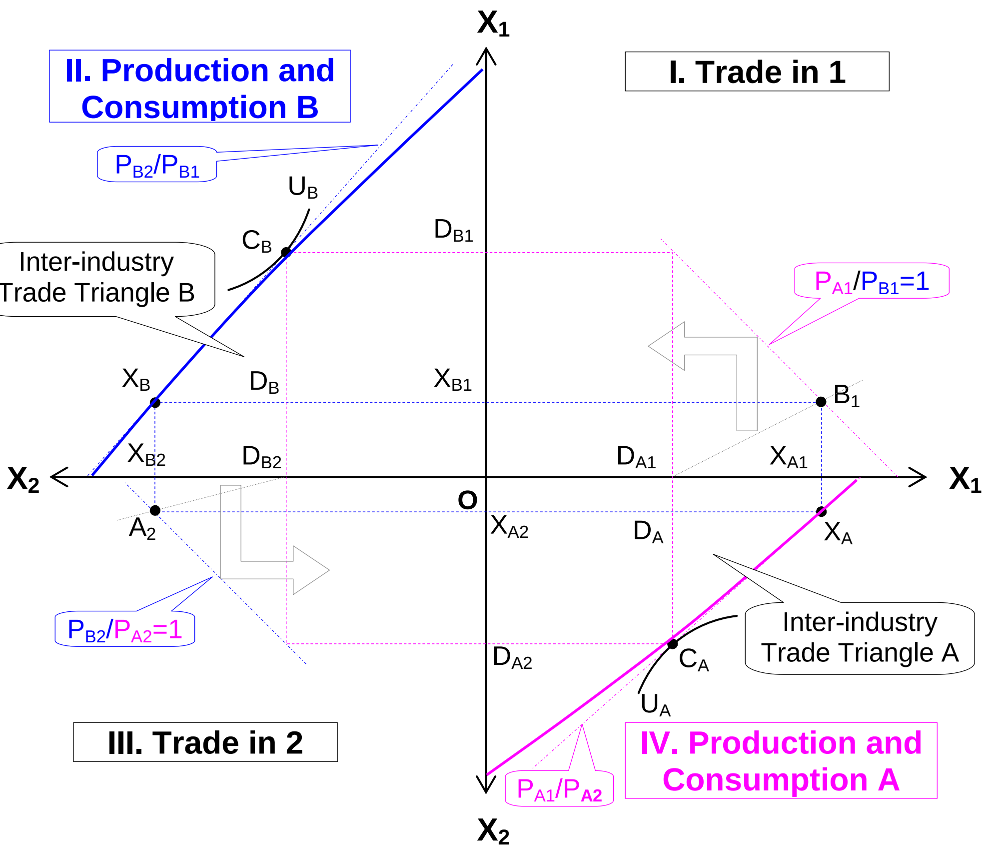
3 Vanek's factor content theory generalises the preceding result from the two goods, two factors, two country model that the exporting goods, on average, contain 'more' of the factors in which the country has a relative abundance, and the imports contain relatively more of those factors which are relatively scarce.

4 As both countries face the same prices and have the same homothetic preferences, in equilibrium, both consume goods in the same proportions and because they consume the goods in the same proportion there are no unexploited gains from trade. Hence, the equilibrium is a Pareto optimum.

indifference curve[5] in quadrant III is therefore a 45-degree line, which coincides with the relative price line (P_{B2}/P_{A2}). Country B imports good 1 from A.

Figure A.4 **Free trade equilibrium in goods markets: the H-O model**

Free trade in goods across countries, factors cannot move between countries



Because the goods produced in both countries are identical, according to the 'Law of One Price', their respective prices should equalise in free trade. This is reflected in the gradient of the slope of price lines in quadrants I and III (P_{A1}/P_{B1} and P_{B2}/P_{A2}), which are both equal and have a magnitude of 1. The relative prices of the two goods produced in each country should also equalise, shown by the relative price lines in quadrants IV and II having the same slope. As good 1 is an exported good and good 2 is an imported good in country A, the slope of the price for good 1 relative to good 2, in quadrant IV, is country A's terms of trade. Similarly, the slope of the price for good 2 relative to that for good 1, in quadrant II, shows B's terms of trade. In this two-country world, this is the inverse of A's terms of trade. In this figure, country A's trade triangle is $C_A D_A X_A$ in quadrant IV, while B's trade triangle is $X_B D_B C_B$ in quadrant II. In equilibrium, the two trade triangles are identical.

[5] The indifference curve is only included here for comparison with the Armington model.

Equilibrium consumption for both countries is represented by the points C_A and C_B. They are above their respective PPFs. This indicates that there are gains from trade. It can be shown that production points X_A and X_B on two countries' PPFs represent a maximum output level for the world as a whole. It is this physical or real gain in world output that makes the gains in consumption and utility for both countries possible. As is shown in chapters 2 and 3, this represents one of the fundamental differences between the H-O model and the Armington model.

A.3 Integrated equilibrium

To verify whether the free trade equilibrium is really an optimum requires an exploration of the properties of the integrated world equilibrium. In the trade literature, the integrated equilibrium is usually defined for a single economy. To be comparable with the Armington model, a different approach is required. Instead of assuming both countries merge into one, it is assumed that all the factors of production are freely mobile between the two countries. Assume also that the owners of factors move with the factors and so the incomes of the countries change as factors move between them. In equilibrium, the two countries can be seen as two regions in an integrated world economy. Factor mobility equalises factor prices between the countries. As a result, both regions produce both goods using the same capital–labour ratios in the same industries. They also produce good 1 and good 2 in the same ratio in each region. A continuum of equilibria is possible with factors in both regions ending up in the same ratio, which is the same ratio as the global endowment. The split of the global endowment of both factors in a region can range from 0 to 100 per cent, with the complement in the other region. Out of the continuum of possible equilibria which are consistent with perfect factor mobility, in this H-O model, an equilibrium is chosen where both regions end up with equal shares of the resources so that the results are directly comparable with the results for perfect factor mobility in the Armington model (see figure A.5).

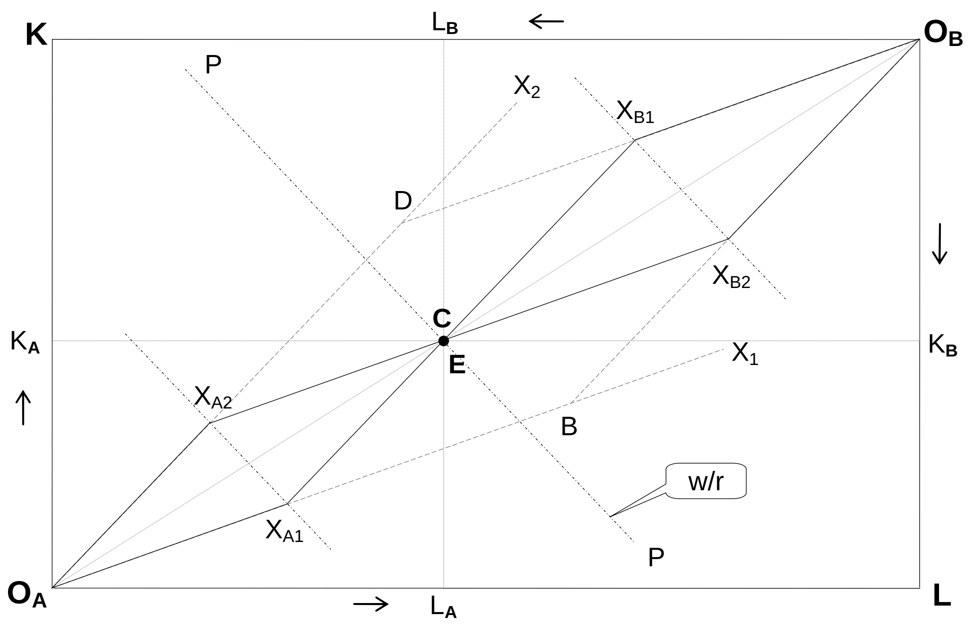
With identical homothetic preferences over the goods, trade is no longer needed — each region consumes only the goods it produces. This free factor mobility equilibrium is equivalent to the equilibrium of a fully integrated world economy.

It can be shown that, compared with free trade equilibrium, essentially nothing is changed in this integrated equilibrium — the equilibrium factor and goods prices are the same as in free trade and, therefore, the equilibrium quantities of factor services in the production of the two goods and the outputs of the two goods are the same as well.

The integrated world equilibrium in factor markets is shown in figure A.5. This box has the same size as before because the world endowments of capital and labour remain unchanged. Factor mobility equalises the relative endowments for two regions so that the world endowment point E moves to the free trade consumption point C on the diagonal of the box. The rays from the origin O_AX_1 and O_AX_2 represent the factor services in production of good 1 and 2. The slopes of these two rays are the same as those from the free trade case. In equilibrium, the two parallelograms representing the factor services in the two industries, $O_AX_{A1}EX_{A2}O_A$ and $O_BX_{B1}EX_{B2}O_B$, are proportional for the two regions. The slope of the line PP is the wage-rental ratio, which is also equal to that prevailing in the free trade case.

Figure A.5 **Integrated world equilibrium: the H-O model**

Factors perfectly mobile between countries



The parallelogram $O_ABO_BDO_A$ is usually referred to as the 'factor price equalisation (FPE) set' (Dixit and Norman 1980). This is because if the endowment point is located within this set, free trade will equalise factor prices for all trading regions. This has already been confirmed in the previous free trade case. In the integrated world, production and consumption of the two regions are located along the diagonal, which is inside the FPE set. In equilibrium, the two regions produce and consume the same proportions of two goods, so factor mobility makes trade in goods unnecessary.

General equilibrium in the goods markets can be shown in the same diagram used for the free trade case. It is not presented here because it is easy to imagine what the equilibrium would look like. Perfect factor mobility will make the PPFs for the both countries proportional. As a result, the production and consumption patterns of both countries are identical and coincide within each country, so trade is not required.

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ATTACHMENT

This page is designed to fold out to compare with corresponding figures in chapters 3, 4 and appendix A.

Figure 2.1 **Free trade equilibrium in goods markets: the Armington model**

A 2-country 2-sector 2-factor model with σ = 0.99

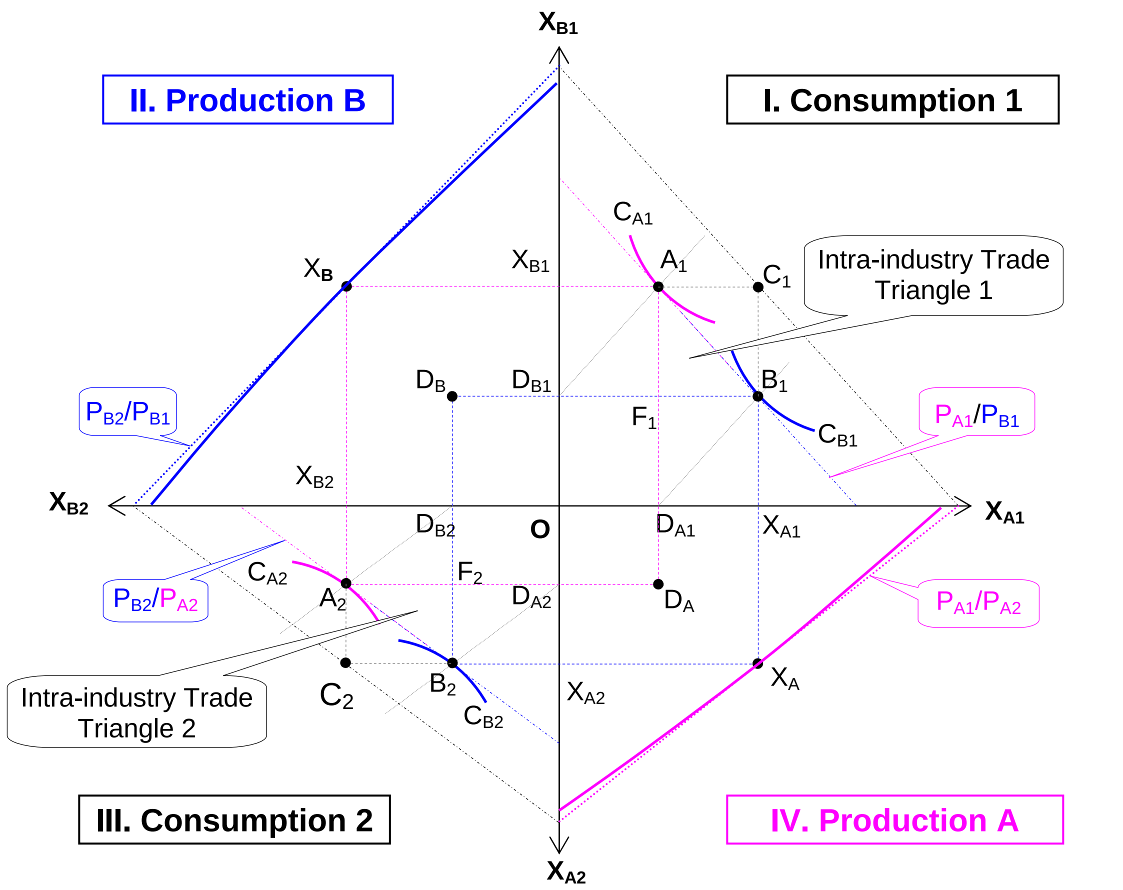

Figure 2.2 **Free-trade equilibrium in factor markets: the Armington model**

A 2-country 2-sector 2-factor model with σ = 0.99