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# The Armington Model

Staff Working Paper

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# Preface

All policy analysts use some type of model in their work. Most of the conceptual models that underlie thinking in the Australian context are built assuming homogeneous goods, use the ‘small country’ assumption and assume that countries differ in endowments or technology. The numerical models of global trade that are based on the Armington formulation, however, depart from this set of assumptions, by positing that each country produces its own set of products, which are somewhat differentiated from the products of similar name, produced elsewhere.

Armington elasticities specify the degrees of substitution in demand between similar products produced in different countries. They are critical parameters which, along with model structure, data and other parameters, determine the results of policy experiments. Especially when many tariffs are small, trade liberalisation simulations can produce positive or negative welfare outcomes depending on the values assumed for Armington elasticities.

The Commission developed a research program on the role of Armington elasticities in quantitative models that are commonly used to analyse trade issues. The research program was designed to improve the effectiveness of models used in analysing various options for unilateral, bilateral and multilateral liberalisation.

The purpose of this paper is to explore how models adopting the Armington formulation differ from traditional models, in their quantitative properties and underlying theory of trade.

In publishing its research in this area, the Commission hopes to clarify issues that arise as single-country and global trade models are increasingly used to assess the potential impacts of various types of trade liberalisation.

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# OVERVIEW

## Key points

- Multi-country computable general equilibrium (CGE) models used to analyse tariff and trade policy changes typically incorporate the Armington structure which differentiates commodities by their country of origin (national product differentiation), and assumes them to be imperfect substitutes for each other.
- In contrast to the well-known Heckscher-Ohlin model, relatively little is known about 'Armington models' and their properties despite their wide acceptance among model builders and policymakers. This makes it difficult to interpret the trade and welfare results that might arise from trade liberalisation simulations that are based on Armington models.
- Introducing the Armington structure changes fundamentally the properties of a trade model regardless of the values assumed for the elasticities of substitution between imported and domestically produced goods. In particular:
  - there is no comparative advantage and hence no gains from trade due to product specialisation;
  - the number of products is fixed and hence there are no gains from trade due to increased product variety; and
  - large terms of trade effects tend to offset other gains from trade.
- As a consequence of these properties, Armington models tend to understate the gains from tariff and trade policy liberalisation.
- A numerical, 3-good, 3-country modification of the Global Trade Analysis Project (GTAP) model is used to illustrate these properties. Compared to a Heckscher-Ohlin model, a unilateral across-the-board cut in tariffs in an Armington model results in:
  - a larger shift in consumption from domestically produced goods to imported goods;
  - a larger decline in terms of trade; and
  - a smaller resource reallocation across industries.
- The paper indicates possible future directions for methodology and practice.

# Overview

Paul Armington (1969) introduced into international trade theory the assumption that final goods internationally traded are differentiated on the basis of the country of origin. He assumed that, in any one country, each industry produces only one product and that this product is distinct from the product of the same industry in any other country. For simplicity, he assumed there is only one consumer in each country. In the eyes of this consumer, the products of one industry which originate from different countries are a group of close substitutes. This set of assumptions is called the Armington assumption. It relates to the demand side of the model.

Soon after the development of computable general equilibrium (CGE) modelling of the world economy in the early 1970s, the Armington assumption was built into single-country and multi-country (or global) CGE models, often used to study trade policy. This was done mainly to facilitate the use of international trade statistics and to avoid unrealistic specialisation in trade liberalisation scenarios.

On the supply side, these models typically incorporated the standard neoclassical assumptions of constant returns to scale and perfect competition in all industries. The term 'Armington model' will be used below to cover any model which makes this combination of demand and supply-side assumptions.

Global CGE models are used to analyse the effects of changes in government policies or of scenarios in the world economy such as oil shocks or changes in technology.[1] While these models differ in their specifications of the national economies and the international trade between them, the great majority of CGE models with international trade are Armington models. For example, the MONASH Model and the models used by the Centre for International Economics (CIE) in Australia, and the Global Trade Analysis Project (GTAP) model used widely around the world, are all Armington models. In fact, Armington models dominate CGE analyses of trade policy issues made for many governments and international organisations such as the World Bank, the International Monetary Fund and the World Trade Organisation.

1 Although this discussion concentrates on multi-country models, many results obtained in this paper are also relevant for single-country models.

Despite their acceptance among model builders and policymakers, uncomfortably little is known about the general properties of Armington models. In contrast, the properties of the Heckscher-Ohlin model and the specific factor model are both well known. They are characterised by a number of theorems such as the Heckscher-Ohlin or factor proportions theorem, the factor price equalisation theorem and the Stolper-Samuelson theorem. These theorems give us our basic understanding of international trade and of the gains from international trade. Given the dominance of Armington models in the general equilibrium analysis of trade issues, the absence of knowledge of the general properties of these models is a major gap in our understanding of international trade modelling results. This paper was written to examine the general properties and to give a better understanding of the peculiarities of Armington models.

The introduction of Armington substitution in the demand for commodities is a departure from the assumption of perfect substitution that underlies traditional trade models. This departure changes fundamentally the properties of a trade model and the well known theoretical results that are based on variants of the Hekscher-Ohlin model. In this paper, the main comparison is between the properties of an Armington model on the one hand and the standard Heckscher-Ohlin model on the other.

## The Armington model and its properties

Section 2 presents the specification of the Armington model. On the demand side, it presents the different ways in which the Armington assumption can be specified by using multi-level utility functions. The same view of national product differentiation has been applied to the demand for intermediate inputs when intermediate inputs and international trade in these inputs are added to CGE models. Increasingly, the production of final goods has become ‘fragmented’ with several nations adding value to the final product.

On the supply side, the specification adopted is as general as possible within the constraints of the maintained assumptions that there are constant returns to scale everywhere and perfect competition in all markets. The specification is sufficiently general to encompass most CGE models with international trade.

Section 3 presents the properties of the Armington model. The approach is analytical. We seek general properties of the model which hold at all times, that is, for any number of goods, products and factors and any numerical values of the parameters of the model or the tariff rates which impede international trade. In this way, the results apply to any CGE model which is based on the Armington assumption and perfectly competitive behaviour of all agents.

The general properties cover:

1. the form in which the Law of One Price holds when all trade is free;
2. the nature of comparative advantage in the model;
3. the gains from trade; and
4. the conditions under which factor price equalisation holds in this model.

An understanding of the general nature of the Law of One Price and factor price equalisation is necessary to understand the workings of the model. For the purpose of understanding the implications of using this model for trade policy analysis, the main focus is on the nature of comparative advantage and gains from trade.

In an Armington model, there is no comparative advantage. One country cannot have a price advantage in producing one product relative to the prices of other products from other countries — that is, it cannot have a comparative advantage — because the sets of products produced by any two countries are disjoint.

Yet, national endowments and technologies clearly play a role in determining the pattern of production and trade in an Armington model. The key to understanding the pattern of trade lies in the distinction between the gross and the net trade in the products of an industry or, if one prefers, between the inter-industry and intra-industry trade. Under certain conditions, the net or inter-industry trade can be explained by factor proportions. This is a version of the Heckscher-Ohlin theorem, but it applies only to inter-industry trade in the Armington model.

The implication is that there are three causes of trade in an Armington model — differences in endowments, technology and national products. The Armington model has some advantages over the Heckscher-Ohlin model. For example, it introduces product differentiation and gains from trade in consuming differentiated products. It also makes it possible to use aggregated trade data.

But introducing differentiated products via the Armington assumption results in a number of biases in the estimation of gains from trade. First, since there is no comparative advantage in an Armington model, gains from trade cannot be due to increasing specialisation. This is a standard source of gains in a Heckscher-Ohlin model and the only source of gains in the Classical model of international trade.

Second, the standard Armington assumption — that the number of varieties of a product is fixed — ignores one of the possible benefits of trade liberalisation, namely, an increase in product variety and in consumer and producer choice. In contrast, the way in which the Michigan model introduces firm-level product differentiation permits benefits from liberalisation through this mechanism.

Third, and possibly most significantly, the small country assumption cannot be made in an Armington model. Each country is large in the sense that its demand and supply affect prices of the goods it trades. Consequently, unilateral reductions in trade barriers by one country lead to a troublingly large worsening in its terms of trade. Large terms of trade effects of tariff changes are a feature of the Armington model, even when the country concerned accounts for a small share of world production and trade. This is due to the Armington assumption itself. The assumption that each country is the sole supplier of the products it supplies to world markets gives it substantial monopoly power. This makes the terms of trade effects of tariff cuts in Armington models much larger than they would be in a standard neoclassical model.

Each of these three aspects of the Armington model contribute to understating the net gains to the liberalising country from unilateral trade liberalisation.

## Numerical illustration

To illustrate the properties of Armington models, we modify a small version of the GTAP model. This version has three product groups (food, manufactures and services), three factors (capital, labour and land) and three countries (the United States, European Union and the Rest of the World). The model has nationally differentiated final consumer goods and nationally differentiated intermediate inputs. With three product groups and three countries, there are nine nationally-differentiated products.

Three situations are considered. The first is a distorted competitive equilibrium in which the three countries have initial tariffs that restrict imports of products from the other two countries. This is illustrative of the real world. The second situation is one of free trade throughout the world economy. This is used to examine the properties of the competitive equilibrium under free trade. The third situation is a variant of the first in which one country unilaterally halves all its tariff rates.

The results of this numerical model illustrate the general properties of the model. In the free trade situation, neither final product prices nor factor prices are equalised. Final product prices are not equalised because the national products are differentiated, and therefore their prices are determined by the nationally differentiated cost structures. Factor prices are not equalised because product prices are not equalised.

Cutting tariffs across-the-board in the European Union shows one especially interesting feature of unilateral tariff reductions, that is, that reallocation in consumption is large, but resource reallocation across industries is small. Reducing tariffs uniformly does not change the relative prices of outputs across industries; therefore, there is little resource reallocation. This is reinforced by the assumption that consumers require at least some of each of the nine goods. Consequently, when the tariff reduction reduces the cost of imports, it leads to large adjustments in consumption in favour of imports, which contrast with the small adjustments in production.

This combination of results cannot be obtained in a Heckscher-Ohlin model where industries are divided into exporters and import-competing. In this situation, an across-the-board tariff reduction affects both types of industries differently and leads to resource reallocation.

## Implications and related issues

The final section contains observations on the limitations of Armington models. Features of Armington models mean that the mechanisms that affect modelled changes in the world economy are different from those that operate in Heckscher-Ohlin models. Most attention has focussed on the large terms of trade effects in Armington models, which are due to the monopoly power of nationally differentiated products.

The Armington assumption means that the number of industries is fixed and demand is usually implemented with constant elasticity of substitution (CES) preferences. This rules out an industry starting up or closing down as market conditions change. Similarly, it is not possible for consumers to stop or start buying particular products. Thus, adjustments by both producers and consumers in the model are understated.

Armington elasticities are sensitive to the equilibrium described in the initial database and to commodity aggregation. As a result, models should use estimates of the elasticities of substitution which are consistent with the base situation equilibrium. Model users should conduct sensitivity analyses routinely with respect to the levels of product and country aggregation as these profoundly affect the nature of trade (for example, the amount of intra-industry trade) as well as the gains from trade and the effects of changes in trade flows on the rest of the economy.

In summary, the peculiarities of the Armington assumption make it difficult to obtain reliable estimates of gains from trade. On the other hand, it makes it possible to use aggregated trade and input–output data in building global economic models.

The Armington assumption might be replaced by a Helpman-type or some other form of product differentiation. Another possibility might be to model trade in both homogeneous and nationally differentiated (Armington-type) products within each industry. Both solutions have the potential to mitigate some of the biases that the Armington assumption introduces in evaluating the gains and other results from trade liberalisation.

# 1 Introduction

In a well-known paper, Armington (1969) introduced the assumption that final products traded internationally are differentiated on the basis of the location of production. He assumed that in any one country each industry produces only one product and that this product is distinct from the product of the same industry from any other country. For simplicity, he assumed there is only one consumer in each country. In the eyes of this consumer, the products of one industry which originate in the different countries are a group of close substitutes. They form a product group which is separable in the consumer's utility function.

This Armington (1969) assumption of nationally differentiated products has been widely adopted in global computable general equilibrium (CGE) models to define demands for domestically produced and imported goods. When imports originate in more than one foreign country, these models usually assume that imports from different countries are differentiated from each other and form a group that is separable from the domestically produced product. CGE models which incorporate traded intermediate inputs also almost invariably assume that the domestically produced and the imported inputs used in one industry are imperfect but close substitutes, nested in a production function for the industry output. Together these assumptions on the demand side give a lot of structure to the trade model.

The Armington structure was introduced in CGE models to overcome problems that arose in early CGE modelling efforts. It accommodates 'cross-hauling', a phenomenon commonly observed in bilateral trade statistics, that is, a country appears to simultaneously export and import the same goods. This phenomenon cannot be explained in traditional trade models with homogeneous goods. Homogeneous goods can either be exported or imported, but not both at the same time.[1] The Armington assumption of product differentiation and imperfect substitution makes the existing trade statistics immediately usable for global trade models. The Armington structure also overcomes the problem that arises in a Heckscher-Ohlin-type model with more goods than factors that countries tend to specialise in only a few of the goods produced. It overcomes this problem by having specialisation in country-specific goods in each industry.

1 This ignores internal transport costs and other factors that may cause a homogeneous commodity to be both exported and imported by one nation.

In a survey of applied general equilibrium modelling, Shoven and Whalley (1984, p. 1046) commented:

> … because of the difficulties in accommodating a wide range of empirical phenomena in model building, there is often a tendency to depart from the essential structure and graft on *ad hoc* portions of the model not rooted in traditional theory … Unfortunately, the problem is, the models that make major departures from known theoretical structure can become difficult to interpret.

The introduction of Armington substitution is a departure from the assumption of perfect substitution that underlies traditional trade theory. The new structure may cause departures from known theoretical results. In fact, we find that the Armington structure does change fundamentally the properties of a trade model.

Chapter 2 specifies the Armington model. Chapter 3 develops the properties of the competitive equilibrium of this model. These are compared with the properties of standard Neoclassical models, especially the Heckscher-Ohlin model. In chapter 4, some illustrations are given from small dimensional CGE models with the Armington assumption. In light of the findings, chapter 5 offers some general comments on the interpretation of results from CGE models that employ the Armington assumption.

# 2 The Armington Model

## 2.1 Demand side

Armington (1969) considered a world in which there are m countries and n goods. Each good is a ‘type’ or ‘kind’ of good, one variant of which may be produced in each country. Consequently there are mn ‘products’ or commodities over which consumers have defined preferences. Thus the goods are differentiated by country of origin. The number of goods is fixed and therefore the total number of products in the model is fixed.

When the consumption of a good is differentiated in this way, it is necessary to have three subscripts attached to quantities and prices of products in order to identify the good itself, the country of origin of the good and the country in which the good is consumed. Let i = 1,…,n denote the type of good, j = 1,…,m the country of origin of the good, and k = 1,…,m the country in which the consumer is located. Thus the first two subscripts identify the product and the third the country in which it is consumed. Then $x_{ijk}$ is the consumption of a product from the i’th group when the product originates from country j and the consumer is located in country k.

In each country there is only one household. With Armington differentiation the utility function of the household of one country, country k, takes the form

$$U_k = U(x_{11k},\ldots,x_{1mk};\ \ldots;x_{i1k},\ldots,x_{imk};\ldots;x_{n1k},\ldots,x_{nmk}) \qquad \forall k$$

$$\equiv U(x_{1k},\ldots,x_{ik},\ldots,x_{nk}) \qquad (1)$$

$$\equiv U(x_k)$$

where $x_{ik} = (x_{i1k},\ldots,x_{imk})$ is the set of products of type i originating in the different countries and consumed in country k and $x_k$ is the set of the quantities consumed in country k of all products in all groups. It is assumed that the utility function is identical across countries, U(.).

Armington (1969) next assumed that, for any country k,

$$U(x_k) = V(v^1(x_{1k}),\ldots,v^i(x_{ik}),\ldots,v^n(x_{nk})) \qquad \forall k \qquad (2)$$

Thus, the utility function is assumed to be weakly separable in the types of goods. The sub-functions $v^i(x_{ik})$ are indices of the consumption in the country of each type of good. Moreover, these sub-functions are linearly homogeneous. Thus, the utility function is homogeneously separable. This has the important consequence that the household allocates its budget in two stages: first at the top stage or level among the groups of products and then at the bottom stage or level among the products within a group.

Armington adopted a CES form for the functions at the second level; *viz.*:

$$v^i(x_{ik}) = [\alpha_{i1k}x_{i1k}^{-\rho i} \ldots + \alpha_{ijk}x_{ijk}^{-\rho i} \ldots + \alpha_{imk}x_{imk}^{-\rho i}]^{-1/\rho i} \quad \alpha_{ijk} \geq 0,\ \rho_i > -1;\ \forall i, k \qquad (3)$$

When the technology for building multi-country CGE models of the world economy became available, the Armington assumption of national product differentiation was incorporated in these models. The first CGE model of a multi-country world economy was that of Shoven and Whalley (1984). Although they did not refer to Armington, they used the Armington differentiation in a single-level CES function for each industry or type of good.

Within a few years several authors adopted the Armington two-level structure and assumed that both the utility function at the top level, V(.), and all the sub-functions at the lower level, $v^i(.)$, were CES in form. This combination of constant upper- and lower-level elasticities of substitution implies the utility function is the two-level CES function introduced by Sato (1968) in production theory. The first to have done this in a fully-specified CGE model of the world economy with several countries were John Whalley in a series of papers starting from the mid-1970s (see Whalley (1985) and references therein), Miller and Spencer (1977) who built a model to analyse the effects of UK entry into the European Economic Community (EEC), and Lloyd (1979a; 1979b) who used the Whalley model to study intra-industry trade. These models were the first models of the world economy to incorporate the Armington assumption, and they were the first general equilibrium models of intra-industry trade. They were followed soon after by Deardorff and Stern (1981) in the first version of the Michigan model and other early multi-country models using the two-stage CES function (for a list, see Shoven and Whalley (1984) table 8).

In subsequent CGE models, a typical Armington structure for final demand assumes a three-stage budgetary allocation procedure. Expenditure is first allocated among goods. Expenditure on each good is then allocated between domestic and imported varieties. (See, for example, the ORANI model outlined in Dixon et al. (1982)

section 14.[1]) Finally, if more than one source of imports is acknowledged, expenditure on imports is allocated among competing national suppliers.[2] The whole structure can be neatly captured in a set of nested functions:

$$U(x_k) = V(v^1(x_{1k}),\ldots,v^i(x_{ik}),\ldots,v^n(x_{nk})) \qquad \forall k \qquad (4)$$

where

$$v^i(x_{ik}) = f^i(g^{ih}(x_{ikk}), g^{if}(x_{ifk}))$$

and,

$$g^{if}(x_{ifk}) = g^{if}(x_{i1k},\ldots, x_{ijk},\ldots, x_{imk})) \qquad j \neq k$$

Here, for convenience, h denotes the home country, the country in which the consumer is located, and f a foreign country. That is, the utility function is separable in types of goods, and the sub-functions for each type of good in turn are separable into the demand for the product originating in the home country and the demand for the products originating in foreign countries. In turn, the demands from foreign sources are themselves functions of the demand for each type of good supplied by each of the foreign countries. All of the functions $v^i$, $f^i$, and $g^{if}$ are linearly homogeneous.

The first stage allocation does not involve Armington differentiation. It is based on an ordinary utility function with preferences allocated over different goods that are substitutable. The form of this utility function can be any well-behaved function used in consumer theory. Winters (1984) argues that the homothetic CES form is too restrictive at this level. In the Armington model, the ‘goods’ in this utility function are not actual goods but composites of domestic and imported goods, which are aggregated from the second stage budgetary allocation. Only the second and third stage allocations are based on the Armington assumption of product differentiation.

Some builders of CGE models have assumed that the functions at all three levels are CES while others have assumed that they are CES at one or two levels. Some have assumed that, at some level, the functions are Leontief or Cobb-Douglas which are of course special cases of the CES function. The crucial assumption that determines an Armington model is that the utility function is separable in the manner of

1 While it treats the country concerned as a small country and is not therefore a model of the world economy, the ORANI model was a major input into the GTAP model of the world economy.

2 In some models a higher stage involving the allocation of the household’s budget between consumption and savings and sometimes the demand for government-supplied goods is added.

equation (2). Differentiation of the products in a group by country of origin makes a model an Armington model. Within this defining structure, there are a number of variations as in equations (2) or (4).

It should be noted that the Armington form of national differentiation of products is only one possible form of national differentiation. The post-Armington literature distinguishes between horizontal and vertical differentiation. The former refers to different varieties of a product and the latter to different qualities of a product. Armington does not discuss the nature of his product differentiation but it resembles the 'love of variety' type found in the models of Krugman (1979; 1980). Another form of horizontal differentiation is the 'most preferred variety' found in the models of Lancaster (1979; 1980) and Helpman (1981). In both the Krugman and Lancaster–Helpman models, countries produce and export differentiated products that are disjoint, that is, nationally differentiated. However, in these models each country produces and exports more than one product (or variety) and the number of varieties produced in each country is endogenously determined. Any of these forms of product differentiation can be introduced into a CGE model. For example, the current version of the Michigan model has adopted the 'love of variety' form along with scale effects and monopolistic competition, in place of the Armington assumption and constant returns to scale.

With the development of CGE models that incorporate intermediate inputs as well as primary inputs and international trade in intermediate inputs, the assumption that final goods are differentiated by nationality has been extended to the demand for intermediate inputs. For example, this assumption has been adopted in the families of models developed by Whalley (1985), Dixon et al. (1982) and Dixon and Rimmer (2002) and Hertel (1997). As with the demand for final goods, it is assumed that foreign-produced intermediate inputs used in any industry may be nested in the industry production function.

The assumption that final and intermediate goods are differentiated by nationality and form separable groups in the utility and production functions will be called the Armington assumption. The assumption, also made by Armington, that the functions at one or more levels are CES, is not regarded as part of the Armington assumption. The CES assumption does, however, affect the properties of the general equilibria that will be noted.

Armington (1969) specified only the demand side of the model.[3] The term 'Armington model' will be used to cover any model which makes the Armington

[3] Indeed, it should be remembered that Armington was not building a model of the world economy. He specified his demand structure in order to understand the changing composition of imports.

assumption on the demand side together with a standard Neoclassical specification of the supply side, that is, constant returns to scale and atomistic competition everywhere. This gives a family of models with a basic commonality in their structure.[4] These models are now dominant in CGE analysis of trade policy issues throughout the world.

## 2.2 Supply side

The supply side of the Armington model may be given compactly by a set of national product functions, one for each of the countries in the world economy. For any one producing country, country j, the function is

$G^j(p_j, e_j)$ where $p_j = (p_{1jj}, \ldots, p_{njj})$ is the vector of prices of the n goods produced and sold in country j.

$p_{ijj}$ is the producer price of the product in the i'th group produced in country j and sold in country j.

Each of the Armington-type products produced by the country is produced in a single-output industry. $e_j$ is the vector of endowments for the country; all factors are in fixed supply but they may be specific or non-specific.

There may be intermediate inputs used in the production of the goods. These intermediate inputs are the outputs of the industries in the home country and the foreign country. They are, therefore, like the final outputs, differentiated by country of origin. It is assumed that there are constant returns to scale in all industries and all product and factor markets are perfectly competitive.

$G^j(p_j,e_j)$ is the maximal value of national output, given the vector of producer prices and the constraints of technology and national endowments. In many Armington models, the technology is stated in terms of industry production functions and the endowments are specified separately. In such cases, the national product function can be derived from this information. This function contains all of the information regarding the technology of the country. It is not assumed, in general, that the technologies are identical across countries.

Assuming certain regularity conditions hold, this national output function is well-behaved. It is well-known that $G^j_p = y_j(p_j,e_j)$ and $G^j_v = w_j(p_j,e_j)$ where $G^j_p$ and

[4] The Armington demand structure has sometimes been combined with increasing returns to scale and/or imperfect competition. However, the simultaneous introduction of either of these features itself substantially changes the properties of the model and it is not, therefore, simple to determine the net effect of introducing the Armington structure.

$G_v^j$ are the vectors of derivatives with respect to product prices and endowments respectively, $y_j$ is the vector of outputs and wj is the vector of primary factor prices. These are the output supply functions and the factor price functions.

If required, the Armington structure may be imposed on the demand for intermediate inputs. The most common assumption is that there is a two-level demand function for intermediate inputs. At the top level, there is a demand in each industry for intermediate inputs supplied by the home country and by foreign countries collectively. At the second level, there is a function specifying the demand for foreign-supplied intermediate inputs from each of the foreign countries. (See, for example, Whalley (1985), Dixon et al. (1982) and Hertel (1997).)

Thus, the Armington model can be viewed as a standard Neoclassical model on the supply side with the addition of the Armington structure on the demand side. This viewpoint highlights the role played by the assumption of national differentiation of products.

This set up is sufficiently general to encompass a very wide variety of models. It has dimensions mn by h by m where mn is the number of products, h is the number of primary (non-produced) factors and m is the number of countries in the world economy. Each of these numbers may be small or large and there is no restriction, in general, on the number of factors (h) in relation to the number of goods produced in any country (n). The number of industries and the number of countries are usually small, though the numbers have tended to increase in revisions of CGE models. With national differentiation of the products of each industry, the number of products will be larger than the number of factors in almost all applications. Most commonly, it is assumed that the number of factors is two (non-specific labour and capital) (for example, Whalley (1985) and Dixon et al. (1982)) or three (non-specific labour, capital and land) (for example, GTAP; see Hertel (1997)), as in the standard Heckscher-Ohlin or specific factor models respectively. However, the Armington model is not a general model of national product differentiation as it has a special form of national horizontal differentiation of products that is exogenous and unexplained.

## 2.3 Competitive equilibrium

A competitive equilibrium for the world economy is one in which a set of prices for products and factors satisfies these supply and demand conditions and clears all markets. Under the assumptions we have made on the demand and supply sides of the model, a competitive equilibrium exists for the world economy. Multiple equilibria are possible but do not seem to arise in computable versions of the model.

In each country, the competitive equilibrium conditions for the production sector are given by the standard zero profit and full employment conditions. For brevity, we will consider a model with no intermediate input usage in each country.

$$A_j^t w_j = p_j \qquad \forall j$$

$$A_j y_j = e_j \qquad (5)$$

$A_j$ is the h by n matrix of physical input coefficients in the competitive equilibrium for the country concerned and $A_j^t$ is its transpose. As before, $p_j$ and $w_j$ are the (column) vectors of product and factor prices respectively, and $y_j$ is the column vector of industry outputs.

These conditions are taken to hold as equalities, rather than as inequalities combined with slackness conditions. The latter allow some zero outputs or some unemployed factors. It is assumed there is sufficient regularity in the technology to rule these out. In any case, the Armington assumption coupled with the assumption that the demand is CES require a strictly positive output of all products in all countries.

The n+h equations in (5) jointly determine the factor prices and industry outputs, $w_j = w_j(p_j, e_j)$ and $y_j = y_j(p_j, e_j)$, in country j. Consumer incomes are determined by the factor prices together with factor ownership. Consumer demand for each product depends on incomes and product prices.

We consider the properties of the competitive equilibrium of this Armington model in the next chapter.

# 3 Properties of the Armington model as a general equilibrium trade model

The properties of the Armington model are quite different in some respects from those of the Heckscher-Ohlin or the specific factor model.

## 3.1 The Law of One Price

Let $p_{ijk}$ denote the price of a product of industry i originating in country j and sold in country k. Assume now that there are zero trade costs in the world economy, that is, zero transport costs, tariffs and other government-imposed restrictions on international trade in the goods in all countries.

PROPOSITION 1.

*When there are zero trade costs in the world economy, the Law of One Price holds in the form;*

$$p_{ijk} = p_{ijl} \qquad \forall\ i, j \text{ and } l \neq k$$

$$\text{but } p_{ijk} \neq p_{ilk} \qquad \forall\ i, k \text{ and } l \neq j.$$

There is one price for each product throughout the world, a world price. This result holds because of perfect competition in a completely integrated world market for the good. However, in an Armington model, the outputs of the industry produced in different countries are imperfect substitutes for each other and will, therefore, sell for different prices throughout the world.

By contrast, in a standard Neoclassical model, such as the Heckscher-Ohlin or the specific factor model, it is assumed that the outputs of an industry originating in different countries are perfect substitutes. Assuming again that each industry produces a single output in each country, there is a single worldwide price for the output of an industry. That is;

$$p_{ijk} = p_{ijl} \qquad \forall\ i, j \text{ and } l \neq k$$

$$\text{and } p_{ijk} = p_{ilk} \qquad \forall\ i, k \text{ and } l \neq j. \qquad (6)$$

Or simply, dropping the second subscript, $p_{ik} = p_{il}$, for all i and all consuming countries k and l and $l \neq k$. Here $p_{ik}$ denotes the price of the product of industry i sold in country k. We can denote the world price of the output of industry i as $p_i$.

## 3.2 Comparative advantage

In the Armington model, one country cannot have a price advantage in producing one product relative to the prices of other products over other countries because the sets of products produced by any two countries are disjoint. There is no comparative advantage in the Armington model, except in the trivial sense that each country specialises in the production and export of the products which it alone can produce. There is nothing in the model to explain how this differentiation arises.

Yet, national endowments and technologies clearly play a role in determining the pattern of trade in the Armington model, as they do in other Neoclassical models. The homogeneous (undifferentiated) goods of the Heckscher-Ohlin model emerge as the limit case of the Armington model as the elasticities of substitution for each group $\sigma_i \rightarrow +\infty$ for all groups of products. This suggests that national endowments and technologies should play a role in determining the pattern of trade even when $\sigma_i < +\infty$.

The key to understanding the pattern of trade lies in the distinction between gross and net trade in the products of an industry. Gross trade is the aggregate value of exports and imports of the products classified in the industry. Net trade is the difference between the aggregate value of these exports and imports. This difference is sometimes called inter-industry trade. The total trade in the products of an industry is the sum of inter-industry trade and intra-industry trade in the products of the industry. That is, for industry i in country j;

$$X_{ij} + M_{ij} = |X_{ij} - M_{ij}| + \{X_{ij} + M_{ij} - |X_{ij} - M_{ij}|\} \qquad (7)$$

$X_{ij}$ and $M_{ij}$ are the value of exports and of imports of the variant(s) of industry i produced by country j and imported by country j respectively. Let $z_{ij}$ and $m_{ij}$ be the quantities of the goods exported and imported respectively. Then, $X_{ij} = (p_{ijj} z_j)$ and $M_{ij} = (\sum_{l \neq j} p_{ijl}\, m_{ij})$. This is the terminology of intra-industry trade theory.

In a model of intra-industry trade in which there are economies of scale, imperfect competition and two primary factors of production–capital and labour, Helpman (1981) proved that net or inter-industry trade is determined by factor proportions. He assumed that the technologies were identical across countries, as is done in the proof of the Heckscher-Ohlin theorem, and added a lot of symmetry among consumers.[1] In his model there are two factors, two goods and two countries. Helpman and Krugman (1985, part III) applied the same technique to the Krugman love-of-variety form of product differentiation in a model with two goods, two factors and two countries.

A similar result can be proven for the Armington model with dimensions mn by h by m. The different differentiated commodities in one group can be aggregated in the Armington model if there exist aggregators in both demand and supply with suitable properties and if the aggregation applies to the same groups in both demand and supply. These groups are the groups of products viewed by consumers as a separable group. For producers we can regard them as the products of an 'industry'.

> In this event the industry is a device which allows us to view the world economy at two levels, the industry level and the commodity or intra-industry level. At the top level the dimensions have been reduced from the number of elementary commodities to the number of aggregated commodities. (Lloyd 1994, p. 102).

Consider a free trade competitive equilibrium.

On the demand side, the utility function of the consumer in each country is homogeneously separable, as in equation (2). We can consider that the consumer in each country maximises utility by choosing the top-level quantities, given the prices in the competitive equilibrium. For each product group there is an aggregator function, $v^{ik}(x_{ik})$ in country k. This is a quantity index of the i'th group. It is now convenient to introduce a notation for these top-level groups. Bold letters will be used to denote aggregated top-level variables or coefficients. The quantity indices are indexed by I = 1,…,n. We have $\mathbf{v^I}(x_{ik})$. Each of these quantity indices has a dual price index;

$$\mathbf{p_{Ik}} = \boldsymbol{\varphi}^{\mathbf{I}}(p_{i1k},\ldots, p_{ink}) \qquad \forall I \text{ and } k \qquad (8)$$

This function is linearly homogeneous in its arguments. As the utility functions have been assumed to be identical across countries, the quantity and price indices

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1 Lloyd (1994) used the same technique in his analysis of the intra-industry model with intermediate inputs constructed by Dixit and Grossman (1982). He presents the results as properties which emerge from an explicit aggregation of supply and demand and a representation of the model at a top level. This reduces the dimensions of the model and allows us to use the familiar results of Heckscher-Ohlin trade theory.

are identical across the countries. With free trade, the prices faced by consumers are the same for each product and for each product group. There is no need, therefore, to attach a country of consumption subscript. Thus, the $p_{ijk}$ are identical in all consuming countries and the price index $\mathbf{p_{Ik}} = \mathbf{p_I}$ applies in all countries. Aggregation in demand is consistent in each country in the sense that expenditure on the aggregate quantity at the top level equals the sum of the expenditures on the products in the group in the full competitive equilibrium, *viz.* $\mathbf{p_I}\ \mathbf{v^{Ik}}(x_{ik}) = \sum_j p_{ijk} x_{ijk}$. This is a consequence of homogeneous separability.

In each consuming country k, the demand for an industry's products is;

$$\mathbf{x_{Ik} = x_{Ik}(p_{Ik}, w_k)} \qquad \forall \text{I and k} \qquad (9)$$

These functions are the product of maximising utility at the top level.

On the supply side, no aggregation is possible if the technologies differ across countries. If, however, the technologies for producing each variant of the good produced in each country are all identical, then the aggregator function is simply the production function for each and every product in the group. Suppose the technology is known in terms of production functions for each product at the bottom level, $y_{ij} = \psi_{ij}(e_{ij}) = \psi_i(e_{ij})$ where $e_{ij}$ is the usage by industry i of the factors available in country j. Then, at the top level, the industry production functions are;

$$\mathbf{y_{Ij} = \psi_I(e_{Ij})} \qquad \forall \text{I} \qquad (10)$$

Each production function has a dual cost function. The producer prices will equal unit costs in the industry;

$$\mathbf{p_{Ij}} = \mathbf{c_{Ij}}(w_j) \qquad \forall \text{I, j} \qquad (11)$$

For aggregation in both demand and supply simultaneously, the prices of these industry outputs in equation (11) must be equal to the prices $\mathbf{p_I}$ given by equation (8). Now $\mathbf{p_{Ij} = p_{Il}}$. But $\mathbf{p_{Ij}} = p_{ij}$, the price of the one product produced by country j and $\mathbf{p_{Il}} = p_{il,}$ the price of the one product produced by country l. Therefore, it must be that $p_{ij} = p_{il}$.

This equality of product prices will hold if, on the supply side of the model, there is factor price equalisation in the model. On the demand side, the equality of product prices will hold if there is complete symmetry of demand in that the $\alpha_{ijk}$ are identical for all j products and all k countries in the group i. In this case, the Law of One Price holds.

We have now all of the elements of a top-level competitive equilibrium. In the production sector, the conditions for a top-level equilibrium are;

$$\mathbf{A_j}^{\mathbf{t}} w_j = \mathbf{p_j}$$

$$\mathbf{A_j y_j} = \mathbf{e_j} \qquad \forall j \qquad (12)$$

where $\mathbf{p_j} = (\mathbf{p_{1j}},\ldots,\mathbf{p_{Ij}},\ldots,\mathbf{p_{nj}})$ whose elements are now regarded as the prices of the industry aggregates and the input shares are defined for these outputs. These conditions have the same form as in equation (5) for the top level but the outputs and prices are now those of the industry rather than of the individual product variant produced in each country. Here, the same equations apply for all countries. This technique of aggregation via the assumptions of identical technologies for producing each of the products within a group and across countries and symmetry of demand is in essence the same technique as that used by Helpman (1981) and by Helpman and Krugman (1985) in their models of intra-industry trade with differentiated products of the Helpman-Lancaster type and the Krugman type.

To verify the aggregation is consistent for trade flows, we define the exports or imports of country j at the top level as;

$$\mathbf{X_{Ij}} \textbf{ or } \mathbf{M_{Ij}} = \mathbf{p_{Ij}\, y_{Ij} - p_{Ij}\, x_{Ij}}$$

$$= p_{ijj}\, y_{ij} - \sum_j p_{ijj} x_{ij} \qquad \text{[by definition of terms]}$$

$$= (p_{ijj}\, y_{ij} - p_{ijj}\, x_{ij}) - (\textstyle\sum_{1 \neq j} p_{ijl}\, m_{ij}) \text{ [since } x_{ij} = m_{ij} \text{ for } j \neq 1 \text{ ]}$$

$$= X_{ij} - M_{ij} \qquad (13)$$

The absolute value of this term is the value of inter-industry trade as defined in equation (7). That is, because both consumption and production of the products can be consistently aggregated, exports or imports (that is, excess demand) can also be consistently aggregated. Inter-industry trade in the full competitive equilibrium is equal to the excess demand for the good at the top level of the model.

This top-level competitive equilibrium is identical to that of a Heckscher-Ohlin model of the same dimensions, namely n goods, h factors and m countries. Consequently, this top-level competitive equilibrium has the same properties as a Heckscher-Ohlin model of the same dimensions.

In particular, the principle of comparative advantage applies at this level.

PROPOSITION 2.

*If there are zero trade costs and the conditions necessary for consistent aggregation are satisfied on both the demand and supply sides of the model, factor proportions determine the trade in top-level goods.*

If we make the additional assumption that the top-level utility function is homothetic, then the Heckscher-Ohlin theorem applies exactly as for a Heckscher-Ohlin model of the same dimensions. For example, if the dimensions of the model are 2 by 2 by 2, as in the standard textbook version of the Heckscher-Ohlin model, and we assume that the top-level utility function is homothetic, then the quantity version of the Heckscher-Ohlin theorem holds — each country has (inter-industry) exports of the good which uses intensively the factor with which it is well endowed. Furthermore, if the factor endowment ratios of the two countries as well as the technologies are identical, there is no inter-industry trade (but there is still intra-industry trade). If, instead, the dimensions of the model are h by m by n and we assume that the top-level utility function is homothetic, then the factor content form of the factor proportions theorem holds as in Vanek (1968) — each country is an exporter/importer of the services of the factor with which it is well/poorly endowed relative to the world.

If nationally differentiated intermediate inputs and international trade in these inputs are introduced, aggregation of the demand for and supply of these intermediate inputs can proceed in the same manner. Net or inter-industry trade is now a combination of trade in final products and trade in intermediate inputs, but there is a top-level representation of the model which is identical to a Heckscher-Ohlin model of the same dimensions with intermediate inputs.

Thus, we use the Heckscher-Ohlin theorem to explain inter-industry trade. Trade in the elementary products, intra-industry trade, is determined by national product differentiation and preferences defined over these differentiated products. Although the conditions required for this result are extremely stringent and require a lot of symmetry in the Armington model, this result gives a fundamental insight into the determination of the pattern of trade in the model. This result is not surprising when we view the Armington model as if it starts with a standard Neoclassical model with homogeneous outputs in each industry and then add Armington differentiation.

In the Armington model there are three causes of trade — differences in endowments, differences in technology and product differentiation. If both the endowments and the technologies are identical across all countries, product differentiation alone is sufficient to cause trade. By comparison, under these conditions, there would be no trade in a Heckscher-Ohlin model.

If production technologies are not identical across countries, the competitive equilibrium cannot be aggregated into a top-level competitive equilibrium. Factor proportions, and differences in technologies and product differentiation will all interact to determine jointly the pattern of trade in the competitive equilibrium.

We can obtain a third proposition concerning the pattern of trade that is related to the Armington elasticities.

PROPOSITION 3.

*Consider an initial competitive equilibrium for the world economy. Assume that there are zero trade costs and that the conditions sufficient for consistent aggregation are satisfied on both the demand and supply sides of the model. If the elasticities of substitution between home and foreign-produced goods increase, then the proportion of intra-industry trade decreases. As* $\sigma_i \rightarrow +\infty$ *for all groups of products, this proportion tends to zero.*

In the initial competitive equilibrium, inter-industry trade is determined by the top-level conditions. These do not change. At the second level, the indifference map is CES. Suppose initially there are only two countries, the home country and the foreign country. If the second-level elasticity of substitution increases, each indifference curve becomes flatter (see Chipman 1966, p. 58).[2] The combination of home and foreign products chosen in the initial equilibrium is no longer optimal. As the indifference curve at the initial equilibrium has become flatter, the consumer in each country will move towards the axes. The consumer will buy more of the product that is cheaper. The consumer will tend to buy from a single source. In contrast, a low elasticity of substitution forces a more even mixture of products.

If there is more than one foreign country, suppose the functions at both the second and third levels of equation (4) are CES. Again intra-industry trade decreases from the level observed in the initial competitive equilibrium. At the third or lowest level, the same kind of substitution occurs. Consumers will buy more of the foreign products of each industry from the cheapest foreign source.

## 3.3 Gains from trade

Measuring gains from trade in an Armington model is problematic. Here the assumption of the functional form of the second-level functions, or of the second- and third-level functions if there is a three-stage budget allocation process,

2 In fact, the indifference map pivots around some ray from the origin — for example, if the $\alpha$ coefficients are equal, this ray is the ray that bisects the quadrant.

is critical. If the form is CES, the final goods sourced from each country in the model are essential in that, in every country, a strictly positive quantity of each good is demanded at anything less than an infinite (relative) price for the product. Geometrically, the isoquants in an indifference map defined in product space do not intersect the axes or intersect them asymptotically.

This property of essentiality in demand has the important consequence that an economy cannot be closed in a world with finite tariff rates. Lloyd and MacLaren (2002, pp. 77–78) demonstrated this numerically in a GTAP model. The usual measure of gains from trade is the elevated level of welfare with free trade, or with some trade in a tariff-distorted equilibrium, compared to the level with no trade. The standard gains from trade propositions do not hold, strictly speaking, in an Armington model. However, one can get arbitrarily close to no trade by continuously raising tariff rates.

In a tariff-distorted situation, there will still be gains from trade liberalisation, as in a Heckscher-Ohlin model with trade costs. CGE models using the Armington assumption have proven a useful vehicle for exploring gains from greater trade. There are, however, several reasons why CGE estimates of these these gains are inaccurate.

In an Armington model, strictly speaking there is no comparative advantage, as noted in the previous chapter. Consequently, gains from trade due to greater specialisation in goods in which a country has a comparative advantage, and greater importing of goods in which it has a comparative disadvantage, cannot arise. These are the standard gains in a Heckscher-Ohlin or a Classical model of international trade.

Gains from greater trade may be understated for a second reason. One effect of trade liberalisation is an increase in the number of varieties. Hertel and Tsigas (1997, p. 41) acknowledge fixity of the number of varieties as a 'fundamental critique' of the Armington assumption. The standard Neoclassical model also has a fixed number of commodities. However, this fixity is more serious in an Armington Model than in a Neoclassical model. In a Neoclassical model, the number of industries or broad product groups can be regarded as fixed. In an Armington model, however, the focus is on differentiation of products within a group. Changes in the composition of the group will occur when relative prices and incomes change with any shock to the general equilibrium. One part of this change is that the numbers of products produced and consumed in each group will normally change. This intra-industry variation is ruled out by the Armington assumption. In comparing the Armington assumption with the treatment of differentiated products in the Krugman model, Feenstra (2004, p. 166) notes that:

> The fact that the number of varieties per country is fixed in the Armington assumption gives it quite different properties than monopolistic competition, where the number of varieties produced in each country varies due to free entry.

Another problem with estimation of gains from greater trade in an Armington model stems from the terms of trade effect of trade liberalisation. In the Armington model, as in the Heckscher-Ohlin model, each country is large in the sense that its demand and supply affect prices of the goods it trades. Consequently, unilateral reductions in trade barriers by one country lead to a worsening in the terms of trade. Large terms of trade effects of tariff changes are a feature of the Armington model, even when the country concerned accounts for a very small share of world production and trade. Zhang (2006) shows that this is due to the Armington assumption itself. The assumption that each country is the sole supplier of the products it supplies to the world markets gives it substantial monopoly power. This makes the terms of trade effects of tariff cuts in Armington models much larger than they would be in a standard Neoclassical model and thereby understates the net gains to the liberalising country from greater trade.

Finally, the gains from trade are sensitive to the degree of country aggregation. If the number of countries increases, other things remaining equal, the number of differentiated products in each group increases and welfare in all countries increases. This property is similar to the property of the Krugman model with horizontally differentiated products and CES utility functions that defined preferences over closely substitutable products but it emerges here solely as a function of the level of aggregation in the model.

## 3.4 Factor price equalisation

Factor prices are not, in general, equalised across countries in the Armington model when product prices are equalised with the absence of trade costs. Non-equalisation of factor prices is a consequence of the Law of One Price not holding in the usual form. With zero trade costs, the price in country k of country k’s output from industry i is not identical to the price in country k of country l’s output from industry i because the products are differentiated, not homogeneous. There is no single world price for the output of an industry and, hence, there can be no unique mapping of goods prices to factor prices that holds across countries. However, we can find a special condition under which factor prices are equalised.

PROPOSITION 4.

*Factor prices are equalised with zero trade costs if the conditions necessary for consistent aggregation are satisfied on both the demand and supply sides of the model, $h = n$, and the country endowments lie in the same cone of diversification.*

Under the first set of conditions, as noted in the previous chapter, the products in the model may be aggregated and we may write the model at the top level as if it is a lower-dimensional Heckscher-Ohlin model. The other conditions reproduce a combination of assumptions that is sufficient for factor price equalisation in the Heckscher-Ohlin model. These conditions are extremely unrealistic.

## 3.5 Other properties

The Stolper-Samuelson and the Rybcynski theorems continue to hold in the same form as they hold for a Heckscher-Ohlin or specific factor model of a given dimensionality. This is true because these two theorems are comparative static theorems that apply to one country in the world economy, given world prices for the outputs it produces. By derivation, these responses in one country have nothing to do with the structure of production in any other country.

# 4 Illustrations from an Armington CGE model

Computable general equilibrium models with the Armington assumption have the properties of the Armington general equilibrium model as they are versions of the Armington model with specific numeric values assumed for all of the parameters of the model. In this chapter we use a small dimensional version of the GTAP model to illustrate the properties of Armington models.

The model used is loosely based on the 3 by 3 GTAP aggregation used for teaching and demonstration purposes. This version has three product groups (food, manufactures, and services), three factors (capital, labour and land) and three countries (the United States, European Union and the Rest of the World). With three product groups and three countries, there are nine products. The dimensions of the model, therefore, are 9 by 3 by 3. The model has nationally differentiated final consumer goods and nationally differentiated imported intermediate inputs. The utility functions and the production functions are two-stage CES functions, as specified in chapter 2. The details of the model are available from the GTAP website at www.gtap.agecon.purdue.edu.

In the GTAP version, there is a government and a private sector. In our modified version, the government sector is dropped and the number of factors is reduced to two: capital and labour. The endowments are reset so that the United States is capital-abundant (capital 2 and labour 1.5), the Rest of the World is labour-abundant (capital 1.5 and labour 2) and Europe has an intermediate relative endowment (capital 1.5 and labour 1.5). The technologies are identical across the three countries.

The competitive equilibrium for any specification of the model can be computed directly using Microsoft Excel. We begin the iteration with product prices around unity. This produces an equilibrium with product prices around unity. By contrast, the GTAP version implicitly sets all goods and factor prices at unity in all countries which does not allow a test of goods and factor price equalisation/convergence as the world economy moves to free trade. However, the changes in endowments and the government sector, as well as in the method of normalisation, mean that the model no longer replicates the flows for the United States, European Union and the

Rest of the World in the base situation of the GTAP version. The main advantage of our version of the model is that it corresponds to the general model of chapter 2.

We are particularly interested in the elasticity of substitution parameters and the tariff rates. These are taken from the GTAP database and are set out in table 4.1. The elasticities of substitution are the top-level elasticities which are assumed to be the same for the use of intermediate inputs in production and for final consumer demand. The bottom-level elasticities are double those of the top level. The production technologies at the top-level are Cobb-Douglas. The tariff rates are higher on food products than on manufactures in all countries and, for each good, they are highest in the Rest of the World and lowest in the United States. The differences in table 4.1 for one country between the tariff rates on imports from each of the other two countries reflect preferences for some imports from particular sources. The share parameters of the utility and production functions are assumed to be the same for all countries. Good 1 is labour-intensive, good 3 is capital-intensive and good 2 is of neutral factor intensity.

Table 4.1 **Elasticities of substitution and *ad valorem* tariff rates**

| *Sector* | *Substitution elasticity* | *US imports from* | | *EU imports from* | | *RoW imports from* | |
|---|---|---|---|---|---|---|---|
| | | EU | RoW | US | RoW | US | EU |
| | | % | % | % | % | % | % |
| Good 1 | 2.4 | 10.0 | 11.7 | 36.9 | 41.6 | 65.6 | 16.8 |
| Good 2 | 2.8 | 9.6 | 9.8 | 5.9 | 7.8 | 9.9 | 11.8 |
| Good 3 | 1.9 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

Three situations are considered. The first is a distorted competitive equilibrium in which all of the three countries have initial tariffs that restrict imports of products from the other two countries. This is illustrative of the real world. The second situation is one of free trade throughout the world economy. This is used to examine the properties of the competitive equilibrium under free trade. The third situation is a variant of the first in which one country unilaterally halves all tariff rates. This illustrates some of the effects of unilateral liberalisation in an Armington model.

In the first tariff-distorted situation, the competitive equilibrium exhibits some interesting features. There is incomplete specialisation in the extreme form — every country produces its national variety of the product from each of the three product groups. Comparative advantage is a more complex notion in a model with three countries and three goods. The United States, the most capital-abundant of the three countries, is a net exporter of the products of industry 3, the most capital-intensive of the three industries, and a net importer of the products of industry 1, the most labour-intensive of the industries. Conversely, the Rest of the World, the most labour-abundant of the countries, is a net exporter of the products of industry 1 and a net importer of the products of industry 3. The net exports of goods by the middle country, the European Union, balance the world demand and supply of these goods. Goods prices are not equalised. The 3 by 3 matrix in table 4.2 shows the set of goods prices, $p_{ijj}$. These are the prices at the border in the country of production, before the imposition of border taxes when the goods are shipped to the two foreign countries. The prices of the goods delivered to the foreign countries can be obtained by multiplying the basic prices by the relevant factor $(1 + t_{ijk})$ where $t_{ijk}$ are the *ad valorem* tariff rates on the product of good i produced in country j and imported by country k, as reported in table 4.1.

Table 4.2 **Changes in basic prices of goods and factors with initial tariff distortions**

Percentage changes

| | *US* | *EU* | *RoW* |
|---|---|---|---|
| Good 1 | 1.00 | 1.07 | 0.92 |
| Good 2 | 0.98 | 1.06 | 0.98 |
| Good 3 | 0.95 | 1.06 | 1.04 |
| Capital | 0.47 | 0.56 | 0.65 |
| Labour | 0.56 | 0.58 | 0.39 |

Table 4.2 also shows the set of factor prices. As expected, factor prices are not equalised. The capital-abundant United States has a low relative rental price of capital while the labour-abundant the Rest of the World has a low relative wage rate for labour.

Trade is a mixture of traditional inter-industry trade and of intra-industry trade. For all countries combined, intra-industry trade comprises 90 per cent of the total value of world trade. This unrealistically high proportion is due to the very high level of aggregation in the model with only three product groups.

We can now use the model to explore the effects of an increase in the elasticities of substitution between domestic and imported products, $\sigma_i \rightarrow +\infty$ for all groups of products. An increase in this elasticity of substitution by a factor of 5 reduces the proportion of intra-industry trade to 80 per cent. This shows that the result in proposition 3, that in a free trade situation, the proportion of intra-industry trade decreases as $\sigma_i \rightarrow +\infty$ for all groups, can also hold in a tariff-distorted situation.

In the second situation (free trade), the competitive equilibrium illustrates the propositions derived in chapter 3. The Law of One Price holds in the form stated in proposition 1. The Vanek factor content theorem holds: through product trade each country exports/imports the services of the factor with which it is well/poorly endowed relative to the world. Table 4.3 reports the prices of capital and labour in each country in the free trade situation. Factor prices are not equalised. (If the endowments of the three countries were also set equal to each other, there would be factor price equalisation. In this case, there is complete symmetry between the three countries. Even though they still produce nationally differentiated products, the countries and the products differ only in name.) Comparing these factor prices with those in the tariff-distorted situation, we see that factor prices converge between the initial tariff-distorted situation and the free trade situation in the capital markets but not in the labour markets. The proportion of intra-industry trade increases from 90 per cent of the total world trade to 95 per cent.

Table 4.3 **Changes in basic prices of goods and factors under free trade**
Percentage changes

| | *US* | *EU* | *RoW* |
|---|---|---|---|
| Good 1 | 0.99 | 0.99 | 0.87 |
| Good 2 | 0.96 | 0.99 | 0.93 |
| Good 3 | 0.92 | 1.00 | 1.00 |
| Capital | 0.46 | 0.57 | 0.64 |
| Labour | 0.59 | 0.55 | 0.39 |

In the third situation, all tariff rates in the European Union alone are halved. We use this example to illustrate the behaviour of Armington models in the kind of tariff-distorted situations for which GTAP and other Armington models are commonly used.

The United States and the Rest of the World gain from this unreciprocated trade liberalisation as the utility levels both increase by 0.7 per cent. The European Union loses slightly from its own trade liberalisation (-0.3 per cent). The loss from own-country liberalisation is a result of the worsening in the terms of trade of the tariff-cutting country. When the European Union halves its tariff rates on products 1 and 2, the terms of trade for industries 1 and 2 decline by 2.25 and 1.19 per cent respectively. This implies a deterioration in the European Union's terms of trade of 1.79 per cent. In this example, the three countries are roughly equal in size in terms of GDP and their shares of world trade.

As well as changes in the terms of trade and utilities, table 4.4 shows the changes in production and consumption of the three products in each country and in the international trade of each product. The changes in exports and imports are of course equal to the difference of the changes in consumption and production.

Table 4.4 **Effects of a 50-per-cent reduction in the European Union's tariff rates**

Percentage changes

| | *US* | *EU* | *RoW* |
|---|---|---|---|
| Terms of trade | -1.05 | -1.79 | 0.58 |
| Utility | 0.74 | -0.29 | 0.69 |
| Output 1 | 2.03 | -2.67 | 1.79 |
| Output 2 | -1.47 | 2.76 | -0.76 |
| Output 3 | -1.44 | 2.56 | -1.54 |
| Domestic consumption 1 | -1.16 | -10.75 | -0.92 |
| Domestic consumption 2 | -1.05 | -2.39 | -0.38 |
| Domestic consumption 3 | -0.58 | -0.08 | -0.06 |
| Export 1 | 9.72 | 7.82 | 7.32 |
| Export 2 | -2.27 | 9.39 | -0.44 |
| Export 3 | -2.38 | 4.81 | -2.74 |

These results show one other interesting feature. When the European Union cuts its tariff rate on imports of the products of industry 1, the most highly protected industry, the response of domestic consumption of the home variety is much larger than the response in domestic production. This is typically the case in simulations of tariff cuts in Armington models.

It is not possible in an Armington model with CES preferences for changes in tariff rates to cause an industry to close down or to start up. Similarly, it is not possible for consumers to stop or start buying particular products. These features understate the adjustments in a model.[1] If the assumption that preferences are CES were dropped, it would then be possible in an Armington model for consumers and producers to change their product mixes. The total number of products that could be produced and consumed would still be fixed, however, fixity of the number and nature of the differentiated products cannot capture the changes in market shares and the welfare effects due to the introduction of new product varieties as tariffs change. This also applies if technologies and other market conditions change.

1 This aspect is exacerbated by the high level of aggregation of goods in most CGE models.

# 5 Limitations of the Armington model

The main builders of the CGE models based on the Armington assumption seem to regard it as a satisfactory approximation to the product differentiation observed in the real world. In discussing his suite of models, Whalley (1985, p. 38) states: ‘though not wholly satisfactory (in that cross-hauling is accommodated rather than explained), the use of the Armington assumption in models here has had some benefits’. In discussing the nature of the GTAP model, Hertel and Tsigas (1997, p. 41) conclude that ‘although we are not particularly happy with the Armington specification, it *does* permit us to explain cross-hauling of similar products and to track bilateral trade flows’. In discussing the Monash model, Dixon and Rimmer (2002, p. 30) state that, ‘the Armington approach is not suitable for all commodities’. They employ a non-Armington specification for production of agricultural commodities and some services. Thus, the consensus view of builders seems to be that the Armington assumption is generally a workable and satisfactory approximation.

As noted above, some authors have commented on the restrictiveness of the property that there is an unchanging number of differentiated commodities in an Armington model. Most attention has focused on the large terms of trade effects of tariff changes in an Armington model (see particularly, Shoven and Whalley (1984, p. 1043); Brown (1987); Shiells and Reinert (1993); Saito (2004); Zhang (2006)). This debate has been about the values of the elasticity of substitution parameters at various levels.

There is, however, very limited recognition of the peculiarity of the Armington assumption and its effects on analyses that use models incorporating this assumption. We have seen that the Armington model has a particular form of national horizontal differentiation of products. The Armington model does introduce product differentiation as a cause of international trade, in addition to factor endowments and differences in technologies. This was a big advance in trade theory. But the exogeneity of Armington differentiation removes comparative advantage in the trade of products. Moreover, it does not allow any changes in product varieties on the global markets as tariffs, technologies and other market conditions change. Adjustments by producers and consumers are understated while the terms of trade effects of policy changes are overstated.

CGE model builders should take a number of steps to mitigate the limitations of Armington models. Builders of existing Armington models should use estimates of the elasticities of substitution parameters which are consistent with the base situation equilibrium and also routinely conduct sensitivity analyses with respect to the levels of product and country aggregation because these profoundly affect the nature of trade and its gains and effects. In the longer term, they should try to endogenise national product differentiation. Another possibility is to combine trade in both homogeneous and nationally differentiated products in each industry. This would overcome the peculiarities of the Armington model and enrich analyses of trade issues. In interpreting the results of simulations from Armington models, all users should be aware of the properties of the model and their implications for modelling policy changes.

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