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# ECEC model specification

This document provides a description of the Commission's early childhood education and care (ECEC) partial equilibrium model. It is also a guide for anyone interested in using the model. Section 1.1 provides a description of the downloadable files and how to install them. Section 1.2 gives a brief outline of the rationale and purpose of the model, and an intuitive explanation of the mechanics of the model. Section 1.3 details the sets, variables and parameters contained in the model. Section 1.4 outlines the objective function and constraints in the model. Section 1.5 describes the data used to calibrate the model.

All demand and supply functions were calibrated with a point elasticity, a price–quantity point, and an assumed linear function.

## 1.1 Using the model

The model comprises a number of files that are to be solved using the General Applied Modeling Software™ (GAMS). GAMS is a high-level modelling system for mathematical programming and optimisation. While it is normally necessary to obtain a license to use GAMS, a free trial version is available for download and use (<http://www.gams.com/download/>).

Experience with mathematical programming is an advantage when using the following files. To download and install **ECEC\_MODEL\_FILES**:

- create a new directory into your GAMS working directory. Give it a name (say, **ECEC model**)
- download and save this document into that directory (**ECEC model**)
- download and save the zipped **ECEC\_MODEL\_FILES** into that directory (**ECEC model**)
- unzip **ECEC\_MODEL\_FILES**
- open GAMS and create a new project in the GAMS working directory. Give the project a name (say, **ECEC model project**)
- you will now be able to run the **ECEC model.gms** in your working directory (**ECEC model**).

The contents of ECEC\_MODEL\_FILES are listed in table 1.1.

**Table 1.1 Contents of ECEC\_MODEL\_FILES**

<i>File name</i>	<i>Type</i>
ECEC model.gms	Contains the main code
HighElasticity6.gms	Data. Contains estimates of high elasticities of labour supply
MidElasticity6.gms	Data. Contains estimates of mid elasticities of labour supply
LowElasticity6.gms	Data. Contains estimates of low elasticities of labour supply
HighOwnPrice6.gms	Data. Contains estimates of high own price elasticities of ECEC demand
MidOwnPrice6.gms	Data. Contains estimates of mid own price elasticities of ECEC demand
LowOwnPrice6.gms	Data. Contains estimates of low own price elasticities of ECEC demand
LowTeacherDemands6.gms	Equations and data. Contains estimates of the proposed additional staffing requirements resulting from the National Quality Standard (NQS).
LowTeacherRecal6.gms	Equations and data. Contains estimates of the proposed effects of the 'working towards' provisions of the NQS.
NPA_ECE_Reform6.gms	Equations and data. Contains estimates of the additional number of children attending preschool programs under the National Partnership Agreement on Early Childhood Education.
SupplyElas6.gms	Intentionally left blank
NoSupplyElas6.gms	Equations and data. Contains estimates of the assumed elasticity of labour supply with respect to the price of early childhood education.
zcalc6.gms	Equations. Used to calculate staffing requirements.
zdata6.gms	Data.

## 1.2 Model structure

The Commission has developed a partial equilibrium model of the ECD sector to provide insights into the impact of policy reforms. Specifically, the model is used to provide broad estimates of the likely employment, price (both to households and suppliers), and expenditure (for Australian, state and territory governments, and households) outcomes that could flow from policy changes. These changes are examined for a range of elasticity and funding assumptions.

The model simulates a market for four kinds of ECD services: preschool; long day care; family day care and in-home care; and occasional care. The Australian Government, as well as state and territory governments, provide subsidies for the consumption of ECD services, such that households do not pay the full cost of the services they consume. Demand for ECD services is assumed to be linear and downward sloping, with no cross-price substitution between the respective services.

Demand is a function of household income, though household income is specified exogenously.

Suppliers produce ECD services from two kinds of inputs: capital and labour. Capital is demanded as a linear function of output and the rental rate, and is supplied infinitely at the market rental rate. Labour is demanded in fixed proportions to output. There are five types of labour in the market (bachelor qualified; diploma qualified; certificate III-IV qualified; certificate I-II qualified; and unqualified) and each has an upward sloping linear supply curve with respect to the wage. Labour supply is also a negative function of the price of ECD services: since a large proportion of ECD services are utilised by working women, an increase in the price is assumed to make it more difficult for these women to work, and therefore reduces their supply of labour. All four types of ECD services draw labour from a common labour market.

Total household expenditure, Australian, state and territory government expenditures were also calculated as part of the modelling.

### 1.3 Model sets, variables and parameters

Table 1.2 Sets

<i>Set</i>	<i>Name</i>	<i>Components</i>	<i>Description</i>
Industry	j	Pre	Preschools
		LDC	Long day care
		FDC	Family day care and in home care
		Occ	Occasional care
Labour type	n	B	Bachelor degree or higher qualification labour
		A	Advanced diploma and diploma qualified labour
		C3	Certificate III and IV qualified labour
		C1	Certificate I and II qualified labour
		U	Unqualified labour

Source: Productivity Commission model

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**Table 1.3 Variables in the model**

<i>Variable</i>	<i>Units</i>	<i>Description</i>
$NW$	\$	Net welfare, sum of Marshallian consumer and producer surplus
$Qd_j$	Children	Quantity of services demanded from industry j
$Qs_j$	Children	Quantity of services supplied by industry j
$Pd_j$	\$	Unit cost of production for industry j
$Ps_j$	\$	Out-of-pocket unit cost of services from industry j to households
$Ld_{j,n}$	Persons	Quantity of labour type n demanded by industry j
$Ls_n$	Persons	Supply of labour type n
$W_n$	\$	Annual wage of labour type n
$Kd_j$	Establishments	Quantity of capital demanded by industry j
$Ks_j$	Establishments	Quantity of capital supplied to industry j
$R_j$	\$	Annual rental rate of capital for industry j
$ExpA$	\$	Total expenditure by the Australian government on ECD
$ExpS$	\$	Total expenditure by State and Territory governments on ECD
$ExpH$	\$	Total expenditure by Households on ECD
$ExpT$	\$	Total expenditure on ECD

Source: Productivity Commission model

**Table 1.4 Parameters in the model**

<i>Parameter</i>	<i>Units</i>	<i>Description</i>
$y$	\$	Annual household income
$aD_j$	\$	Constant for the inverse linear demand curve for industry j
$bD_j$	\$/child	Slope ( $Qd_j$ ) for the inverse linear demand curve for industry j
$cD_j$	..	Slope ( $y$ ) for the inverse linear demand curve for industry j
$aL_n$	\$	Constant for the inverse linear supply curve for labour type n
$bL_n$	\$/person	Slope ( $Ls_n$ ) for the inverse linear supply curve for labour type n
$cL_{j,n}$	..	Slope ( $Pd_j$ ) for the inverse linear supply curve for labour type n
$mL_{j,n}$	Children/person	Fixed proportions labour requirement per unit of output
$aK_j$	Establishments	Constant for the linear capital demand curve from industry j
$bK_j$	Establishments/child	Slope ( $Qs_j$ ) for the linear capital demand curve from industry j
$cK_j$	Establishments/\$	Slope ( $R_j$ ) for the linear capital demand curve from industry j
$rbar$	\$	Fixed rental rate of capita
$\sigma S_j$	..	Ad valorem subsidy from State and Territory Governments
$\sigma A_j$	..	Ad valorem subsidy from the Australian Government
$sCCB_j$	\$	Specific subsidy rate (child care benefit)
$sOTH_j$	\$	Other specific subsidies

Source: Productivity Commission model

## 1.4 Model specification

### Objective function

$$\begin{aligned}
 NW = & \sum_j \left( aD_j + \frac{1}{2} bD_j \cdot Qd_j^2 + cD_j \cdot y \right) \\
 & - \sum_n \left( aL_n \cdot Ls_n + \frac{1}{2} bL_n \cdot Ls_n^2 + \sum_j cL_{j,n} \cdot Pd_j \cdot Ls_n \right) - \sum_j R_j \cdot Ks_j
 \end{aligned}$$

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## Output constraints

Demand–supply balance in the output market:

$$Qd_j - Qs_j \leq 0$$

Demand price identity:

$$Pd_j = aD_j + bD_j \cdot Qd_j + cD_j \cdot y$$

Unit cost of production identity:

$$Ps_j = \frac{\sum (W_n \cdot Ld_{j,n}) + R_j (Kd_j - aK_j - cK_j \cdot R_j)}{Qs_j}$$

## Labour constraints

Demand–supply balance in the labour market:

$$\left( \sum_j Ld_{j,n} \right) - Ls_n \leq 0$$

Labour supply identity:

$$Ls_n = -\frac{aL_n}{bL_n} + \frac{1}{bL_n} \cdot W_n - \sum_i \frac{cL_{i,n}}{bL_n} \cdot P_i$$

Labour demands in fixed proportion to output:

$$Ld_{n,j} = \frac{1}{mL_{j,n}} \cdot Qs_j$$

## Capital constraints

Demand–supply balance in the capital market:

$$Kd_j - Ks_j \leq 0$$

Capital supply: unlimited supply of capital available at the exogenous rental rate:

$$R_j = rbar_j$$

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Capital demand identity:

$$Kd_j = aK_j + bK_j \cdot Qs_j + cK_j \cdot R_j$$

### Policy constraints

Australian, state and territory and government subsidies:

$$Pd_j = (Ps_j(1 - \sigma S_j) - sCCB_j - sOTH_j) \cdot (1 - \sigma A_j)$$

### Expenditure identities

Australian government expenditure:

$$ExpA = \sum_j (\sigma A_j \cdot (Ps_j(1 - \sigma S_j) - sCCB_j - sOTH_j) \cdot Qd_j + (sCCB_j + sOTH_j) \cdot Qd_j)$$

State and territory government expenditure:

$$ExpS = \sum_j (\sigma S_j \cdot Ps_j \cdot Qd_j)$$

Household expenditure:

$$ExpH = \sum_j (Pd_j \cdot Qd_j)$$

Total expenditure:

$$ExpT = \sum_j (Ps_j \cdot Qd_j)$$

## 1.5 Calibration data and parameter values

Table 1.5 Output demand calibration data

<i>Industry</i>	<i>Quantity</i>	<i>Price</i>	$\varepsilon_P$	$\varepsilon_Y$
	Children	\$		
Pre	213 446	742	-0.65	0.25
LDC	543 539	3 250	-0.65	0.25
FDC	93 738	2 530	-0.65	0.25
Occ	28 515	3 109	-0.65	0.25

Table 1.6 Capital demand calibration data

<i>Industry</i>	<i>Quantity</i>	<i>Price</i>	$\varepsilon_Q$	$\varepsilon_R$
	Establishments	\$		
Pre	4 809	19 374	1.00	-0.1
LDC	5 781	377 021	1.00	-0.1
FDC	329	485 684	1.00	-0.1
Occ	298	10 638	1.00	-0.1

Table 1.7 Labour supply calibration data

<i>Labour</i>	<i>Quantity</i>	<i>Wage</i>	$\varepsilon_W$	$\varepsilon_{Pre}$	$\varepsilon_{LDC}$	$\varepsilon_{FDC}$	$\varepsilon_{Occ}$
	Persons	\$					
B	15 393	46 020	1.50	-0.1	-0.15	-0.15	-0.15
A	27 570	40 710	2.50	-0.1	-0.15	-0.15	-0.15
C3	30 629	31 978	5.00	-0.1	-0.15	-0.15	-0.15
C1	2 578	30 680	7.50	-0.1	-0.15	-0.15	-0.15
U	31 113	30 680	7.50	-0.1	-0.15	-0.15	-0.15

Table 1.8 Labour use by industry

<i>Industry</i>	<i>B</i>	<i>A</i>	<i>C3</i>	<i>C1</i>	<i>U</i>	<i>Total</i>
Pre	7 936	3 671	3 640	612	5 782	21 640
LDC	6 680	21 299	21 262	1 339	17 217	67 797
FDC	655	2 189	4 488	430	5 778	13 539
OCC	123	411	1 239	196	2 336	4 306
Total	15 393	27 570	30 629	2 578	31 113	107 282

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**Table 1.9 State and Territory and Australian Government Subsidies**

	$sOTH_j$	$sCCB_j$	$\sigma A_j$	$\sigma S_j$
	\$	\$	(share)	(share)
Pre	0	0		0.83
LDC	0	0	0.61	0.02
FDC	0	0	0.61	0.02
OCC	0	0	0.36	0.04