GGS

A.C.N. 080 175 908

**GLOBAL GAMING SERVICES PTY. LTD.**

3rd August 99

Gambling Inquiry
Productivity Commission
P.O. Box 80
Belconnen
ACT 2616

Attention: Mr Ross Wilson

Dear Ross,

I congratulate you on an objective draft report.

The material I reference below may be of interest:

1. The referenced paper (written and presented in 1993) espouses some of the themes alluded to in your report: Toneguzzo, S. (1996) "Socially Responsible Introduction of Gaming Machine Technology" in J.McMillen, M.Walker and S.Strurevska (eds.) Lady Luck in Australia, NAGS, Sydney University.

2. Attached is a paper, presented in 1996, explaining the mathematical relationships between turnover, time, volatility, etc. and drawing conclusions which impact social behavior and machine play. It is titled, "Relationship between gaming machine prize limits and turnover, a mathematical model." I do caution that whilst I believe the theory is sound, the equation interrelating "everything" has been long sought after by game developers...I don't believe I have it quite right and the example toward the end of the report has never been empirically proven. In any event, I note the draft report alludes to the concept of "player time" and presents some mathematical analysis of this concept, so my paper may be of interest in further developing those ideas and mathematically proving some of your findings.

3. It is one thing to promote people driving automobiles responsibly and to ensure quick response times from police and ambulances to address the carnage in a timely manner. However, this probably has less of an affect than ensuring the cars themselves have seat-belts, air-bags, etc. A holistic approach is required in approaching road-safety. Using this analogy, I now move to responsible gambling campaigns the industry in its various forms is pursuing. Most forms of venue gambling are technology based. I observe with interest that no-one involved in the problem gambling industry reference groups (e.g.

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NSW) would appear to have any appreciation of the design theory and technology behind the gambling devices. Probably most know that the devices make money, but do they know why? Unless the technology and the companies that supply this technology are a major consideration I would question the effectiveness of any strategy for responsible gambling. The builder of the roads (e.g. club) can design the roads for safety (analogy: don't stick an ATM next to a poker machine), they can provide proper warning signs on the road (analogy: Responsible advertising of gambling), but unless someone also takes responsibility for the standard of car (read poker machine) on the road, the rest is arguably window dressing.

If the first transition from being a gambler to a problem gambler occurs at the game (machine) interface, then it is at the game interface that the provision of responsible gambling services commences.

Of equal importance is player awareness / education.

The game is central to all advertising, promotions, and related gaming activities - controlling such measures are secondary.

4. A Gambling License? Just as a driver's license is needed because drivers can cause harm to themselves and others, so might a gambling license be required. Requiring such a license could provide the opportunity for player education, research statistics, cashless (less crime and overhead gambling) and another source of income for government (licence fees).

This concept could be taken one step further to not permit a machine to operate unless a player has their "license" inserted. In most states and territories at least one wide-area monitoring system links legal poker machines, and would provide a means to facilitate this. Of course there are in reality potential privacy implications, civil rights, and infrastructure costs. Nevertheless, it may provide a means of sustaining venue profitability, and minimise the occurrences, or assist with the detection of, problem gamblers.

5. I have prepared a paper, prior to your draft report being released, and submitted the paper to the Senate Select Commitee in response to their call for submissions. The Committee have instructed that my submission not be disclosed to any other party, however, you may wish to approach them directly for a copy.

Yours sincerely,

for Steve Toneguzzo
(Managing Director)

# RELATIONSHIP BETWEEN GAMING MACHINE PRIZE LIMITS and TURNOVER, a MATHEMATICAL MODEL.

Seventh National Conference of the National Association for Gambling Studies.
Stamford Grand Hotel, Glenelg, SA.
18 - 21 November, 1996.

*We must rediscover the distinction between hope and expectation.* -*Ivan Illich*

# CONTENTS

## 1. ABSTRACT

The objective of this dissertation is to explore a quantifiable relationship between a game's prize and the effect adjusting this prize has upon the expected turnover. Along the way a number of other key relationships are quantified.


## 2. PERCENTAGE RETURN

*Percentage Return to Player*, or *Player Return,* is the theoretical amount of money returned to the player from a wager. For example, if one wagered $1.00 on a game with an 85% return, their theoretical expectation is to receive 85c returned to them.

*Theoretical Player Return* (R) is the return of wagers realised in theory, and is for a typical game, the sum of the probability of prizes occurring multiplied by the value of the associated prize.

$$R = \sum\{p(\text{occurrence of winning combination}) \,.\, (\text{associated prize})\} \qquad (1)$$

$$R = \sum\{p(i) \,.\, P(i)\} \qquad (2)$$

Where i = 1 to the Maximum number of prizes offered.

The *continuing actual player return* is the return realised in practice and may be defined to be the ratio of total win to the total bet for a game. In a game of chance, given a sufficient sample base (i.e. number of games), the actual player return will converge to the theoretical player return.

## 3. RELATIONSHIP: RETURN TO TURNOVER

If **Cb** credits are placed into a machine, and these are bet, **T|O**, then given a large enough sample, **R%** of these credits (that is, the theoretical percentage return of the machine) will be accumulated as a residual and wagered again (re-cycled). This process may be considered as being recursive with the limit being the quantity of the residual (amount collected by the patron) - it must be greater than zero. Hence, there is a quantitative relationship between initial coins wagered, percentage return and the sum of wagers (which invariably is related to the number of games played). This may be described by the polynomial:

$$\mathbf{T|O} = Cb + Cb.R + Cb.R^2 + Cb.R^3 + Cb.R^4 + ... + Cb.R^n$$

$$\mathbf{T|O} = Cb\ (R + R^2 + R^3 + R^4 + ... + R^n)$$

$$\mathbf{T|O} = Cb\ .\ \Sigma\{\ R^i\ \} \quad \text{Where } i = 1 \text{ to total games played, } n \qquad (3)$$

Where $Cb.R^n$ = the minimum residual, which may be credits collected, or zero.

It must be appreciated that in the case where the cycle continues until $Cb..R^n = 0$, Equation (3) gives us the maximum theoretical turnover possible from an initial wager.

### 3.1 CONCLUSION 1

What Equation (3) tells us is that **the greater the return, the greater the theoretical turnover** (given an initial wager).

## 4. PRACTICAL INTERPRETATION OF Eqn (3)

In practice Cb.Rn must always be ≥ 1 (because in a practical sense we can only have integer credits. Hence Eqn. (3) becomes:

$$\mathbf{T|O} = Cb \,.\, \Sigma\{ R^i \} \quad \text{Where } i = 1 \text{ to } n \text{ and } Cb.R^{n+1} < 1. \qquad (4)$$

Additionally, in practice, the Eqn (4) does not run its full cycle due to the finite resources (time or money) of a given $\text{patron}_{(1)}$, the integer value of credits and the fact that during a finite period of time a real game will return an amount as dictated by the prize table. This may be money back or greater, but almost never R% of the bet. Further, the frequency with which these prizes occur is not each game, rather it is roughly inversely proportional to the prize. Hence Eqn. (4) still represents a near theoretical maximum.

In an attempt to approximate a formula for **T|O** based on re-cycling of the wager which tends toward the practical rather than the theoretical, I believe it is reasonable to consider the variance on the return for a finite period of a given play.

## 5. PERCENTAGE RETURN VARIANCE

Given that the equation for population standard deviation, SD, as it applies to a gaming machines is:

$$SD = \sqrt{\ } \Sigma\{ (\text{frequency} * \text{Prize}^2) \,/\, (\text{Game Cycle} - \text{Return}^2) \} \qquad (5)$$

Where Game Cycle is the total number of possible symbol combinations that may occur in a game, and:

Given that the equation for sample standard deviation, S is:

$$S = SD / \sqrt{g} \qquad \text{Where } g = \text{number of games in the sample.} \qquad (6)$$

Now, for a 95% confidence (the table for the Central Limit Theorem) gives us 1.96, hence

$$R - 1.96S \leq R \leq R + 1.96S \qquad (7)$$

Over the extended period the expected return is R, but in finite period Cb could be lost very quickly given a few losses in sucession. To try to provide an approximation, we shall assume, given typical prize table structures and "infrequent" awards of prizes, that R is reduced by the standard deviation so that, $R = R - 1.96S$,

Hence Equ (4) becomes:

$$\mathbf{T|O} = Cb \,.\, \sum\{ (R - 1.96 \,.\, (SD / \sqrt{g}))^{i} \} \qquad (8)$$

Where $i = 1$ to $n$ and $Cb.R^{n+1} < 1$.
$g$ = finite number of games in the sample.

## 6. RELATIONSHIP: TURNOVER TO TIME

**T|O** divided by the average number of coins bet, **Nb**, provides an average number of games, **G(av).**

The average time duration, **Tp**, for the play of a single game is constituted by the duration of machine controlled game functions and the rate at which patrons initiate and make determinations pertaining to games.

Hence the theoretical amount of time it will take to spend an initial wager may be estimated as:

$$T = Tp(Cb \,.\, \Sigma\{ R^i \})/Nb \qquad \text{Where } i = 1 \text{ to } n \qquad (9)$$

or in general terms:

$$T = (Tp.\mathbf{T|O}) / Nb \qquad (10)$$

## 6.1 CONCLUSION 2

What Equation (10) tells us is that **the greater the return, the greater the turnover, the greater the number of games and therefore the greater the period of play** (given an initial wager). Theoretically, T is the maximum time it would take to loose all Cb credits.

To use an industry term, this equates to "Bums on seats".

Referring back to Eqns (3) and (10) we deduce that for an initial wager, the theoretical turnover **T|O**1 for a given game is realised in time T1.

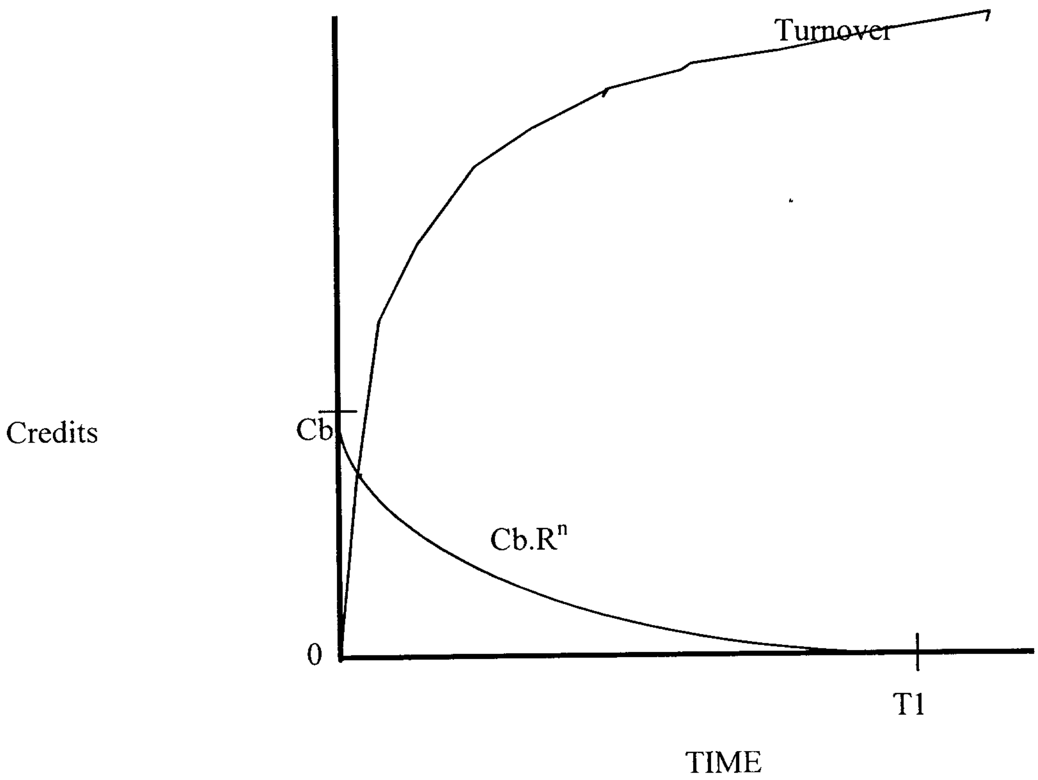

Figure 1: Turnover vs. Time (theoretical maximum) - not to scale.

Likewise equation (8) would provide a turnover, **T|O2**, in time T2 which will be less than T1. However if we ignore T2, and assume an endless supply of samples (that is after one patron has finished playing another inserts more credits), then Eqn (8) will produce a lower re-cycled turnover per initial credits bet than does (3), which means that the wager Cb is lost in quicker time, therefore the turnover during the same period T1 will be higher.

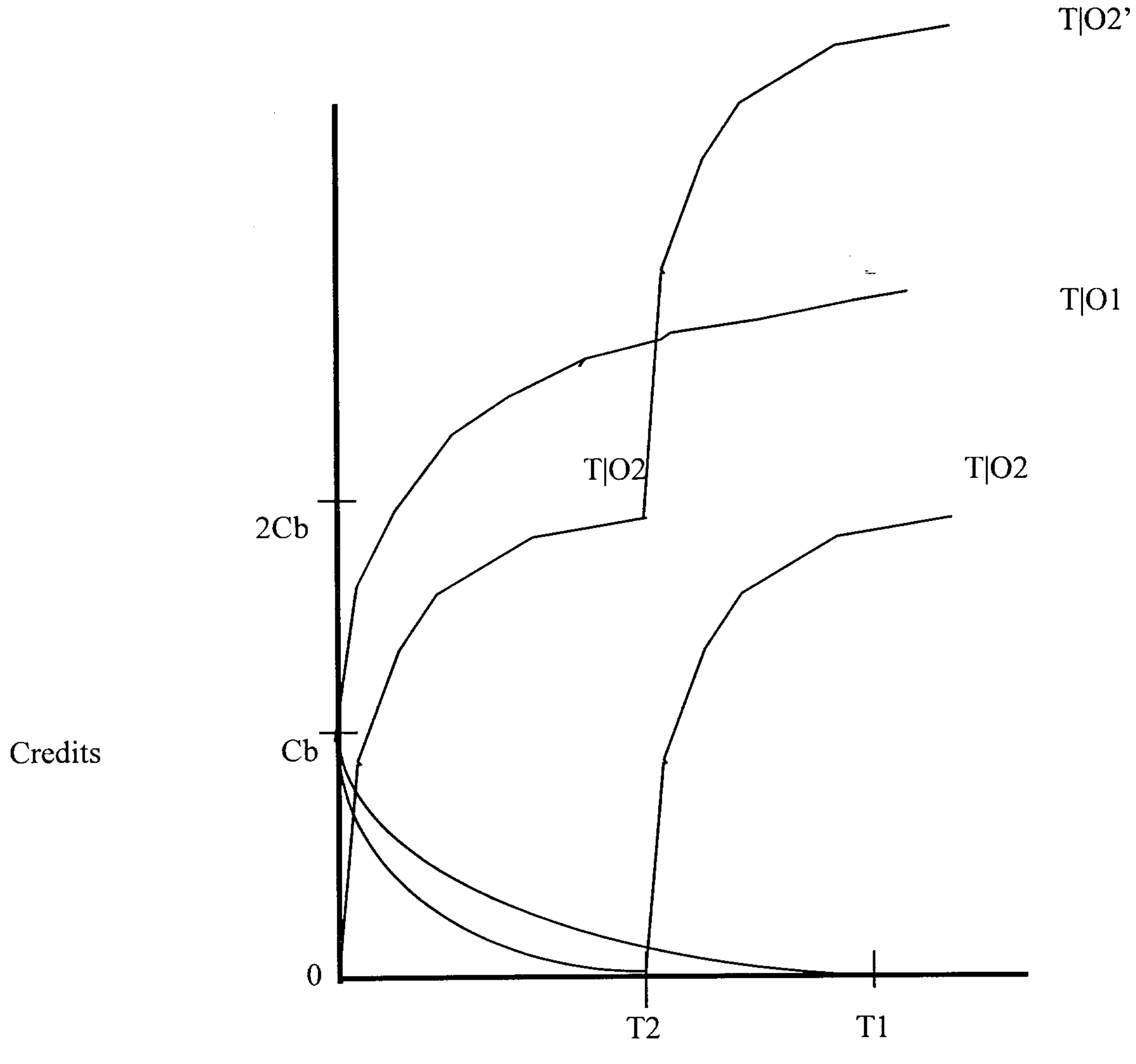


Figure 2: Turnover vs. Time (theoretical maximum vs practical) - not to scale.

## 6.2 CONCLUSION 3

The more "volatile" a game is over a finite (short) period of play, the higher the expected turnover. To take this to an extreme, a game with an infinite number of double-ups would in all probability be a "Gold Mine".

# 7. RELATIONSHIP BETWEEN MAX. PRIZE and RETURN

Following from Eqn. (2), if we increase the maximum prize,Pm, there must be an inversely proportional decrease in the probability of that prize, pm, occurring in order that R remain constant.

Let us call the increase factor, I. Ergo; If P = Pm . I, $R = R_I$ when p = pm / I.

# 8. RELATIONSHIP: PRIZE and VOLATILITY

Referring back to Eqn (5):

$$SD = \sqrt{\sum\{ (\text{frequency} . \text{Prize}^2) / (\text{Game Cycle} - \text{Return}^2) \}}$$

Assuming, as is most typically the case: P(m) occurs once in the prize table, frequency = 1.

Return = R

Game cycle = total of all possible combinations for that game.

Then increasing the maximum prize for a game will affect the standard deviation as follows:

$= SD^2 - (fm . Pm^2) / (Game Cycle - R^2) + (fm/I.Pm^2 .I^2) / (Game Cycle - R^2)$

$= SD^2 - (fm.Pm^2) / (Game Cycle - R^2) + (fm.Pm^2 .I) / (Game Cycle - R^2)$

$= SD^2 - \{(fm.Pm^2) / (Game Cycle - R^2)\} . (1 - I)$ (11)

Substituting, $K = (fm.Pm^2) / (Game Cycle - R^2)$

$= SD^2 + K . (I - 1)$ (12)

## 9. RELATIONSHIP: VOLATILITY to TURNOVER

Substituting (12) into (8)

$\mathbf{T|O} = Cb . \sum\{ (R - 1.96 . \sqrt{(SD^2 + K . (I - 1))} / \sqrt{n})^i \}$ (13)

Where:

$i = 1$ to $n$ and $Cb.R^{n+1} < 1$.

$n$ = finite number of games in the sample.

$K = (fm.Pm^2) / (Game Cycle - R^2)$

N.B. Pm = Pm prior to increase.

### 9.1 CONCLUSION 4

Increasing the maximum prize, whilst maintaining the same percentage return will produce a lower turnover per initial credits bet, but a higher overall turnover during the same time period T1. **Therefore for a given period of play, the greater the prize, the greater the volatility of a game, the greater the Turnover.**

## 10. ESTIMATION OF INCREASED TURNOVER PER GAME

1. For a given Cb, Substituting (8) into (10) will provide a reference time, T1 for turnover **T|O**1.
2. For that same Cb, substituting (13) into (10) will provide a reduced reference time, T2 for turnover **T|O**2.

3. The theoretical turnover discussed is the turnover attributed to an initial wager being re-cycled down to nothing. However, our practical approach dictates that the initial wager is not 'nibbled' away, but rather 'torn off in chuncks'. Hence the total turnover, **TT|O** is estimated to be:

   Initial credits available (Cb) + Re-cycled turnover (**T|O**) (14)

4. Calculate the time differential; Td = T1 - T2
5. Calculate **TT|O**1 and **TT|O**2
6. Pro-rata TT|02 to estimate turnover for time period T1 using:

   T|O = (TT|02 / T2).Td
7. Total the turnover estimated to be obtained from a prize increase T|OP= T|O + TT|O2
8. [ (T|OP - TT|O1) / TT|O1 ] . 100%, gives an approximation of the percentage increase in turnover attributable to increasing the maximum prize of a game assuming a constant play during a given time period.

## 11. EXAMPLE

Assume a representative game has the following parameters, which would be best obtained by investigating empirical data but for this example:

Return, R = 87%

Av game duration, Tp = 3sec

Av credits bet per game, Nb = 2.3 credits

Game Cycle = 3,200,000

Initial Credit injection, Cb = 1000 credits

Games played, n = 1024

The paytable is structured so that the game's SD = 8

The maximum prize, Pm = 1000 credits

The maximum prize occurs once, fm = 1.

I = 1 to n

Substitution into Eqn (8) gives:

$$T|O = 1000 \,.\, \sum\{ (R - 1.96 \,.\, (8 / \sqrt{1024}))^{i} \}$$

$$= 1000 \,.\, \sum\{ (R - 0.49)^{i} \}$$

$$= 612.9$$

From (10), we obtain:

TT|O1 = 612.9 recycled + initial 1000 lost = 1612.9

T1 = 3 x 1612.9 / 2.3 = 2103.7 sec.

Now, increase the maximum prize by a factor of 10, so that Pm becomes 10 000 credits.

Substitution into Eqn (13) gives:

$$T|O = 1000 \,.\, \sum\{ (R - 1.96 \,.\, \sqrt{(8^2 + K \,.\, (10 - 1))} / \sqrt{1024}))^{i} \}$$

$$K = (1000^2) / (3{,}200{,}000 - .87^2)$$

$$= 0.31$$

$$T|O = 1000 \,.\, \sum\{ (R - 1.96 \,.\, \sqrt{(8^2 + 0.31 \,.\, 9)} / \sqrt{1024}))^{i} \}$$

$$= 1000 \cdot \Sigma\{ (R - 1.96 \cdot 8.17 / \sqrt{1024}))^{i} \}$$

$$= 1000 \cdot \Sigma\{ (R - .596)^{i} \}$$

$$= 586.3$$

T|O = 586.3

TT|O2 = 586.3 recycled + initial 1000 lost = 1586.3

From (10), we obtain:

T2 = 3 x 1586.3 / 2.3 = 2069 sec.

T1 - T2 = 34.74 s

(34.74 x 1586.3) / 764.7 = 72.06 cred.

Total T|O for period T1, using increased maximum prize:

= 1586.3 + 72.06

= 1658.6

Thus a x10 increase in top prize represents an increase of :

100% . (1658.6-1612.9)/1612.9

= approx 3 % increase in turnover.

Repeating the exercise for a x100 increase in top prize represents an increase of :

= approx 39% increase in turnover.

## 12. CONCLUSION

The procedures and calculations presented above apply to games on a case by case basis as it is impractical to develop a finite equation to cater for ever type of game distribution available. Of note, issues such as double up and feature games have an additional effect, though these have not been considered as the intent is to provide an approximation and not a definitive solution for every type of game available. Indeed, one could boldly suggest that if the data for an "average" game be extrapolated to apply to all games in a venue, one may have an approximate indication of the net effect of a population increase in turnover for an associated increase in maximum prize.

The reader should note that the assumptions stated and the logic and equations that are derived based on said logic and assumptions have not been empirically proven.

It should also be noted that this paper is not supported by the IGT game development department.

It is the Author's request that copies of work be provided to the Author, should any person decide to further develop in theory or practice, or to provide an alternate "solution", to the concepts presented herein.

# 13. APPENDICES

## 13.1 THE AUTHOR

Stephen J. Toneguzzo possesses a Masters Degree in Engineering Science, a Bachelor of Electronic Systems Engineering and Graduate Diploma in Computer Science from the Queensland University of Technology. Stephen has been a participant in the gaming industry both in Australia and abroad for many years, specialising in gaming technology, policy and regulation. He is currently managing the Software Engineering, Technical Compliance and Engineering Quality Assurance Departments within I.G.T. (Australia) Pty Limited. IGT manufacture and sell poker machines and electronic gaming systems.

## 13.2 REFERENCES


1. Toneguzzo, Stephen: *Socially Responsible Introduction of Gaming Machine Technology*, 1993. Discussion paper on policy matters relating to gaming technology.
2. C.W.D. Radcliffe: *Probability and Statistics*, William Brooks & Co., Sydney. 1981.