FERTMOD: Software for making demographic projections under different fertility assumptions

Technical Paper
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1.1 What is FERTMOD?

*FERTMOD* is an Excel-based program that projects Australia’s population, given assumptions about fertility, mortality, and net migration.\(^1\) It is simple to use and requires little input from users. In many cases, users can simply enter one parameter to conduct experiments — but it also allows considerable flexibility in making demographic projections.

In most instances, users should be able to run the program in Excel without changing their Excel settings. However, in testing we sometimes found users encountered a message ‘Can’t find project or library’. In that instance, please see section 1.7 at the end of this document about how to solve this problem.

1.2 Purpose of the model

The goal of the model is to indicate how changes in the model’s demographic assumptions affect Australia’s population and its age structure. *FERTMOD* uses the standard cohort component model. This is a deterministic model, which for given settings for mortality, net migration, fertility and a starting population, will exactly calculate future population numbers.

At this point, it is important to distinguish projections from forecasts.

**Projections**

Projections do not aim to predict the future, so that there is no particular view that the underlying settings will be realised. In that sense, it is a demographic ‘laboratory’ that allows users to conduct ‘what if’ experiments. For example, what would happen were government to significantly increase migration intakes or to raise fertility through family policies? The outcomes of these experiments may inform policymakers about whether a given set of policies are sensible. That information will mean that those settings are not actually used, thus preventing the realisation of that projection. In its work on projections in its analysis of the

\(^1\) The Commission has also developed a much more sophisticated projection tool, MoDEM, which allows users even greater control over their demographic scenarios, and which also projects key economic variables, such as labour force participation rates and GDP. However, because of its sophistication, this model is larger and requires more knowledge to run properly than *FERTMOD*. MoDEM and its documentation can be found on the Commission’s website at www.pc.gov.au/research/commissionresearch/nationalreformagenda/modem.
implications of population ageing, the Productivity Commission (2005) gave an analogy to this. A large rock is lying on a rail track and 10 kilometres away a train is hurtling towards a collision. The projection is that the train will crash, with all of its tragic consequences. The forecast, taking account of this projection, is that the train will stop and the rock removed from the tracks. No collision occurs. The lack of realisation of the projection is not a fault of the method, but in this context, an obvious strength.

It is important to undertake projections over a long horizon to identify ‘steady-state’ outcomes (when the age structure and population growth has stabilised). Hence, FERTMOD is a long-run model, which provides projections on a year-by-year basis to 2251. The likelihood of realising projections as far forward as 2251 is obviously low. This is because the economic and social circumstances are likely to be fundamentally different from today, altering the mortality, net migration and fertility settings underpinning these projections. Nevertheless, the projections show what would happen were the government to preserve its policy settings over the long run, and this can usefully indicate their sustainability and realism. For example, a very substantial increase in family policy might raise fertility above replacement levels to 2.2 babies per woman. With net migration of say, 170 000 per year, population growth and long-run population numbers would be very high. This might suggest that the policy settings underpinning the high fertility or/and net migration could not realistically be sustained.

**Forecasts**

Forecasts attempt to predict what will actually transpire – taking into account future government policies (for example, migration policy). Accurate forecasts of population numbers can only be made over relatively short horizons. Users can employ FERTMOD to produce such forecasts if they are confident about the accuracy of their underlying assumptions about mortality, fertility and migration.

Even in the presence of uncertainty, FERTMOD can indicate some likely bounds on Australia’s future demographic characteristics by undertaking scenario analysis. For example, users may be reasonably certain about the likely maximum and minimum life expectancy outcomes over the next fifty years, and can then see, for given fertility and migration settings, what this implies for Australia’s future age structure. These are not point forecasts, but they can enable users to test whether certain qualitative outcomes are likely or not – such as the probable magnitude of population ageing. Judicious use of scenario analysis of this kind can emulate some of the advantages of stochastic forecasts.
1.3 The model’s options

The model is relatively flexible. Users can specify:

- long-run total fertility rates (which, by definition, are also completed fertility rates);
- life expectancy of males and females (separately);
- net migration levels;
- the transition periods to these long-run values;
- the shape of the age distribution of fertility; and
- whether net migration is endogenous. In this case, users specify a long-run population target and then net migration levels adjust to achieve that target (given all other demographic assumptions). This is an appropriate experiment where governments alter migration policy to achieve a sustainable long run population.

1.4 How to use the model

On opening the FERTMOD excel spreadsheet, there are two (default) worksheets.

The ‘Start’ worksheet

The most important sheet is the Start worksheet, and this is the one that the program opens in. (The other worksheet – the Input worksheet – is described later, but most users need never look at it.)

The Start worksheet (figure 1.1) serves two purposes:

- It documents the key inputs that feed into the experiments (the ‘key inputs’ part of the sheet) and results of each experiment (the Key Results area of the sheet, including the long-run age-specific fertility rates and a graph showing how they have changed); and
- Through the ‘Click here to Start’ button, it provides the entry point to the model – where users can specify the parameters they wish to use and the form of outputs generated by the model.
### Figure 1.1 The Start worksheet

#### The top part of the sheet

**Key inputs used (shows parameter values used in model)**

<table>
<thead>
<tr>
<th>Input</th>
<th>Value</th>
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</tr>
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<td>Theta (adjusts shape of fertility distribution for TFR)</td>
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#### The bottom part of the sheet

**Age-specific fertility**

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<th>Long run ASFR</th>
<th>Shift in S distribution due to timing</th>
<th>Shift in S distribution due to TFR</th>
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</table>

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6  FERTILITY TRENDS
On clicking the ‘Click here to Start’ users see a variety of model options (figure 1.2).

**Figure 1.2 Setting model options**

The form has default settings for all of its options, making it easy for users to select just one option, while retaining other settings at their default values. For the purposes of this fertility project, the key option is the long-run value of the total fertility rate. Most of the other options are self-evident, giving options for setting life expectancy, net migration (inwards) and the length of transition periods to the long-run settings.

There are several output options. The default option is the minimalist one. It produces the ‘key results’ shown in the Start form and no other results. The program runs almost instantaneously when this minimalist option is selected.
However, sometimes users may want more detailed data. The particular options are:

- ‘Do you want to draw pyramid graphs?’ If yes, this will provide a sheet with graphs of the share of the population by age (so-called ‘pyramid’ graphs). (A sample of part of the output is shown in figure 1.3). It will also provide the data underlying these graphs. The graphs and data are provided for 2007, 2051, 2101, 2151 and 2251.

Figure 1.3  Pyramid graphs in FERTMOD

- ‘Do you want a dependency graph?’ If yes, this will produce a graph of so-called ‘dependency’ ratios from 2007 to 2251 (A sample is shown in figure 1.4.) Dependency ratios are ratios of various population age groups (0 to 14 and 65+ years) to the population in which workforce participation rates are highest (15 to 64 years). The term ‘dependency’ need not imply that particular people in these young or old populations are financially or otherwise dependent. But it nevertheless will indicate the relative numbers of people who are intensively engaged in labour markets.

- ‘Store detailed population numbers?’ If yes, this will provide year by year data on population numbers by age and gender, and also life expectancies for each year.
Documenting some key inputs: the ‘Input’ worksheet

The ‘Input’ worksheet is a less important sheet, whose role is only to document the key input data. These data are common to all projections. Users should not change these data unless they wish to update the base numbers. If they wish to do so, the relevant cells should be unlocked by removing worksheet protection — available under the ‘tools’ menu in Excel. We do not recommend this for most users.

The worksheet includes by initial population numbers, death probabilities ($Q_{x,t}$), net migration flows, the current stock of Australians born overseas, and the change in mortality given by $\log(Q_{x,2050}) - \log(Q_{x,2004})$ from the ABS B series projections — all by age and sex. The worksheet also shows the most recent age-specific fertility rates.

1.5 Model construction

The standard features of the cohort-component model are well documented (for example, in PC 2005). However, it is important to indicate how FERTMOD:

- determines future age-specific fertility rates;
- translates life expectancy settings into $Q_x$ over time; and
- estimates net migration levels to achieve a given population target, where net migration is endogenous rather than exogenous in the model.
**Age-specific fertility rates**

Empirically, increases or decreases in the total fertility rate (TFR) do not scale up or down the existing age-specific fertility distribution, but change the shape of the distribution. Accordingly, decreasing TFRs have usually resulted in much bigger shifts down in age-specific fertility rates for young women than in older women. In some countries, the age-specific fertility rates of the latter have often risen as women have postponed fertility.

As an illustration, Italy (a low fertility country) and France (a high fertility country) provide revealing contrasts, with quite different distributions of their age-specific fertility rates (ASFRs) (figure 1.5). It is clear from the distribution of age-specific fertility rates that older Italian women have fertility rates similar to French women at those ages. However, younger Italian women have fertility rates much lower than do their French counterparts. Consequently, older women account for a greater share of total fertility in Italy (the right-hand panel of figure 1.5). The age-specific fertility share distribution (the S distribution) for a low TFR country shifts to the right.

**Figure 1.5 Italian and French age-specific fertility**

As an illustration, Italy (a low fertility country) and France (a high fertility country) provide revealing contrasts, with quite different distributions of their age-specific fertility rates (ASFRs) (figure 1.5). It is clear from the distribution of age-specific fertility rates that older Italian women have fertility rates similar to French women at those ages. However, younger Italian women have fertility rates much lower than do their French counterparts. Consequently, older women account for a greater share of total fertility in Italy (the right-hand panel of figure 1.5). The age-specific fertility share distribution (the S distribution) for a low TFR country shifts to the right.

Data source: Eurostat data for Italy and France.

Changing patterns in the timing of children also affect the S distribution, even when the total fertility rate stays the same. For example, Australia’s S distribution moved significantly to the right from 1995 to 2006 (as the role of older women in childbearing increased), although the TFR was about the same (figure 1.6).
So in modelling the future S distribution (and its associated age-specific fertility rates), it is important to take account of the likely continuation of these timing effects, as well as the separate impact of changing TFRs.

Figure 1.6  **Australia’s age share of fertility is shifting towards older women**  
Share of total fertility by age group

![Graph showing the shift in age share of fertility](image)

Data source: From unpublished data provided by the ABS (based on Births, Australia, various issues, Cat. No. 3301.0).

This is achieved in several steps in FERTMOD.

*The first step: timing effects*

In ABS projections of future age-specific fertility rates, the S distribution shifts rightwards as in figure 1.6 (ABS 2006). This shift can be depicted by the ratio between the two S distributions (R):

\[
R_{age} = \frac{ASFR_{age, 2018}}{ASFR_{age, 2014}} \div \frac{1000 \times TFR_{2018}}{1000 \times TFR_{2014}} = \frac{ASFR_{age, 2018}}{ASFR_{age, 2014}} \times \frac{TFR_{2014}}{TFR_{2018}}
\]

where the ASFR is the age-specific fertility rate and TFR the total fertility rate. The ABS provides the data for calculating \(R_{age}\), but only for five-year age groups (figure 1.7).
In order to interpolate the value of $R_{age}$ for all ages between 15 and 49 years, we ran a regression of $R_{age}$ on age, $age^2$ and $age^3$:

$$R_{age} = 1.0 + \lambda \times \{2.338288 - 0.308566 \times age + 0.0103884 \times age^2 - 0.00009140508 \times age^3\}$$

$R_{age}$ provides a close approximation to the observed ABS value of $R$, as shown in figure 1.8. It is used in FERTMOD to determine the long-run impact of timing effects on the $S$ distribution, independent of any effects on that distribution arising from changes in the TFR.

$\lambda$ is a parameter such that $0 \leq \lambda \leq 0$, which allows users to determine how much weight they wish to give to shifts in the $S$ distribution due to timing changes in the TFR. If $\lambda = 0$, any shift is disregarded. With $\lambda = 1$ (the default), the full impact of changes to the $S$ distribution arising from timing are taken into account. Users can specify amounts between 0 and 1 if they wish to choose an intermediate position, or specify $1 < \lambda < 2$ if they wish to accentuate shifts in the $S$ distribution due to timing.

**The second step: effects of changing TFRs**

FERTMOD uses the relationship between the $S$ distributions of France and Italy as the basis for estimating the effects of variations in the TFR on the $S$ distribution. The ratio ($Y$, shown below in figure 1.9) can be approximated as:

$$\hat{Y}_{age} = 1.0 + \{\mu + \gamma \times age + \beta \times age^2 - \alpha \times age^3\}$$
\[ \hat{Y}_{age} = 1.0 + \{2.167 - 0.3038 \text{age} + 0.01159 \text{age}^2 - 0.0001277 \text{age}^3\} \]

**Figure 1.8  Ratio of age shares, Australia**  
2018 age share distribution on 2004 age share distributions

**Figure 1.9  Ratio of Age shares**  
Italy compared with France

Data source: As in figure 1.5.

\( \hat{Y}_{age} \) is the ratio of the S distributions for two countries whose TFRs are 1.32 and 2.004 — that is, where the low-fertility country’s TFR is 66 per cent of the high-fertility country’s TFR.
The most recent official estimate of the TFR for Australia is 1.814 for 2006 (TFR₂₀₀₆). In an Australian context, the position of Italy relative to France is akin to comparing an alternative level of TFR (Alt_TFR) of 0.66 of the current TFR, that is:

\[ Alt\_TFR = \left( \frac{TFR_{Italy}}{TFR_{France}} \right) \times TFR_{2006} = 1.2 \]

We assume that were the Australian TFR to fall to 1.2 in the long run, the S distribution would shift by exactly the same as Italian S distribution currently does with respect to the French distribution (i.e. by \( \dot{Y}_{age} \)). But what might happen for other values of the TFR? To approximate that, we scale \( \alpha, \beta, \mu \) and \( \gamma \) by \( \Omega \), where:

\[ \Omega = \frac{(LRT - TFR_{2006})}{Alt\_TFR - TFR_{2006}} \]

where LRT is the Longrun_TFR (the projected long-run TFR selected by the user) or 3.35, whichever is the smaller. It is apparent that if the TFR does not change over time \( \Omega = 0 \). Were LRT to be equal to Alt_TFR (that is, around 1.2) then \( \Omega = 1 \) and the scaled values of \( \alpha, \beta, \mu \) and \( \gamma \) would be the same as estimated above.

The reason for specifying a maximum of LRT is that if Longrun_TFR is much greater than 3.35, some age-specific fertility rates become negative at older ages. Setting a limit prevents this. Users can still nominate a value of the long-run TFR above 3.35. If they do, then age-specific fertility rates will still add up to that higher TFR. So setting Longrun_TFR = 4.0 will still result in age-specific fertility rates that add up to 4.0 (when divided by 1000). However, the shape of the distribution of age-specific fertility rates (the S distribution) is fixed for Longrun_TFR >= 3.35.

Given \( \Omega \), the appropriate ratio to shift the S distribution in line with a new TFR is: \( AdjRatio_{age} = 1.0 + \theta \times \Omega \times \{ \mu + \gamma\ \text{age} + \beta \\text{age}^2 + \alpha \text{age}^3 \} \). Given the definition of \( \Omega \) above, other than the effects of timing, the S distribution will stay the same as in 2006 if the TFR does not change from its current level.

\( \theta \) fulfils a similar function to \( \lambda \) for timing effects. \( \theta \) is a parameter such that \( 0 \leq \theta \leq 2 \) that allows users to determine how much weight they wish to give to shifts in the S distribution due to changes in the TFR. If \( \theta = 0 \), any shift is disregarded, regardless of the choice of the longrun_TFR. This is the modelling option implicit in ABS projections.\(^2\) With \( \theta = 1 \) (the default), the full impact of changes to the S distribution arising from changes in TFR are taken into account. Users can specify

\(^2\) Since, the age-specific fertility shares of the TFR in the ABS projections are invariant to the long-run choice of the TFR.
amounts between 0 and 1 if they wish to choose an intermediate position, or specify $1 < \theta < 2$ if they wish to accentuate shifts in the S distribution.

Notably, if $\theta=\lambda=0$, then both timing and TFR effects on the S distribution are ignored and the long-run age specific shares of the TFR will remain at their 2006 settings. In FERTMOD, this setting means that any changes in the TFR will simply scale up or down proportionately the current age-specific fertility rates.

**Step 3 bringing the steps together**

First, we calculate the current age-specific shares of the TFR for Australia:

$$AUSH_{\text{age}} = \frac{\text{ASFR}_{\text{age},2006}}{(1000 \times \text{TFR}_{2006})}$$

Then the long-run age specific fertility rates resulting from the combined effects of timing and any changes in the TFR are calculated, and are normalised so that they must sum to one:

$$\text{LR}_{\text{ASFR}}_{\text{age}} = \frac{\sum_{\text{age}=15}^{49} \text{AdjRatio}_{\text{age}} \times \text{R}_{\text{age}} \times \text{AUSH}_{\text{age}} \times \text{Longrun}_ \times \text{TFR} \times 1000}{\sum_{\text{age}=15}^{49} \text{AdjRatio}_{\text{age}} \times \text{R}_{\text{age}} \times \text{AUSH}_{\text{age}}}$$

Finally, the path to the long-run age-specific fertility rate is calculated, where PeriodM is the time it takes to reach the long-run values:

If $(\text{year} \leq \text{PeriodM} + 2006)$ Then

$$\text{ASFR}_{\text{age,year}} = \text{ASFR}_{\text{age,year-1}} + \left\{\text{LR}_{\text{ASFR}}_{\text{age}} - \text{ASFR}_{\text{age,2006}}\right\} / \text{PeriodM}$$

Else

$$\text{ASFR}_{\text{age,year}} = \text{LR}_{\text{ASFR}}_{\text{age}}$$

**Life expectancies**

Historically, age-specific probabilities of death have declined steeply — and the default presumption of FERTMOD is that this continues unabated, albeit over a protracted period.

In FERTMOD the user nominates the long-run life expectancies of males (desired_M) and females (desired_F).

FERTMOD then finds a set of long-run mortality probabilities by age ($Q_x$, where $x$ is an age from birth to 100+) consistent with these nominated life expectancies. A simple solution would be a fixed scaling up or down of the $Q_x$ values for 2006.
However, historically, probabilities of death have declined more for some ages than others. For example, males aged 65-69 have experienced larger reductions in death probabilities than males aged 35-39. Consequently, the achievement of a given life expectancy figure in any future year will involve larger reductions in mortality for some ages than others. ABS projections of future populations assume the continuation of this pattern. Figure 1.10 shows $DABSQ_x = \log(Q_{x,2050}) - \log(Q_{x,2004})$.

**Figure 1.10  Reductions in the probability of death**

*By age, 2004 to 2050, ABS B projections*

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*Data source:* Unpublished data from the ABS.

FERTMOD finds a scale factor, $\Phi$ (for males and females separately) to apply to $DABSQ_x$, such that the resulting long-run $Q_x$ series (in 2251) are consistent with the nominated life expectancy. This means that $\log(Q_{x,2251}) - \log(Q_{x,2005}) = \Phi \times DABSQ_x$.

The long-run value of $Q_x$ for a given gender is then given by:

$$LRQ_x = e^{(\Phi \times DABSQ_x)} \times Q_{x,2005}$$

$Q_{x,2005}$ is used as the base, since the most recent lifetables relate to 2005. We find $\Phi$ for any given life expectancy using a non-linear numerical technique (the secant method). We increase the speed of the routine by seeding the routine with close approximations to the solution. These close approximations are based on an estimated generalised logistic curve of the relationship between life expectancies (desired_M and desired_F) and the scale factor.
As in the case of fertility, FERTMOD allows users to determine the transition period to the realisation of the life expectancies (T).

If year < T+2005 then

\[ Q_{x,\text{year}} = Q_{x,2005} \times e^{ \left( \frac{(\text{year - 2005}) \times \Phi \times \text{DABSQ}_x}{T} \right) } \]

else \[ Q_{x,\text{year}} = Q_{x,2005} \times e^{ \Phi \times \text{DABSQ}_x } \]

Endogenous net migration

The Australian Government has a considerable capacity to choose the level of net migration into Australia. Making choices in this area depend, on the one hand, any sustainability issues associated with larger populations, and on the other, concerns to bolster the size of the workforce or the commercial desire for a bigger population.

As a consequence, were the TFR to fall, the government may well counter its dampening effect on population growth by increasing net migration. Conversely, were the TFR to rise, the government may well reduce its emphasis on net migration to achieve a sustainable population.

FERTMOD allows users to consider the demographic outcomes if governments choose net migration levels to ultimately achieve a certain population target (for given fertility levels).

The model is solved iteratively for differing net migration levels to achieve the population target. As in modelling mortality, a non-linear numerical technique (again the secant method) is used to find the solution.

1.6 Model code

The full model code is provided below.

Module 1 code
Public Life_Exp As Double
Public PopM(100, 250) As Double
Public PopF(100, 250) As Double
Public BornOS_male_age_dist(101) As Double
Public BornOS_female_age_dist(101) As Double
Public NOM_male_age_dist(101) As Double
Public NOM_female_age_dist(101) As Double
Public Age_specific_TFR(49, 250) As Double
Public year, age As Integer
Public PeriodM As Integer
Public longrun_TFR As Double
Public PopM_ABorn(100, 250) As Double, PopF_ABorn(100, 250) As Double ' note need to declare type of each variable, even when on the same line
Public TotPopM(250) As Double, TotPopF(250) As Double
Public NOM_Fix As Double
Public DQx_Male(101) As Double
Public DQx_Female(101) As Double
Public Qx_ABSLR_Male(101) As Double, Qx_ABSLR_Female(101) As Double
Public Qx_Male(101, 250) As Double
Public Qx_Female(101, 250) As Double
Public Transition_yrs_mortality_male As Double, Transition_yrs_mortality_female As Double
Public i As Integer
Public desired_M As Double, desired_F As Double
Public scale_male As Double, scale_female As Double
Public x0 As Double, x1 As Double, x2 As Double, Dx As Double
Public k As Integer
Public scale_M As Double, scale_F As Double
Public endogenous_migration As String, Transition_yrs_NOM As Double
Public AgeShare_NOM_Male(100) As Double, AgeShare_NOM_Female(100) As Double, POPNOM_Male(100, 250) As Double, POPNOM_Female(100, 250) As Double
Public SURVIVE_POPNOM_Male(100, 250), SURVIVE_POPNOM_Female(100, 250) As Double
Public Target_pop As Double
Public Survivors_male_Aborn(100, 250) As Double, Survivors_female_Aborn(100, 250) As Double, Survivors_male(100, 250) As Double, Survivors_female(100, 250) As Double
Public Births(49, 250) As Double, TotalBirths(250) As Double
Public Pop(250) As Double, Aust_Born(250) As Double, Prime(250) As Double, Aged(250) As Double, Under10(250) As Double, Youth(250) As Double, Aged_Dependency(250) As Double, Youth_Dependency(250) As Double, Population(250) As Double
Public LambDa, ThetaVal As Double
Public Const Male_share As Double = 0.501477272727273
Public Const mig_share As Double = 23.8398553979533
Public Const male_birthshare As Double = 105.5 / 205.5
Public Const TFR2006 As Double = 1.81442959142374
Public BigL_m(101) As Double, BigL_f(101) As Double
Public T_m As Double, T_f As Double
Public Life_m As Double, Life_f As Double
Public POP2555(250) As Double, POP2555m(250) As Double, POP2555f(250) As Double
Public PopLR0 As Double, PopLR1 As Double, PopTotal As Double
Public LR_Age_specific_TFR(49) As Double
Public epsilon As Double
Public TFR_Ratio As Double, Alpha As Double, Beta As Double, Gamma As Double
Public LR_Age_specific_TFR(49) As Double

Sub Auto_Open()
    MsgBox "Welcome to FERTMOD!"
    Range("H6").Select
End Sub

Sub start()
    StartFrm.Show
End Sub
Startfrm code

Private Sub CmdOK_mode_Click()
    Call Cohort_component_model
End Sub

Private Sub CmdCancel_Mode_Click()
    Unload Me
End Sub

Public Sub Cohort_component_model()
    Application.ScreenUpdating = False
    Application.DisplayAlerts = False
    If SheetExists("detailed") = True Then
        Sheets("detailed").Delete
    End If

    If SheetExists("Pyramids") = True Then
        Sheets("Pyramids").Delete
    End If

    If SheetExists("Dependency") = True Then
        Sheets("Dependency").Delete
    End If
    Application.DisplayAlerts = True

    'Input variables from form
    Transition_yrs_NOM = Val(TxtTransNOM) ' years taken for net migration to shift from current level to new level
    Target_pop = Val(TxtPOP)
    desired_M = Val(TxtMaleLE)
    desired_F = Val(TxtFemaleLE)
    PeriodM = Val(TxtTransTFR)
    longrun_TFR = Val(TxtTFR)
    Transition_yrs_mortality_male = Val(TxtTransLE_male)
    Transition_yrs_mortality_female = Val(TxtTransLE_female)
    LambDa = Val(TxtBoxLambda)
    ThetaVal = Val(TxtBoxTheta)
    If LambDa < 0 Then
        LambDa = 1#
    End If

    If ThetaVal < 0 Then
        ThetaVal = 1#
    End If

    Call input_start 'input some starting values of the population
    Call fertility 'Fertility calculations

    '----------------------------------------------
    'Mortality calculation
    'Qx in 2005 (the known Qx values from the ABS life tables) and
    'Qx in 2251 were the ABS life expectancy projections for 2050 to be fixed thereafter (these will be scaled up or down later)
    'Define life expectancy at 2251

    'Richard’s curve (or generalised logistic) parameters for aprox solutions to scale variable
    male_M = 113.698766239251
    male_T = 0.938136023303275
    male_B = 0.069632942708231
    male_A = -1.9713940319551
    male_C = 27.4843883317946
    guess_male = male_A + male_C / (1# + male_T * Exp(-male_B * (desired_M - male_M)) ^ (1# / male_T))
female_M = 144.2013437038
female_T = 0.288362741928448
female_B = 1.88346551267379E-02
female_A = -3.51761174917085
female_C = 56.9098369841528

\[
guess\_female = female\_A + female\_C / (1# + female\_T \times \text{Exp}(female\_B \times (desired\_F - female\_M)) ^ (1# / female\_T))
\]

epsilon = 0.000001

'----------------------------------
'Calcs for males (secant method)

x0 = guess\_male * 0.95
x1 = guess\_male * 1.05
k = 0
Do While Abs(LifeGoal(x1, DQx_Male, Qx_Male) - desired_M) > epsilon And k < Nmax
   k = k + 1
   If Abs(LifeGoal(x1, DQx_Male, Qx_Male) - LifeGoal(x0, DQx_Male, Qx_Male)) < 0.0000001 Then
      MsgBox("Values of life expectancy are not changing any more.")
      Exit Sub
   End If
   err1 = LifeGoal(x1, DQx_Male, Qx_Male) - desired_M
   err0 = LifeGoal(x0, DQx_Male, Qx_Male) - desired_M
   x2 = x1 + err1 * (x1 - x0) / (err0 - err1)
   x0 = x1
   x1 = x2 ' use x2 as the solution
Loop
scale_M = x2

'Calcs for females

x0 = guess\_female * 0.9
x1 = guess\_female * 1.1
k = 0
Do While Abs(LifeGoal(x1, DQx_Female, Qx_Female) - desired_F) > epsilon And k < Nmax
   k = k + 1
   If Abs(LifeGoal(x1, DQx_Female, Qx_Female) - LifeGoal(x0, DQx_Female, Qx_Female)) < 0.0000001 Then
      MsgBox("Values of life expectancy are not changing any more.")
      Exit Sub
   End If
   err1 = LifeGoal(x1, DQx_Female, Qx_Female) - desired_F
   err0 = LifeGoal(x0, DQx_Female, Qx_Female) - desired_F
   x2 = x1 + err1 * (x1 - x0) / (err0 - err1)
   x0 = x1
   x1 = x2 ' use x2 as the solution
Loop
scale_F = x2

'------------------------------------
'Now work out scaled values of Qx for all intervening years

For i = 0 To 101
   For year = 2006 To 2251
      If (year - 2005) < Transition_yrs_mortality_male Then
         Qx_Male(i, year - 2005) = Qx_Male(i, 0) * Exp((year - 2005) * scale_M * DQx_Male(i) / Transition_yrs_mortality_male) ' Qx_Male(i, 2)
      Else
         Qx_Male(i, year - 2005) = Qx_Male(i, 0) * Exp(scale_M * DQx_Male(i))
      End If
   Next year
   Next i

For i = 0 To 101
   For year = 2006 To 2251
      If (year - 2005) < Transition_yrs_mortality_female Then
         Qx_Female(i, year - 2005) = Qx_Female(i, 0) * Exp((year - 2005) * scale_F * DQx_Female(i) / Transition_yrs_mortality_female)
      Else
         Qx_Female(i, year - 2005) = Qx_Female(i, 0) * Exp(scale_F * DQx_Female(i))
      End If
   Next year
   Next i
```
Else
    Qx_Female(i, year - 2005) = Qx_Female(i, 0) * Exp(scale_F * DOx_Female(i))
End If
Next year
Next i
'----------------------------------------------------
'Net migration
If OptButNo_Mode = False Then
    Call secant
Else
    NOM_Fix = Val(TxtNOM)
End If
Call LongRunPop(NOM_Fix) 'does cohort model
'display parameter settings for model
If OptButYes_Mode = True Then
    Worksheets("Start"),Cells(4, 5).Value = Target_pop
    Worksheets("Start"),Cells(11, 5).Value = x2
Else
    Worksheets("Start"),Cells(4, 5).Value = "Population endogenous"
    Worksheets("Start"),Cells(11, 5).Value = Val(TxtNOM)
End If

Worksheets("Start"),Cells(5, 5).Value = longrun_TFR
Worksheets("Start"),Cells(6, 5).Value = PeriodM
Worksheets("Start"),Cells(7, 5).Value = desired_M
Worksheets("Start"),Cells(8, 5).Value = desired_F
Worksheets("Start"),Cells(9, 5).Value = Transition_yrs_mortality_male
Worksheets("Start"),Cells(10, 5).Value = Transition_yrs_mortality_female
Worksheets("Start"),Cells(12, 5).Value = Transition_yrs_NOM
Worksheets("Start"),Cells(13, 5).Value = LambDa
Worksheets("Start"),Cells(14, 5).Value = ThetaVal
'get useful output results
For year = 2007 To 2251
    For age = 0 To 100
        If age >= 15 And age <= 64 Then
            If age >= 25 And age <= 55 Then
            End If
        ElseIf age < 15 Then
            If age < 10 Then
            End If
        End If
    Next age
    Aged_Dependency(year - 2005) = Aged(year - 2005) / Prime(year - 2005) * 100#
    Youth_Dependency(year - 2005) = Youth(year - 2005) / Prime(year - 2005) * 100#
    AgedShare(year - 2005) = Aged(year - 2005) / Pop(year - 2005) * 100#
    YouthShare(year - 2005) = Youth(year - 2005) / Pop(year - 2005) * 100#
    OS_Share(year - 2005) = (1# - Aust_Born(year - 2005) / Pop(year - 2005)) * 100#
    POP2555Share(year - 2005) = POP2555(year - 2005) / Pop(year - 2005) * 100#
```

Next year

<table>
<thead>
<tr>
<th>Year Range</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007-2005</td>
<td>( \frac{\text{Pop}(2007 - 2005)}{1000000} )</td>
</tr>
<tr>
<td>2007-2005</td>
<td>( \text{AgedShare}(2007 - 2005) )</td>
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<tr>
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<td>( \text{YouthShare}(2007 - 2005) )</td>
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</tr>
<tr>
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<td>( \frac{\text{Prime}(2007 - 2005)}{1000000} )</td>
</tr>
<tr>
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</tr>
<tr>
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<td>( \text{Under10}(2007 - 2005) \times \frac{100}{\text{Pop}(2007 - 2005)} )</td>
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<tr>
<td>2007-2005</td>
<td>( (\text{Pop}(2007 - 2005) / \text{Pop}(2007 - 2005 - 1) - 1) \times 100 )</td>
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</tr>
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</tr>
<tr>
<td>2007-2005</td>
<td>( \text{Under10}(2007 - 2005) \times \frac{100}{\text{Pop}(2007 - 2005)} )</td>
</tr>
</tbody>
</table>

If OptButNo_StoreMode = False Then
   Sheets.Add.Name = "Detailed"
Call MoveSheets
Worksheets("detailed").Cells(309, 1).Value = "Total population"
For year = 2007 To 2251
    Worksheets("detailed").Cells(1, year - 2005).Value = year
    ' Calculates Life expectancy from Qx values
    ' It assumes first value of Q is Qb, then Q0 to Qmax
    BigL_m(0) = (1# - Qx_Male(0, year - 2005)) * 100000#  'L0 = (1-Qb)*100,000 noting that Qxd(0) is Qb
    BigL_f(0) = (1# - Qx_Female(0, year - 2005)) * 100000#  'L0 = (1-Qb)*100,000 noting that Qxd(0) is Qb
    For i = 1 To 100
        BigL_m(i) = (1# - Qx_Male(i, year - 2005)) * BigL_m(i - 1)  'L1=(1-Q0)L0 to L100=(1-Q99)L99
        BigL_f(i) = (1# - Qx_Female(i, year - 2005)) * BigL_f(i - 1)  'L1=(1-Q0)L0 to L100=(1-Q99)L99
    Next i
    BigL_m(101) = BigL_m(100) * (1# - Qx_Male(101, year - 2005)) / Qx_Male(101, year - 2005)
    BigL_f(101) = BigL_f(100) * (1# - Qx_Female(101, year - 2005)) / Qx_Female(101, year - 2005)
    T_m = 0#
    T_f = 0#
    For i = 0 To 101
        T_m = BigL_m(i) + T_m
        T_f = BigL_f(i) + T_f
    Next i
    Life_m = T_m / 100000#
    Life_f = T_f / 100000#
    Worksheets("detailed").Cells(350, year - 2005).Value = Life_m
    Worksheets("detailed").Cells(351, year - 2005).Value = Life_f
    For age = 0 To 100
    Next age
    For age = 15 To 49
    Next age
Next year
For age = 0 To 100
    Worksheets("detailed").Cells(age + 105, year - 2005).Value = age
    Worksheets("detailed").Cells(age + 208, year - 2005).Value = age
Next age
For age = 15 To 49
Next age
Worksheets("detailed").Cells(2, 1).Value = "Male population (End June)"
Worksheets("detailed").Cells(104, 1).Value = "Female population (End June)"
Worksheets("detailed").Cells(207, 1).Value = "Total population(End June)"
Worksheets("detailed").Cells(311, 1).Value = "Age-specific fertility rates (babies per 1000 women)(End June)"
Worksheets("detailed").Cells(350, 1).Value = "Male life expectancy (years)"
Worksheets("detailed").Cells(351, 1).Value = "Female life expectancy (years)"
End If
If OptButPyramid_No = False Then  'Chart pyramids
Sheets.Add.Name = "Pyramids"
Call MoveSheets
Worksheets("Pyramids").Cells(1, 1).Value = "Population pyramids"
Worksheets("Pyramids").Cells(3, 2).Value = "2007"
Worksheets("Pyramids").Cells(3, 4).Value = "2051"
Worksheets("Pyramids").Cells(3, 6).Value = "2101"
Worksheets("Pyramids").Cells(3, 8).Value = "2151"
Worksheets("Pyramids").Cells(3, 10).Value = "2251"
Worksheets("Pyramids").Cells(4, 2).Value = "Males"
Worksheets("Pyramids").Cells(4, 3).Value = "Females"
Worksheets("Pyramids").Cells(4, 4).Value = "Males"
Worksheets("Pyramids").Cells(4, 5).Value = "Females"
Worksheets("Pyramids").Cells(4, 6).Value = "Males"
Worksheets("Pyramids").Cells(4, 7).Value = "Females"
Worksheets("Pyramids").Cells(4, 8).Value = "Males"
Worksheets("Pyramids").Cells(4, 9).Value = "Females"
Worksheets("Pyramids").Cells(4, 10).Value = "Males"
Worksheets("Pyramids").Cells(4, 11).Value = "Females"

For age = 0 To 100
Worksheets("Pyramids").Cells(age + 5, 1).Value = age
Worksheets("Pyramids").Cells(age + 5, 2).Value = -PopM(age, 2) / Pop(2)
Worksheets("Pyramids").Cells(age + 5, 3).Value = PopF(age, 2) / Pop(2)
Worksheets("Pyramids").Cells(age + 5, 4).Value = -PopM(age, 46) / Pop(46)
Worksheets("Pyramids").Cells(age + 5, 5).Value = PopF(age, 46) / Pop(46)
Worksheets("Pyramids").Cells(age + 5, 6).Value = -PopM(age, 96) / Pop(96)
Worksheets("Pyramids").Cells(age + 5, 7).Value = PopF(age, 96) / Pop(96)
Worksheets("Pyramids").Cells(age + 5, 8).Value = -PopM(age, 146) / Pop(146)
Worksheets("Pyramids").Cells(age + 5, 9).Value = PopF(age, 146) / Pop(146)
Worksheets("Pyramids").Cells(age + 5, 10).Value = -PopM(age, 246) / Pop(246)
Worksheets("Pyramids").Cells(age + 5, 11).Value = PopF(age, 246) / Pop(246)
Next age
Call Pyramid_Charting
End If

If OptButDepend_No = False Then
Sheets.Add.Name = "Dependency"
Call MoveSheets
Worksheets("Dependency").Cells(1, 1).Value = "Dependency"
Worksheets("Dependency").Cells(2, 2).Value = "Aged Dependency"
Worksheets("Dependency").Cells(2, 3).Value = "Youth Dependency"
Worksheets("Dependency").Cells(2, 4).Value = "Total Dependency"
For year = 2007 To 2251
Worksheets("Dependency").Cells(year - 2007 + 3, 1).Value = year
Next year
Call Dependency_chart
End If

Application.ScreenUpdating = True
CmdCancel_Mode = True
Worksheets("start").Select
End
End Sub

Function SheetExists(SheetName As String) As Boolean
' returns TRUE if the sheet exists in the active workbook
SheetExists = False
On Error GoTo.NoSuchSheet
...
If Len(Sheets(SheetName).Name) > 0 Then
    SheetExists = True
    Exit Function
End If
NoSuchSheet:
End Function

Sub secant()
Dim z0 As Double, z1 As Double
x0 = 100000
x1 = 110000
k = 0
epsilon = 50 / 30000000 * Target_pop ' small % error around target pop allowed
Do While Abs(LongRunPop(x1) - Target_pop) > epsilon And k < Nmax
    k = k + 1
    If Abs(LongRunPop(x1) - LongRunPop(x0)) < 0.000000001 Then
        MsgBox ("Population is no longer changing")
        Exit Sub
    End If
    z1 = LongRunPop(x1)
z0 = LongRunPop(x0)
err1 = z1 - Target_pop
err0 = z0 - Target_pop
x2 = x1 + err1 * (x1 - x0) / (err0 - err1)
x0 = x1
x1 = x2 ' use x2 as the solution
Loop
NOM_Fix = x2
End Sub

Public Function LongRunPop(Nom As Variant) As Double 'this is intended to model the long run population
'Overall cohort modelling
'cohort behaviour of migrants
For year = 2008 To 2251
    If year - 2007 < Transition_yrs_NOM Then
        POPNOM(year - 2005) = (Nom - POPNOM(2)) / Transition_yrs_NOM + POPNOM(year - 2005 - 1) ' popNOM(3) is 2008
    Else
        POPNOM(year - 2005) = Nom
    End If
Next year

For age = 1 To 99
    POPNOM_Male(age, year - 2005) = AgeShare_NOM_Male(age) / 100# * Male_share * POPNOM(year - 2005) ' year 3 is 2008
    POPNOM_Female(age, year - 2005) = AgeShare_NOM_Female(age) / 100# * (1# - Male_share) * POPNOM(year - 2005)
    SURVIVE_POPNOM_Male(age, year - 2005) = 0.5 * POPNOM_Male(0, year - 2005) * (1# - 0.5 * Qx_Male(1, year - 2006)) ' year 3 is 2008
    SURVIVE_POPNOM_Female(age, year - 2005) = 0.5 * POPNOM_Female(0, year - 2005) * (1# - 0.5 * Qx_Female(1, year - 2006))
Next age

POPNOM_Male(100, year - 2005) = AgeShare_NOM_Male(100) / 100# * Male_share * POPNOM(year - 2005) ' year 3 is 2008
POPNOM_Female(100, year - 2005) = AgeShare_NOM_Female(100) / 100# * (1# - Male_share) * POPNOM(year - 2005)
SURVIVE_POPNOM_Male(100, year - 2005) = 0.5 * POPNOM_Male(99, year - 2005) * (1# - 0.5 * Qx_Male(100, year - 2006)) + POPNOM_Male(100, year - 2005) * (1# - 0.5 * Qx_Male(101, year - 2006)) \( \text{year 3 is 2008} \)

SURVIVE_POPNOM_Female(100, year - 2005) = 0.5 * POPNOM_Female(99, year - 2005) * (1# - 0.5 * Qx_Female(100, year - 2006)) + POPNOM_Female(100, year - 2005) * (1# - 0.5 * Qx_Female(101, year - 2006)) \( \text{year 3 is 2008} \)

Next year

`end of cohort behaviour of migrants`

'-----------------------------------------------------------------------------------

'cohort component calcs of whole population

For year = 2008 To 2251

TotalBirths(year - 2005) = 0#

For age = 1 To 99

Survivors_male(age, year - 2005) = (1# - Qx_Male(age, year - 2006)) * PopM(age - 1, year - 2006)

Survivors_female(age, year - 2005) = (1# - Qx_Female(age, year - 2006)) * PopF(age - 1, year - 2006)

PopM(age, year - 2005) = Round(SURVIVE_POPNOM_Male(age, year - 2005) + Survivors_male(age, year - 2005), 0) \( \text{year 3 is 2008} \)

PopF(age, year - 2005) = Round(SURVIVE_POPNOM_Female(age, year - 2005) + Survivors_female(age, year - 2005), 0) \( \text{year 3 is 2008} \)

Survivors_male_Aborn(age, year - 2005) = (1# - Qx_Male(age, year - 2006)) * PopM_ABorn(age - 1, year - 2006) \( \text{noting Qx(1,2) is Qx for age=0 in 2007} \)

Survivors_female_Aborn(age, year - 2005) = (1# - Qx_Female(age, year - 2006)) * PopF_ABorn(age - 1, year - 2006)

PopM_ABorn(age, year - 2005) = Round(Survivors_male_Aborn(age, year - 2005), 0)

PopF_ABorn(age, year - 2005) = Round(Survivors_female_Aborn(age, year - 2005), 0)

If age >= 15 And age <= 49 Then

Births(age, year - 2005) = 0.5 * (Age_specific_TFR(age, year - 2006) * PopF(age, year - 2005) + Age_specific_TFR(age, year - 2005) * PopF(age, year - 2005) / 100# births(15,3) are births for 15yr olds in 2008

TotalBirths(year - 2005) = TotalBirths(year - 2005) + Births(age, year - 2005)

End If

Next age

Survivors_male(100, year - 2005) = (1# - Qx_Male(100, year - 2006)) * PopM(99, year - 2006) + (1# - Qx_Male(101, year - 2006)) * PopM(100, year - 2006)

Survivors_female(100, year - 2005) = (1# - Qx_Female(100, year - 2006)) * PopF(99, year - 2006) + (1# - Qx_Female(101, year - 2006)) * PopF(100, year - 2006)

PopM(100, year - 2005) = Round(SURVIVE_POPNOM_Male(100, year - 2005) + Survivors_male(100, year - 2005), 0) \( \text{year 3 is 2008} \)

PopF(100, year - 2005) = Round(SURVIVE_POPNOM_Female(100, year - 2005) + Survivors_female(100, year - 2005), 0) \( \text{year 3 is 2008} \)

Survivors_male_Aborn(100, year - 2005) = (1# - Qx_Male(100, year - 2006)) * PopM_ABorn(99, year - 2006) + (1# - Qx_Male(101, year - 2006)) * PopM_ABorn(100, year - 2006) \( \text{noting Qx(100,2) is Qx for age=99 in 2007} \)

Survivors_female_Aborn(100, year - 2005) = (1# - Qx_Female(100, year - 2006)) * PopF_ABorn(99, year - 2006) + (1# - Qx_Female(101, year - 2006)) * PopF_ABorn(100, year - 2006) \( \text{noting Qx(100,2) is Qx for age=99 in 2007} \)

PopM_ABorn(100, year - 2005) = Round(Survivors_male_Aborn(100, year - 2005), 0)

PopF_ABorn(100, year - 2005) = Round(Survivors_female_Aborn(100, year - 2005), 0)

Survivors_male_Aborn(0, year - 2005) = (1# - Qx_Male(0, year - 2006)) * PopM_ABorn(99, year - 2006) + (1# - Qx_Male(101, year - 2006)) * PopM_ABorn(100, year - 2006) \( \text{noting Qx(100,2) is Qx for age=99 in 2007} \)

Survivors_female_Aborn(0, year - 2005) = (1# - Qx_Female(0, year - 2006)) * PopF_ABorn(99, year - 2006) + (1# - Qx_Female(101, year - 2006)) * PopF_ABorn(100, year - 2006) \( \text{noting Qx(100,2) is Qx for age=99 in 2007} \)

PopM_ABorn(0, year - 2005) = Round(Survivors_male_Aborn(0, year - 2005), 0)

PopF_ABorn(0, year - 2005) = Round(Survivors_female_Aborn(0, year - 2005), 0)

Next year

' long run population

Dim Tot As Double

Tot = 0#

For age = 0 To 100

Tot = Tot + PopM(age, 2251 - 2005) + PopF(age, 2251 - 2005)

Next age

LongRunPop = Tot
'End of cohort model

End Function

Sub input_start()

With Worksheets("Inputs")
    For i = 0 To 101
        Qx_Male(i, 0) = .Cells(4 + i, 3).Value ' male Qx IN 2005 running from birth, age 0, age 1 to age 100+ (ie year 0 is 2005)
        Qx_Female(i, 0) = .Cells(108 + i, 3).Value
        DQx_Male(i) = .Cells(4 + i, 6).Value 'This is the value of Dlog(Qx) males to 2050 from the ABS
        DQx_Female(i) = .Cells(108 + i, 6).Value
    Next i
End With

For age = 0 To 100
    PopM(age, 2) = Worksheets("Inputs").Cells(5 + age, 2).Value   ' 2007 end June pop of males
    PopF(age, 2) = Worksheets("Inputs").Cells(109 + age, 2).Value  ' 2007 end June pop of females
    TotPopM(2) = TotPopM(2) + PopM(age, 2) 'pop of males in 2007
    TotPopF(2) = TotPopF(2) + PopF(age, 2) 'pop of males in 2007
Next age

For age = 0 To 100
    BornOS_male_age_dist(age) = Worksheets("Inputs").Cells(5 + age, 5).Value  ' share of males born overseas by age
    BornOS_female_age_dist(age) = Worksheets("Inputs").Cells(109 + age, 5).Value ' share of females born overseas by age
    NOM_male_age_dist(age) = Worksheets("Inputs").Cells(5 + age, 4).Value  ' share of new male migrants by age
    NOM_female_age_dist(age) = Worksheets("Inputs").Cells(109 + age, 4).Value ' share of new female migrants by age
    PopM_ABorn(age, 2) = PopM(age, 2) - BornOS_male_age_dist(age) / 100# * TotPopM(2) * mig_share / 100# ' POPF_OSBorn(age,2) is data for end 2007
    PopF_ABorn(age, 2) = PopF(age, 2) - BornOS_female_age_dist (age) / 100# * TotPopF(2) * mig_share / 100# ' POP_POBorn(age,2) is data for end 2007
Next age

For age = 0 To 100
    AgeShare_NOM_Male(age) = Worksheets("Inputs").Cells(age + 5, 4).Value
    AgeShare_NOM_Female(age) = Worksheets("Inputs").Cells(age + 109, 4).Value
Next age

POPNOM(2) = 177600  ' NOM in 2007 (ie year=2 is 2007, year =0 is ignored as not needed)

End Sub

Public Function LifeGoal(scale_guess As Variant, arrayDQx As Variant, arrayQx As Variant) As Double ' this undertakes the calculations for the 2251 life expectancy for 2251

Dim L(101) As Double
Dim T As Double
Dim i As Integer
Dim Qx_ABSLR(101) As Double

For i = 0 To 101
    Qx_ABSLR(i) = Exp(scale_guess * arrayDQx(i)) * arrayQx(i, 0) ' Qx IN 2251 with scalar adjustment of the (Dlog of ABS life expectancy from 2005 to 2050) - running from birth, age 0, age 1 to age 100+
Next i

L(0) = (1# - Qx_ABSLR(0)) * 100000# * L0 = (1-Qb)*100,000 noting that Qx_ABSLR(0) is Qb
For i = 1 To 100
    L(i) = (1# - Qx_ABSLR(i)) * L(i - 1) * L1=(1-QO)L0 to L100=(1-Q99)*L99
Next i

L(101) = L(100) * (1# - Qx_ABSLR(101)) / Qx_ABSLR(101)
T = 0#
For i = 0 To 101
    T = L(i) + T

End Function
Next i
LifeGoal = T / 100000#

End Function

Sub Pyramid_Charting()
'
'
Application.ScreenUpdating = False
Sheets("Pyramids").Select
Dim Rangename As String
Dim Titlename As String

For i = 1 To 5

If i = 1 Then
  Rangename = "a4:a105,b4:c105"
  Titlename = "2007"
ElseIf i = 2 Then
  Rangename = "a4:a105,d4:e105"
  Titlename = "2051"
ElseIf i = 3 Then
  Rangename = "a4:a105,f4:g105"
  Titlename = "2101"
ElseIf i = 4 Then
  Rangename = "a4:a105,h4:i105"
  Titlename = "2151"
ElseIf i = 5 Then
  Rangename = "a4:a105,j4:k105"
  Titlename = "2251"
End If

Range(Rangename).Select
Charts.Add
ActiveChart.ChartType = xlBarClustered
ActiveChart.SetSourceData Source:=Sheets("Pyramids").Range(Rangename), _
PlotBy:=xlColumns
ActiveChart.Location Where:=xlLocationAsObject, Name:="Pyramids"
With ActiveChart
  .HasTitle = True
  .ChartTitle.Characters.Text = Titlename
  .Axes(xlCategory, xlPrimary).HasTitle = True
  .Axes(xlCategory, xlPrimary).AxisTitle.Characters.Text = "age"
  .Axes(xlValue, xlPrimary).HasTitle = False
End With
With ActiveChart.Axes(xlCategory)
  .HasMajorGridlines = False
  .HasMinorGridlines = False
End With
With ActiveChart.Axes(xlValue)
  .HasMajorGridlines = False
  .HasMinorGridlines = False
End With
ActiveChart.HasLegend = False

ActiveChart.Axes(xlValue).Select
With ActiveChart.Axes(xlValue)
  .MinimumScale = -0.01
End With

End Sub
ActiveChart.ChartArea.Select
Selection.Interior.ColorIndex = xlAutomatic
Selection.AutoScaleFont = False
With Selection.Font
    .Name = "Arial"
    .FontStyle = "Regular"
    .Size = 8
End With

Next i
Call ArrangeMyCharts
Application.ScreenUpdating = True
End Sub

Sub ArrangeMyCharts()
    Dim iChart As Long
    Dim nCharts As Long
    Dim dTop As Double
    Dim dLeft As Double
    Dim dHeight As Double
    Dim dWidth As Double
    Dim nColumns As Long

    dTop = 75      ' top of first row of charts
    dLeft = 600    ' left of first column of charts
    dHeight = 225  ' height of all charts
    dWidth = 275   ' width of all charts
    nColumns = 3   ' number of columns of charts
    nCharts = ActiveSheet.ChartObjects.Count

    For iChart = 1 To nCharts
        With ActiveSheet.ChartObjects(iChart)
            .Height = dHeight
            .Width = dWidth
            .Top = dTop + Int((iChart - 1) / nColumns) * dHeight
        End With
    Next i
End Sub
Sub Dependency_chart()
    Dim iChart As Integer
    Dim nColumns As Integer
    nColumns = Cells(2, 1).Width
    For iChart = 2 To 247
        .Left = dLeft + ((iChart - 1) Mod nColumns) * dWidth
        Next
    End Sub

Sub Dependency_chart()
    * Dependency_chart Macro
    Range("A2:D247").Select
    ActiveWindow.LargeScroll Down:=-6
    Charts.Add
    ActiveChart.ChartType = xlLine
    ActiveChart.SetSourceData Source:=Sheets("Dependency").Range("A2:D247"), _
        PlotBy:=xlColumns
    ActiveChart.Location Where:=xlLocationAsObject, Name:="Dependency"
    With ActiveChart
        .HasTitle = True
        .ChartTitle.Characters.Text = "Dependency ratios"
        .Axes(xlCategory, xlPrimary).HasTitle = False
        .Axes(xlValue, xlPrimary).HasTitle = True
        .Axes(xlValue, xlPrimary).AxisTitle.Characters.Text = "%"
    End With
    With ActiveChart.Axes(xlCategory)
        .HasMajorGridlines = False
        .HasMinorGridlines = False
    End With
    With ActiveChart.Axes(xlValue)
        .HasMajorGridlines = False
        .HasMinorGridlines = False
    End With
    ActiveChart.HasLegend = True
    ActiveChart.Legend.Select
    Selection.Position = xlBottom
    ActiveChart.PlotArea.Select
    With Selection.Border
        .ColorIndex = 16
        .Weight = xlThin
        .LineStyle = xlContinuous
    End With
    Selection.Interior.ColorIndex = xlNone
    ActiveChart.ChartArea.Select
    With Selection.Border
        .Weight = 2
        .LineStyle = 0
    End With
    Selection.Interior.ColorIndex = xlAutomatic
    ActiveChart.SeriesCollection(3).Select
    With Selection.Border
        .ColorIndex = 1
        .Weight = xlMedium
        .LineStyle = xlContinuous
    End With
    With Selection
        .MarkerBackgroundColorIndex = xlNone
        .MarkerForegroundColorIndex = xlNone
        .MarkerStyle = xlNone
        .Smooth = False
        .MarkerSize = 3
        .Shadow = False
    End With
    ActiveChart.SeriesCollection(1).Select
End Sub
Private Sub OptButNo_Mode_Click()
    If OptButNo_Mode = False Then
        StartFrm.TxtNOM.Visible = False
        StartFrm.LblNOM.Visible = False
    Else
        StartFrm.TxtNOM.Visible = True
        StartFrm.LblNOM.Visible = True
    End If
End Sub

Private Sub OptButNo_StoreMode_Click()
End Sub

Private Sub OptButYes_Mode_Click()
    If OptButYes_Mode = True Then
        StartFrm.TxtPOP.Visible = True
        StartFrm.LblNOM.Visible = True
    Else
        StartFrm.TxtPOP.Visible = False
        StartFrm.LblNOM.Visible = False
    End If
End Sub

Private Sub TxtTransLE_Change()
End Sub

Private Sub TFRFrame_Click()
Private Sub TxtBoxLambda_Change()
  Lmb = Val(TxtBoxLambda)
  If Lmb < 0# Then
    MsgBox("Lambda cannot be less than zero! FERTMOD is a bit bossy. It will set Lambda to one")
    TxtBoxLambda.Value = 1#
    Exit Sub
  ElseIf Lmb > 2# Then MsgBox("Sorry, but Lambda cannot exceed 2. FERTMOD is intolerant of this error. Lambda will be set to one")
    TxtBoxLambda.Value = 1#
    Exit Sub
  ElseIf Lmb > 1# And Lmb <= 2 Then MsgBox("FERTMOD hopes you know what you are doing - be cautious with Lambda in range 1 to 2")
  End If
End Sub

Private Sub TxtBoxTheta_Change()
  Theta = Val(TxtBoxTheta)
  If Theta < 0# Then
    MsgBox("Theta cannot be less than zero! Theta will be set to unity")
    TxtBoxTheta.Value = 1#
    Exit Sub
  ElseIf Theta > 2 Then MsgBox("Theta cannot exceed 2! Sorry Dave, but FERTMOD has taken control and will set Theta to one")
    TxtBoxTheta.Value = 1#
    Exit Sub
  ElseIf Theta > 1# And Theta <= 2 Then MsgBox("FERTMOD hopes you know what you are doing - be cautious with Theta > 1")
  End If
End Sub

Private Sub TxtTransNOM_Change()
End Sub

Private Sub UserForm_Click()
End Sub

Sub MoveSheets()
  ActiveSheet.Move _
    After:=ActiveWorkbook.Sheets(ActiveWorkbook.Sheets.Count)
  'Moves active sheet to end of active workbook.
End Sub

Sub fertility()
  Dim tot_sh, LRT, Omega As Double
  Const Italy As Double = 1.32 'Italian TFR in 2005 (Eurostat)
  Const France As Double = 2.004 'French TFR in 2006 (Eurostat)
  Dim ASFR_share(49) As Double
  Dim AusF_share(49) As Double
  Dim ABS_ratio(49) As Double
  Dim AdjRatio(49) As Double
  Dim ABS_ratio(49) As Double 'This indicates the extent to which, all other things being equal,
  Dim ABS_ratio(49) As Double 'age-specific fertility rates can be expected to shift in the long-run even if fertility stays at
    * its 2006 value(ie 1.81)
  Dim AdjRatio(49) As Double 'ABS_ratio is put into effect by multiplying the share of each ASFR in the TFR by ABS_ratio.
  Dim AdjRatio(49) As Double 'This is an adjustment ratio that shifts the long-run ASFRs as the TFR deviates away from its 2006 value.
  Dim AdjRatio(49) As Double 'It is based on the difference in the ASFR shares of the TFR between Italy (a low fertility country) and France (a high fertility country).
  Dim AdjRatio(49) As Double 'This difference is used to provide a function that indicates how the shares change as TFR moves away from the TFR in 2006 (TFR2006)
  Dim AdjRatio(49) As Double 'If TFR long-run is at TFR2006, then AdjRatio is 1 for all ages. It grows with age if TFRs fall below TFR2006
  Dim AdjRatio(49) As Double 'or it goes down with age if the TFR rises. This is the typical pattern revealed by time series and cross-country ASFRs of countries with different fertility levels.
mu = 2.166916836
Gamma = -0.303822102
Beta = 0.011590909
Alpha = -0.000127709
LRT = Application.Min(longrun_TFR, 3.35)
Omega = (LRT - TFR2006) / (Italy / France * TFR2006 - TFR2006)
tot_sh = 0#

For age = 15 To 49
    ABS_ratio(age) = 1# + LambDa * (2.338288 - 0.308566 * age + 0.0103884 * age ^ 2# - 0.00009140508 * age ^ 3#)
    AdjRatio(age) = 1# + ThetaVal * Omega * (mu + Gamma * age + Beta * age ^ 2# + Alpha * age ^ 3#)
    Age_specific_TFR(age, 1) = Worksheets("Inputs").Cells(age - 11, 8).Value ' age specific fertility rate in 2006 - (15,1) is 15yr olds in 2006
    AusF_share(age) = Age_specific_TFR(age, 1) / (1000# * TFR2006) ' proportion of total TFR of each asfr in 2006
    ASFR_share(age) = AdjRatio(age) * ABS_ratio(age) * AusF_share(age)
    tot_sh = tot_sh + ASFR_share(age)
Next age

For age = 15 To 49
For year = 2007 To 2251
    If (year - 2006 <= PeriodM) Then
        Age_specific_TFR(age, year - 2005) = Age_specific_TFR(age, year - 2006) + (LR_Age_specific_TFR(age) - Age_specific_TFR(age, 1)) / PeriodM
    Else
        Age_specific_TFR(age, year - 2005) = LR_Age_specific_TFR(age) 'Age_specific_TFR(15, 2) is ASFR for 15yr olds in 2007
    End If
Next year
Next age
End Sub

1.7 Troubleshooting

First, it is important to ensure macros are enabled in Excel. When the macro security level in Excel is set to Low, macros can be run without prompting. When macro security is set to Medium, Excel displays a dialog box asking if you want to enable macros. When macro security is set to High, Excel allows you to run only those macros that are digitally signed or stored in the Excel startup (XLStart) folder. FERTMOD is not digitally signed and will not run in High security mode. To change macro enabling, go to the ‘Tools’ menu in Excel, select Options, select the ‘Security’ tab, click the ‘Macro security’ button, and choose ‘Medium’ or ‘Low’.

Second, some users have reported that they have encountered the error message ‘Can’t find project or library’ when running the program. The problem appears to be caused by the program not being able to reference (i.e locate) the ‘Solver’ object.
To restore this or any other missing references take the following steps.

1. Open the Visual basic editor (select the Tools menu, then Macros, and then Visual Basic Editor) (figure 1.11). Open the Tools menu and select ‘References’

2. The References box will open (figure 1.12). Check whether Microsoft Excel has nominated any references as ‘missing’. If so, the label ‘missing’ will appear alongside the relevant reference. For example in the screen shown in figure 1.12, if ‘SOLVER’ was missing the box next to it would be unchecked and ‘missing’ would be printed beside it. This may mean that the Solver Add-In has not been installed on your computer.
3. To install the Solver Add-In, close the VBA editor. This will return you to the standard Excel worksheet screen. Then select the Tools menu, Add-ins and ensure that the Solver Add-in is ticked (figure 1.13). Click OK.

4. Then return to Visual Basic Editor as before, open ‘Tools’ menu, References and tick the box next to SOLVER.

The program should now run.
Figure 1.13  **Checking Add-ins**

Choose Tools and ‘Add_Ins’  
Check Solver Add-in is ticked
References
