
Incorporating household survey data into a CGE model

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Abstract

Incorporating multiple household types in a computable general equilibrium (CGE) model framework can enhance the capability of conventional CGE models to analyse issues that have to be analysed at the individual household level, such as income distribution and the impacts of differential taxes or subsidies on individual households. Traditionally, these issues have been analysed with microsimulation (MS) models. However, as household surveys usually have a very large sample size, including many thousand households, and CGE models are often structurally complex, incorporating a full sample of households into a CGE framework has been regarded as difficult and costly. Some researchers have used a dual-model approach. So far, only few attempts have been made to produce a fully integrated model.

In fact, incorporating multiple households into a CGE model is conceptually straightforward. With a simple structured CGE model, a large number of households can be incorporated by modifying only a few equations and the integrated model can be solved efficiently. This allows researchers to concentrate on the time-consuming task of reconciling the differences between the CGE and the survey databases.

This paper uses an Australian Household Expenditure Survey (HES) data to describe a simple procedure to integrate the entire sample of households (8,389) into the CGE framework. The paper focuses on the basic structures of the CGE model and of the database required for the household survey data to be incorporated. The CGE model is a conventional one based on the assumptions and theories commonly used in other models, so that the same procedure can be readily adapted to other CGE models with minimum modifications.

The integrated model eliminates three types of errors associated with the dual-model approach: partial equilibrium errors, due to the lack of feedback from household responses in the partial equilibrium MS model to the CGE model; aggregation errors due to the single representative

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household in the CGE model and the multiple households in the MS model; and inconsistency errors caused by the two different databases used by the two models.

The resulting model provides an efficient framework to analyse the distributional effects of policy changes, while accounting for behavioural and CGE effects. It can also be used to analyse the effects of detailed tax and transfer policies on individual households.

Incorporating household survey data into a CGE model

Incorporating multiple household types in a computable general equilibrium (CGE) model framework can greatly enhance the capability of conventional CGE models to analyse issues that have to be analysed at the individual household level, such as the distributional effects of policy changes and the impacts of differential taxes or subsidies for individual households. The ultimate goal for this effort is to integrate the income and expenditure details for a full sample of households, as recorded in national surveys.

Household surveys usually have a large sample size, ranging from thousands to tens of thousands of individual households. Incorporating such a large sample of households into an already complex CGE model framework has been a challenging task. Early efforts were hampered by computational constraints and complex model structures. As a result, the number of households incorporated in most integrated models was limited to no more than a few dozen representative household groups.¹

The behavioural micro-simulation (MS) model has been considered a more appropriate tool for income distribution analysis and to capture the behaviour of individual households. This is because behavioural MS models have simple and flexible structures, which make it easy to handle a large number of individual households from survey data sets. That said, behavioural MS models are partial equilibrium in nature: they cannot account for broader and feedback effects such as the changes in goods and factor prices, and other macro variables. This led researchers to adopt a two-model approach.² In this approach, a CGE model is linked with a MS model that is based on household survey data. Output from the CGE model, such as changes in goods and factor prices, are imposed on the MS model to estimate the effects on household budgets. Feedback to the CGE model can also be implemented if iterative links are established between a set of selected variables used in both models.

However, this two-model approach has its own limitations, due to the fundamental differences between the two models and between their databases. First, the two models differ in nature: CGE models are general equilibrium while the MS models are partial

¹ See Lofgren *et al.* 2004 for a summary of this approach.

² The two-model approach, also called “layered approach”, remains popular, particularly, among researchers who wish to retain the key ingredients of the MS models (see Davies, 2009, for a survey of this approach to income distribution analysis).

equilibrium. The two models also differ in their assumptions about household behaviour: CGE models with representative households assume that all households within a group behave uniformly, while in MS models there is much more diversity in household behaviour. These differences may cause the households in the two models to respond differently to the same policy change. Moreover, the two models have their own databases, which differ in size and composition. These differences and inconsistencies create problems in the interpretation of simulation results, even for the models that are linked iteratively. For example, one is unable to distinguish which part of the simulated impacts results from the feedback effects and which part results from data inconsistencies or aggregation errors.

As the differences between the two models are fundamental, the only solution to resolve the inconsistency problem is to make two models and their databases consistent with each other. If a two-model approach is preferred, a MS model and its database (survey data) may be taken as given and a CGE model needs to be built around the given MS model and the survey database to ensure full consistency. Alternatively, an integrated CGE-MS model approach may be an option, in which, a CGE model and its database are taken as given and household survey data need to be adjusted so that they can be fully integrated into the CGE framework. Unlike the two-model approach, the integrated approach is a single model approach: the MS model is absorbed into the CGE framework and becomes part of a general equilibrium model. That said, this usually involves losing some of the detail in the MS model.

The first attempt at an integrated approach was Decaluwé, *et al.*, (1999), which used a stylised CGE model with fictitious household data to test the feasibility of an integrated model. This study inspired a number of studies on income distribution in the early 2000s to integrate full samples of households into more sophisticated CGE models for policy analysis. Cogneau and Robillard (2001) integrated 4,508 households into a simple, three-sector model of Madagascar to analyse the impacts of different growth strategies on poverty and income distribution.³ Cockburn (2006) integrated a sample of 3,373 households from a Nepal Living Standard Survey into a CGE model with 15 sectors and 3 regions to analyse the impacts of trade liberalisation on poverty.⁴ Cororaton and Cockburn (2007) built a model of the Philippines with 12 sectors and added a sample of 24,797 households from the Family Income and Expenditure Survey, to analyse the impacts of eliminating all import tariffs on household incomes. This 12-sector model was later extended to 35 sectors in Cockburn *et al.* (2008).⁵

The integrated model has not been widely adopted. This is probably because the data reconciliation and model modifications required have been costly, especially if the CGE

³ Both models are highly stylised and not for detailed policy applications. The Decaluwé, *et al.*, (1999), model is used as a proof of concept and the Cogneau and Robillard (2001) model does not include a government sector or taxes.

⁴ Cockburn's work was first reported in two working papers published in the early 2000s (2001, 2002).

⁵ The original work on this model was presented by Corong and Cororaton at a conference in 2007.

structure is large and complex. The integration procedure can result in a model too big to solve in a timely fashion.⁶

In fact, incorporating a large number of households into a CGE model is conceptually straightforward. With a simple structured model, a large number of households can be incorporated by modifying only a few equations to link individual households with the aggregated household in the CGE model; the integrated model can maintain the simplicity of the original structure and be solved efficiently. Although we refer to the model used here as ‘simple’, it contains many more of the features of a ‘standard’ CGE model based on a national input-output table than the models mentioned above⁷ – it is similar to the structure of the ORANI suite of models.⁸ The relative simplicity of the code used to implement it allows researchers to concentrate on the more time-consuming task of reconciling the differences between the CGE and the survey databases.

This paper uses an Australian Household Expenditure Survey (HES) dataset as an example to describe a simple procedure to integrate an entire sample of households (8,389) into a CGE model with 54 sectors and 8 labour occupations. As a proof-of-concept paper, the main focus is on the simple structure of the CGE model and database that facilitate the integration of household survey data. Although structurally simple, the model used is similar to many available CGE models, so that the procedure proposed can be readily implemented in other CGE models. That said, the relative simplicity of the CGE implementation is a key factor in facilitating the process described here.

The remainder of the paper is organised as follows. The next section outlines the structure and the database of the CGE model used for integration. It is followed, first, by the description of a simple and practical procedure for reconciling the CGE and household survey datasets, and then, the modifications of the conventional CGE model required for household integration. Test simulations are conducted to compare results from the integrated model and from the two-model approach (the original single-household conventional model, linked with the same household data). The differences in the simulations reveal aggregation and inconsistency errors, inherent to the two-model approach. Some concluding remarks are made in the last section with suggestions on future directions for the development of integrated models.

⁶ For example, the original version of an integrated Philippine model, PHILGEM, with 240 sectors, 10 labour occupations and 42,094 households is considered to be “far too big to solve” (page 85, Corong and Horridge, 2012).

⁷ Some simplifications are also made to focus on the method and results of integrating household data. Other assumptions might be preferable in a model to be used in policy analysis.

⁸ Developed by the Centre of Policy Studies at the Victoria University, Australia.

A simple CGE model

In this section, we use data from an existing CGE model database to build a simple structured national model to use as a base to incorporate multiple households.

Database

The model used in this paper is built on the basis of a national input-output table, aggregated from an early version of the MMRF model of the Australian economy (Peter, *et al.* 1996).⁹ The input-output table includes 54 single-output industries and 8 labour occupations. An aggregated version of this table is shown in figure 1.

Industries (aggregated in the first column) use domestically produced and imported intermediate goods in production. Domestically produced goods (the first row) are used as intermediate inputs by industries and by other users: investors by industry, a representative household and government. Some products are exported, and some can be used as transport margins, which are added to the basic price of products.

Figure 1 **Aggregated input-output table in basic prices (AUD million)**

		Ind	inv	hou	gov	exp	mgn	Total
Products	dom	258463	60849	169428	77688	67578	97716	731723
	imp	48544	11712	17684	159			78099
Margins	dom	22558	2137	33206		8994		66895
	imp	11542	3734	15545				30821
Taxes	dom	8593	427	14252		360		23632
	imp	2362	492	3133				5987
Factors	lab	231339						231339
	cap	102271						102271
	Ind	3787						3787
	oct	42263						42263
Total		731723	79351	253248	77847	76932	97716	1316818

Notes: 1. Basic values of imports (78099) include import tariffs (5751).

2. oct is the unspecified 'other cost ticket'.

Source: Derived from a MMRF database.

As the main owner of primary factors, the household receives factor incomes. Household primary income is divided between consumption expenditure and savings. Government collects various taxes and uses this income to pay for expenditures on goods and services.

The input-output table reveals a government budget deficit of 42,477,^{10,11} which needs to be financed. In the absence of additional information about how the budget deficit is

⁹ This database, derived from 1993 national account data, is chosen to be consistent with the 1993 Household Expenditure Survey data used in this paper. These were chosen on the basis of ease of access.

¹⁰ All figures in the text are in AUD millions, unless indicated otherwise.

financed, a simplifying assumption is made that government's budget deficit is fully financed through a lump-sum transfer from households.¹² Household saving is, therefore, the difference between disposable income (primary factor income, net of the lump-sum transfer) and private consumption expenditure. As government has zero saving by assumption, household saving is equivalent to national saving.

Investment is partly funded by domestic savings. The gap between investment and saving is net foreign investment (NFI) inflow,¹³ which should be equal to the country's trade deficit.

The information in the input-output table and the assumed transfer payments from household to government can be combined in a social accounting matrix (SAM). An aggregated version of this SAM is shown in Figure 2. Household primary income is 379,660 (row 4 sum), taken entirely from factor income. Government income is 77,847 (row 5 sum), consisting of tax revenues 35,370 (cell 5,1) and lump-sum transfer 42,477 (cell 5,4). The expenditures of household and government are shown as the sums of columns 4 and 5, respectively. The figure also shows that total investment of 79,351 (cell 1,6) is funded by household savings of 83,935 (cell 6,4) and the excess savings is recorded as an outflow of NFI 4,584 (cell 6,8).

Model

The database described above contains all the essential information required for a CGE model. A simple structured database makes the model structurally simple.

The core of the model consists of 38 equations, which are essential for model's general equilibrium solution.¹⁴ A full list of the core equations is provided in Appendix table A2. This section highlights only some key components.

¹¹ The deficit is the difference between tax revenues and current expenditures.

¹² This assumption is made for convenience. Alternative assumption can also be made.

¹³ Although referred to as net foreign investment inflow, this item may include the items of the balance of payments that add up to the trade deficit, including net remittances abroad and net returns to foreign capital. That said, in this model, foreign ownership of capital is not accounted for explicitly and the corresponding income is allocated to households.

¹⁴ This list includes all aggregate macro accounting balancing conditions, but does not include variables and equations that are used purely for result presentational purpose, such as those aggregating quantities or averaging prices, which can be added to the models without affecting its solution.

Figure 2 **Aggregated social accounting matrix (AUD million)**

		Goods & Services	Production	Factor income	Household	Government	Investment	RoW, CurAcc	RoW, CapAcc	
		1	2	3	4	5	6	7	8	
Goods/Services	1	97716	352063		253248	77847	79351	76932		937158
Production	2	731723								731723
Factor income	3		379660							379660
Household	4			379660						379660
Government	5	35370				42477				77847
Investment	6					83935				79351
RoW, CurAcc	7	72348								72348
RoW, CapAcc	8								-4584	-4584
		937158	731723	379660	379660	77847	79351	72348	-4584	

Source: Figure 1.

The model specifies the behaviours of three types of economic agents:

- A representative firm in each industry, which purchases intermediate inputs and factor services to produce goods or services. For given demand for its products and input prices, the firm uses constant-return-to-scale (CRTS) technology to minimise the costs of production.
- A representative household, which plays three roles: as factor owner, it receives factor incomes from firms; as consumer, it spends its income on goods and services to maximise its utility; and as investor, it allocates its savings across industries to maximise total returns.
- A single government sector, which collects revenue from various taxes on goods/services and from household and spend its income on transfers to households and on purchasing goods and services.

The price system of domestically produced goods consists of three layers:

- Basic price = unit cost of production, including intermediate input costs and value-added.
- Producer price = basic price plus transport and sales margin services.
- Purchaser's price = producer price plus indirect taxes.

The basic structure of the model can be summarised in the behaviours of two type of users: users of intermediate inputs (producers) and users of final products (household, government and investor).

Industrial firm behaviour has five components (refer to table A2).

- *Industry demand for domestic and imported intermediate inputs*: a CES cost-minimising demand function of two variables – the given purchasers' prices and the industry demand for the composite good (eq. 1).

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- *Industry demand for composite intermediate inputs*: a Leontief cost-minimising demand function of a given industry's output (eq. 2).
 - *Industry demand for factor inputs*: a CES cost-minimising demand function of two variables – their basic prices and the industry output (eqs. 6-9).
 - *Industry output*: determined by the total demand for its product, due to the properties of CRTS technology (eq.11).
 - *Basic price of industry output*: determined by the unit cost of production, due to the properties of CRTS technology (eq.16).

Final users' behaviours (household, government and investor) have three components.

- *Final user's demand for domestic and imported goods*: a CES cost-minimising demand function of two variables – the given purchasers' prices and final user's demand for the composite good (eq. 17).
- *Final user's demand for composite goods*: the functional form varies depending on the final user concerned. For investor in an industry, it is usually a Leontief cost-minimising demand function of a given quantity of capital formation, allocated by the investor (household). For government, its demand could be determined either endogenously or exogenously. For households, a number of cost-minimising demand functions can be chosen, ranging from Cobb-Douglas, CES to Linear Expenditure System. They are normally a function of two variables – the purchaser's prices of the composite goods, to be consumed, and household spending income (eq. 19).
- *Final user's income and expenditure*: as indicated earlier in the model data section, three final users all have their expenditures fully paid for by their respective incomes, and, therefore, all have balanced budgets (eqs. 26-28).

There are a number of options for the treatment of exports, depending on the assumption about the nature of exported goods. If exported goods are assumed to be identical to that designated for domestic consumption, the foreign demand for exported goods could be exogenously fixed or endogenously determined by a foreign demand function, which may be a function of the given price of exported good and a given foreign income, expressed in foreign currency.¹⁵

In this model, all endogenous variables are uniquely defined by a single equation, except the basic prices of primary factors: labour (by occupation), capital and land. If the supplies of factors are specified exogenously, each of these basic factor prices can be determined uniquely by its market clearing condition (eqs. 33-38). As a result, the model has equal

¹⁵ The assumption of homogenous exports is adopted in this study, see eq. 22 in Appendix table A2. Alternatively, if exported goods are assumed to be differentiated with that designated for domestic consumption, an export supply function may be specified. For example, a CET supply function could be used to specify the supplies of domestic and exported goods. The prices of exported goods will then be determined endogenously by their market clearing conditions (see Appendix table A3 for details).

numbers of endogenous variables and equations, implying a valid closure and a unique solution. This can be used as the first macro closure for the model.¹⁶

Reconciling CGE and household survey data

CGE models are based mainly on official input-output data, while household survey data are based on individual household responses to survey questions. The input-output data are processed or adjusted to be internally consistent with national account information, while the household survey data are un-processed original data and, therefore, unlikely to be consistent with national account data in the input-output table. To incorporate the household survey data into the input-output data of a CGE model requires careful data adjustments on one of the datasets, or both of them.

There is no consensus among researchers on which dataset should be adjusted to fit the other. Unlike processed national account data, either in the form of an input-output table or a SAM, the household survey data inevitably contain errors and omissions.¹⁷ One may be inclined to adjust the household survey data to fit the national account data. In practice, the issues about the data reconciliation are far more complex than those stylised perceptions. The perceived errors and omissions, or the accuracy of household survey data, vary across countries. The choice of which dataset to adjust, to a large extent, depends on one's experience with and understanding of the datasets.¹⁸

In fact, many of the perceived errors in household survey data cannot be easily identified, let alone corrected. More importantly, any attempt to adjust one side of the household account could distort the balance for the entire dataset, creating serious problems of incoherence between the expenditure and the income sides. Since the purpose of integrating the household data into a CGE framework is to model the behaviour of individual households, it seems desirable to keep the original structure of individual households' budgets. As a principle, the integrity of the survey dataset should be respected and, therefore, any adjustment to the data be kept to a minimum, avoiding particularly any distortion to inter-household relativities and intra-household balances.

¹⁶ This closure is designed to be simple, as it is used only to generate model's first solution for checking and validation. Alternative macro closures can be readily implemented by swapping between endogenous and exogenous variables, or adding new variables or equations, as required.

¹⁷ For example, in household surveys, it is more difficult to collect reliable information on income than on consumption, although consumption data also have their own problems (Deaton, 1997).

¹⁸ See Cockburn (2006) and Cockburn, *et. al*, (2008) for examples of data reconciliation.

HES data extraction and manipulation ¹⁹

The data on household income and expenditure are extracted from the 1993-94 Household Expenditure Survey (HES, ABS 1994d). The HES data are contained in two unit record data files: one file contains expenditure data on 8,389 sample households; the other file contains data on income sources for the 25,660 individual members of the sample households. The household data are mapped to the income sources and commodity detail of the CGE model.

On the income side, 34 types of personal income are recorded in the HES, including private incomes, and government benefits and taxes. In households with multiple income earners, incomes are consolidated according to sources within each household.

Private income sources include wages and salaries from eight occupations, and non-wage income from investment and business. Government transfer payments (including family allowance, unemployment benefits and age pensions), and direct tax payments are listed. Table 1 displays the allocation of the HES sources of income to the CGE database items.

In the HES data set, there is no income source matching “land” and “oct” shown in the input-output table. To insert such incomes into the household data set, a simplified assumption is adopted that these incomes are proportionate to capital income. This implies that all capital earning households also receive, as a fixed proportion to their capital income, incomes from “land” and “oct”. These proportions are determined by their respective ratios to capital in the input-output table.²⁰

Table 1 Sources of household income in CGE model and HES

<i>CGE model</i>	<i>Household Expenditure Survey</i>
Wages for eight occupations (same as those in HES)	Wages for Managers and administrators; Professionals; Para-Professionals; Tradespersons; Clerks; Salespersons and personal service workers; Plant and machine operators and drivers; Labourers and related workers
Capital	Interest; Investment; Property rent; Superannuation; Business;
Other private income or government benefits	Workers compensation; Accident compensation; Maintenance; Other regular sources; Private scholarship; Government scholarship; Overseas pensions, Unemployment benefits, Sickness benefits; Family allowance; Veterans' pensions; Age pensions; Widows' pensions; Disabled pensions; Supporting parenting benefits; Wives' pensions; Other Australian government benefits; AUSTUDY support; Carers' pensions; Other overseas government benefits
Direct taxes	Direct taxes

On the expenditure side, there are 772 items recorded under the Commodity Code (ComCode) system in the HES data file. These items include purchases of durable goods

¹⁹ This section is based partly on Verikios and Zhang 2005.

²⁰ Inserting additional incomes will inevitably distort the HES data set. However, as these income sources are relatively small in size, the distortion created may not be significant. Moreover, as the overall sizes of incomes in the two data sets need to be reconciled later, the initial distortions are expected to reduce further.

and the repayment of principal and interest on loans (ComCode 736-772), which do not form part of current consumption. As result, the 735 items of consumer goods and services are aggregated to 54 sectors, consistent with the commodity detail in the CGE model.²¹

Reconciling HES and CGE data

Once the preliminary processing of the HES data is complete, a five-step procedure can be adopted to reconcile the differences between the two databases. As the HES expenditure and income data are expressed in purchaser prices, the input-output data used should also be in purchaser prices.

1. Use the HES weights to inflate the sample household data to the level for the whole population.
2. Scale the aggregated HES expenditure to be consistent with the total household expenditure in the input-output table.
3. Scale the aggregated HES primary incomes to be consistent with the total factor income in the input-output table.
4. Adjust each HES household's expenditure shares, while keeping total expenditure on each product constant, so that the aggregation of all HES expenditures is consistent with the commodity consumption pattern of the representative household shown in the input-output table.
5. In each industry in the input-output table, adjust factor income shares (capital and labour by occupation), while keeping the total value of each factor income constant, so that the aggregated factor income pattern in the input-output table is consistent with the aggregated income pattern derived from all HES households.

The aim of the second and third steps of adjustments is to preserve the original relativities between households in the survey data. The aim of the fourth and fifth steps is to keep the budget balances of individual households unchanged. A bi-proportional adjustment technique, such as RAS, or some entropy-based method, is required to carry out the last two steps. Figure 3 illustrates how these are carried out.

On the right hand side is the input-output table (A), in which two parts are highlighted: the vector for the representative household consumption expenditure and the matrix for factor incomes. On the left hand side are the two tables of the scaled HES household expenditure and income data (B), generated with the first three steps of adjustments. The first three steps matched aggregate expenditures and incomes across both database. The next two steps are used to make individual items of expenditure and income consistent across the two databases.

²¹ These items are assumed to add to household savings. This treatment is consistent with the simplicity of the input-output database. In a more sophisticated model, these items might be modelled explicitly as banking or financial transactions.

Figure 3 Illustration of reconciling the two databases

A. CGE input-output table in purchasers' prices					B. Scaled HES household data				
<i>Product</i>	ind01	← ind →	ind54	←othusr→	hou	h0001	← hou →	h8389	<i>Expense</i>
com01		...				↑	↑	↑	com01
com02			com02
...	No	Adjust	Adjust	Adjust	...
com53		...			change	shares	shares	shares	com53
com54		...				↓	↓	↓	com54
<i>FacInc</i>									<i>FacInc</i>
lab1	↑	↑	↑				...		lab1
...
lab8	Adjust	Adjust	Adjust			No	No	No	lab8
cap	shares	shares	shares			change	change	change	cap
lnd			lnd
oct	↓	↓	↓				...		oct

On the expenditure side, the vector for household expenditure in the input-output table is used as a row target to adjust the expenditure shares for each HES household. In this process, the total expenditure for each HES household is used as a column target, and therefore, remains unchanged. The result of this adjustment is a 54 x 8389 matrix of household expenditures, which can be inserted in the input-output table to replace the vector of single household expenditure.

On the income side, the aggregated factor incomes for all HES households are used as a row target to adjust the factor income shares for each industry in the input-output table.²² In this process, the total value-added for each industry in the input-output table is used as a column target and, therefore, remains intact.

This procedure minimises the changes required in both data sets and preserves as much as possible the original balances in the HES budgets and in the industries in the CGE input-output table. The completed input-output table in purchaser prices has a structure shown in figure 4. Compared with figure 1, the only differences are an increase in the number of households from one to 8,389 and the changes in the value-added shares in the industries (shaded areas). The aggregated expenditure for each household and the total value-added for each industry remain unchanged. Therefore, the original costs of industrial outputs and the total sales of goods and services produced remain unchanged.

²² A similar bi-proportional adjustment cannot be carried out on the HES household income side. This is because many HES households have only one primary income source, or have no factor income at all, and rely on government benefits. Under such conditions, a bi-proportional adjustment cannot keep the incomes of those households constant.

Figure 4 Integrated input-output table in purchasers' prices (AUD million)

	ind	inv	←	hou	→	gov	exp	mgn	Sales
com	352063	79351	←	253248	→	77847	76932	97716	937158
lab	292938								292938
cap	59859								59859
lnd	2265								2265
oct	24597								24597
Costs	731723	79351	←	253248	→	77847	76932	97716	1316818

Source: Figures 1 and 3.

The HES data also include some information on transfers between households and the government, which is not captured in the original input-output table. This additional information can be shown in a SAM (figure 5).

Compared with the original SAM in figure 2, this new SAM includes 8,389 households. As their total expenditure and factor income remain unchanged, the two SAMs have identical values in all cells, except those shown in the shaded area, transfers between households and the government:

Figure 5 Aggregated SAM of the integrated CGE database (AUD million)

		Goods & Services	Production	Factor income	Households	Government	Investment	RoW, CurAcc	RoW, CapAcc	
		1	2	3	4	5	6	7	8	
Goods/Services	1	97716	352063		253248	77847	79351	76932		937158
Production	2	731723								731723
Factor income	3		379660							379660
Household	4			379660		58460				438120
Government	5	35370			100937					136307
Investment	6				83935				-4584	79351
RoW, CurAcc	7	72348								72348
RoW, CapAcc	8							-4584		-4584
		937158	731723	379660	438120	136307	79351	72348	-4584	

Source: Figure 4.

- Cell 4,5 is the sum of various benefits that households receive from the government, which originates from the HES household income data (58,460).
- Cell 5,4 is the sum (100,937) of total income tax (77,430) and the assumed lump-sum payment (23,508) that households pay to the government. The information on income tax is obtained from the HES,²³ while the lump-sum payment is what is required to balance the government's budget.

²³ The HES includes information on income taxes on wages and compensation and on capital.

It should be emphasised that the approach does not address the issue as to which database is more accurate and which one should be adjusted more than the other. As mentioned earlier, it is likely that neither the household survey nor the input-output data sets are free from errors. No attempt has been made to correct possible data problems. Instead, the two databases are taken as given. The adjustment proposed are only those required for reconciling the differences between the two data sets, not for correcting any errors within each data set.

Modifying the model to incorporate sample households

Household Expenditure Survey (HES) data contain much more information on individual household expenditure and income than what is normally contained in an input-output table. Particularly valuable information is the various benefit payments from the government to households and the income tax from households to the government. To incorporate this additional information into the integrated model, a new “household module”, needs to be created in the core equation system (table 2). This module specifies the behaviour of individual households based on the income and expenditure sides of the data.

Household module

On the income side, the module aggregates each household’s incomes from various sources, as reported in the HES data set. Four types of household income need to be redefined (refer to table 2).

- *Household primary income*, including income from different occupation categories, capital and land that are supplied by households (eqs. 39 and 40).
- *Household taxable income*, including household primary income plus various benefits received from the government (eq. 41).
- *Household disposable income*, defined as household income, net of a household specific income tax, derived from the HES data set and the assumed lump-sum payment to the government (eq. 42).
- *Household spending income*, referring to the part of household disposable income that is used for current consumption. The remainder is household saving (eq. 43).

Table 2 Core equations for the household module

7. Household income and expenditure (39-44)

(39) Primary income from source c for household h

$$Y_{(c,h)}^{hpri} = \begin{cases} P_{(c)}^{lab} X_{(c,h)}^{hfac} & (c \in OCC; h \in HALL) \\ P_{-i(c)}^{fac} X_{(c,h)}^{hfac} & (c = cap, lnd; h \in HALL) \\ P_{-cs}^{f2} ("hou") X_{(c,h)}^{hfac} & (c = oct; h \in HALL) \end{cases}$$

where $X_{(c,h)}^{hfac}$ is household h endowment of factor f , and $P_{-i(f)}^{fac}$ is the price index for factor f ,

$$P_{-i(f)}^{fac} = \frac{1}{\sum_i Q_{(f,i)}^{fac}} \cdot Q_{(f,i)}^{fac} P_{(f,i)}^{fac} \quad (f \in FAC)$$

(40) Primary income for household h

$$Y_{-c(h)}^{hpri} = Y_{(c,h)}^{hpri} \quad (h \in HALL)$$

(41) Taxable income for household h

$$Y_{(h)}^{htxb} = \frac{Y_{-c(h)}^{hpri}}{1 - b_{(h)}^{hou}} \quad (h \in HALL)$$

where $b_{(h)}^{hou}$ is the rate of benefits on household h taxable income.

(42) Disposable income for household h

$$Y_{(h)}^{hdis} = Y_{(h)}^{htxb} (1 + b_{(h)}^{hou} - t_{(h)}^{hou}) \quad (h \in HALL)$$

where $t_{(h)}^{hou}$ is the rate of a tax on household h taxable income.

(43) Spending income for household h

$$Y_{(h)}^{hspd} = Y_{(h)}^{hdis} (1 - s_{(h)}^{hou}) \quad (h \in HALL)$$

where $s_{(h)}^{hou}$ is household h saving rate.

(44) Household h demand for composite good c

$$Q_{(c,h)}^{hou} = f(Y_{(h)}^{hspd}, P_{-s(c,"hou")}^{f2}) \quad (c \in COM; h \in HALL)$$

8. Linking household endowments with factor supplies (45-48)

(45) Linking national capital supply to household endowments of capital

$$X_{cap} = \sum_h X_{("cap",h)}^{hfac}$$

(46) Linking occupational labour supply to household endowments of labour

$$X_{(o)}^{lab} = \sum_h X_{(o,h)}^{hfac} \quad (o \in OCC)$$

(47) Leontief demand of industry i for household endowments of land

$$X_{(i)}^{lnd} = Leontief \left(\sum_h X_{("lnd",h)}^{hfac} \right) \quad (i \in IND)$$

(48) Leontief supply of OCT by household h

$$X_{("oct",h)}^{hfac} = Leontief \left(Q_{(i)}^{oct} \right) \quad (h \in HALL)$$

On the expenditure side, it is straightforward to incorporate multiple households in the original CGE framework. The households' demands for composite goods can be defined as the same as that of the representative household in the original model, that is, assuming the

same functional form as that in eq. 44.²⁴ Although facing the same set of prices for goods, each household will respond differently to a policy change, due to their unique pattern of consumption. Similarly, although facing the same returns to factors, each household responds differently, according to their different endowments as described by their sources of factor incomes.

Household endowments of factors can be aggregated and linked to the variables of factor supplies in the original model (eqs. 45-48). Since both the factor supplies in the original model and the household factor endowments in the household module are assumed constant in the initial closure, adding these linking equations does not change the original model closure.²⁵

Switching to the integrated model

The new variables and equations are introduced as an independent module to define the behaviours of individual households from the HES database when facing goods and factor price changes. Once the household module is completed, the demands of all households for composite goods can be aggregated and linked to the variables for the aggregate household in the main CGE model to determine the optimal combination of the demands for domestically produced goods and imports.

Similarly, individual household incomes, once calculated in the household module, can be aggregated and linked to the relevant variables in the main model. In particular,

- Income tax revenue and benefit payments are aggregated to be linked to the government account to determine the lump-sum payment rate and the government expenditure.
- Household savings are aggregated to be linked to the investment account to determine total capital formation.²⁶

To make the modification transparent and simple, we now introduce some equation swaps to integrate the household module with the original model. In fact, the integration requires only replacing three variables in the core system: the representative household's demand for composite goods, $Q_{-s(c, "hou")}^2$, the government's disposable income $Y_{("gov")}^{dis}$ and investment incomes $Y_{("inv")}^{spd}$. The original equations defining the three variables, together with the new equations for replacement, are listed in Table 3.

²⁴ Alternative behavioural assumptions can also be introduced for different households. See below for more detailed discussion.

²⁵ A labour supply module can be added to make it endogenous. In this paper, however, we set this as exogenous to keep the illustration of data reconciliation simple.

²⁶ Remember that in this model, all capital is assumed to be owned by households. Again, while this is consistent with the input-output table and assumed for simplicity, foreign capital ownership can be introduced.

Table 3 Equation swaps to implement the integrated model

9. Switching to the integrated model by replacing three variables in equations 19, 27 and 28

(19) Household demand for composite good c	$Q_{-s(c,"hou")}^2 = f(Y_{("hou")}^{spd}, P_{-s(c,"hou")}^{f2})$	$(c \in \text{COM})$
<i>Switch to the integrated model</i>		
	$Q_{-s(c,"hou")}^2 = \sum_h Q_{(c,h)}^{hou}$	$(c \in \text{COM})$
(27) Government disposable income	$Y_{("gov")}^{dis} = Y_{("gov")}^{pri} + Y_{("hou")}^{pri} \text{ tinc}$	
<i>Switch to the integrated model</i>		
	$Y_{("gov")}^{dis} = Y_{("gov")}^{pri} + \sum_h Y_{(h)}^{htxb} (t_{(h)}^{hou} - b_{(h)}^{hou})$	
(28) Investment income	$Y_{("inv")}^{spd} = Y_{("hou")}^{dis} S_{("hou")} + Y_{("gov")}^{dis} S_{("gov")} + P_{-cis}^{f2} Q_{nfi}$	
<i>Switch to the integrated model</i>		
	$Y_{("inv")}^{spd} = \sum_h Y_{(h)}^{hdis} s_{(h)}^{hou} + Y_{("gov")}^{dis} S_{("gov")} + P_{-cis}^{f2} Q_{nfi}$	
where $s_{(h)}^{hdi}$ is household h saving rate, defined as,		
	$s_{(h)}^{hdi} =$	
in which $S_{("hou")}$ is an average saving rate for all households, an <i>undefined</i> variable.		

- Equation 19 in the original model (Appendix table A2) defines the demand of the representative household for composite goods, $Q_{-s(c,"hou")}^2$. To switch to the integrated model, this behavioural equation is replaced by an aggregation of individual household demands, which is defined in eq. 44 in the household module.
- Equation 27 in the original model (Appendix table A2) defines the government's disposable income as the sum of goods tax revenue and a lump-sum payment from the representative household. In the integrated model, income tax revenue is replaced by a bottom-up aggregation of the income taxes and the lump-sum payments paid by individual households, net of the benefits received.
- Equation 28 in the original model (Appendix table A2) defines investment income as the sum of household and government savings plus net foreign investment (NFI) inflow. In the integrated model, the representative household's saving is replaced by an aggregation of individual households' savings, calculated using variables from the household module in table 2.

To close the model, it is also assumed that households' saving rates change proportionally with a national average saving rate, $s_{("hou")}$, which is an *undefined* endogenous variable,²⁷ required to balance the investment account for the country as a whole.

In the integrated model, the original equations defining the incomes for the representative household (eqs. 26.1, 27.1 and 28.1 in Appendix table A2) become redundant: they are superseded by the multiple households' behavioural equations defined in the household module in table 2.

The way in which equations can be swapped easily to produce either type of model allows us to conduct test simulations to compare the results from the two specifications. An obvious question is to what extent the results from the multi-household model differ from those of the single household model.²⁸

Test simulations: what difference does it make?

The integrated model produces true general equilibrium results for each individual household; it is expected to eliminate three types of errors associated with the single-household model and the two-model approach:

- Aggregation errors, due to the different ways in which households are aggregated in the two models.
- Partial equilibrium errors, due to the lack of feedback from household responses in the partial equilibrium MS model to the CGE model.
- Inconsistency errors, caused by the different sizes and compositions of the household data used by the two models.

In this section, an increase of one per cent in labour productivity is imposed on the electricity industry to test the responses of individual households in the integrated and the two models.

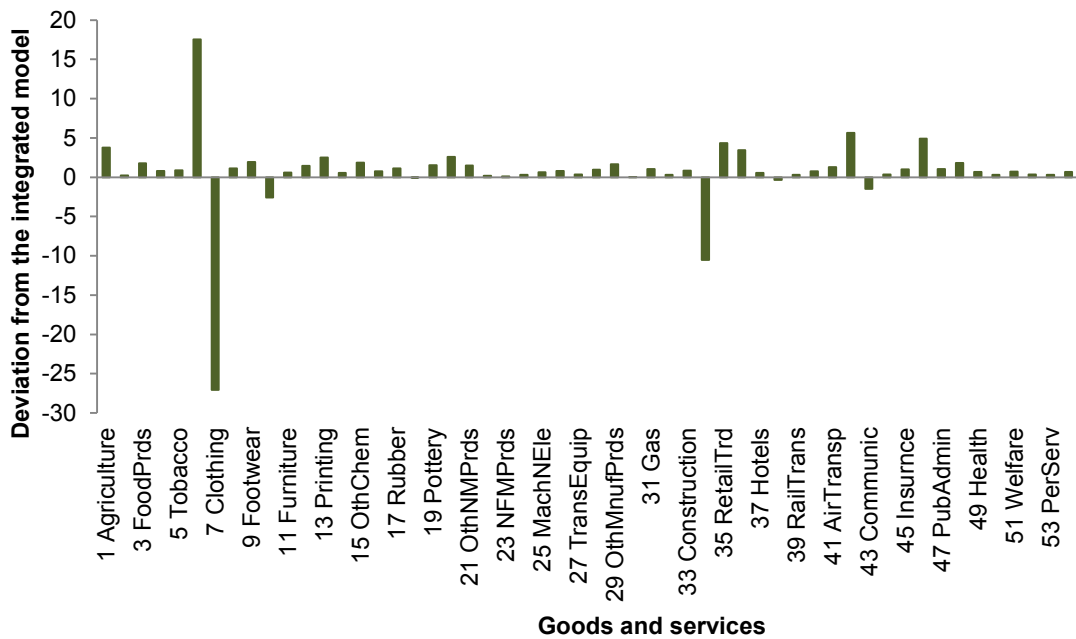
Figure 6 shows the aggregation errors by comparing the results from the integrated model and a single-household model. It shows the differences in household demands for composite goods. The results from the integrated model aggregate the demands of individual households during the simulation, while the single-household model uses a representative household, aggregated in the database prior to the simulation. As shown in

²⁷ An *undefined* variable is one that is not defined by any equation in the core system. Its value is determined by an associated *market clearing condition*. See appendix table A2 and Zhang 2013.

²⁸ Incorporating such a large number of households only doubles the computing time. Using GEMPACK software on a personal computer with an Intel Core i5 CPU and 4 GB RAM, an Euler one-step solution takes 55 seconds for the original single-household model, with no condensation, and 1 minute 40 seconds for the integrated model.

the figure, the demands for composite goods in the single-household model are generally larger than those from the integrated model. The differences are largely due to the lack of interactions between individual households in the single-household model: for example, some households receive benefits and others pay taxes; this does not happen in the single-household model.

Figure 6 **Aggregation errors: household demands for goods and services^a**



^a See table A4 for a complete list of commodities.

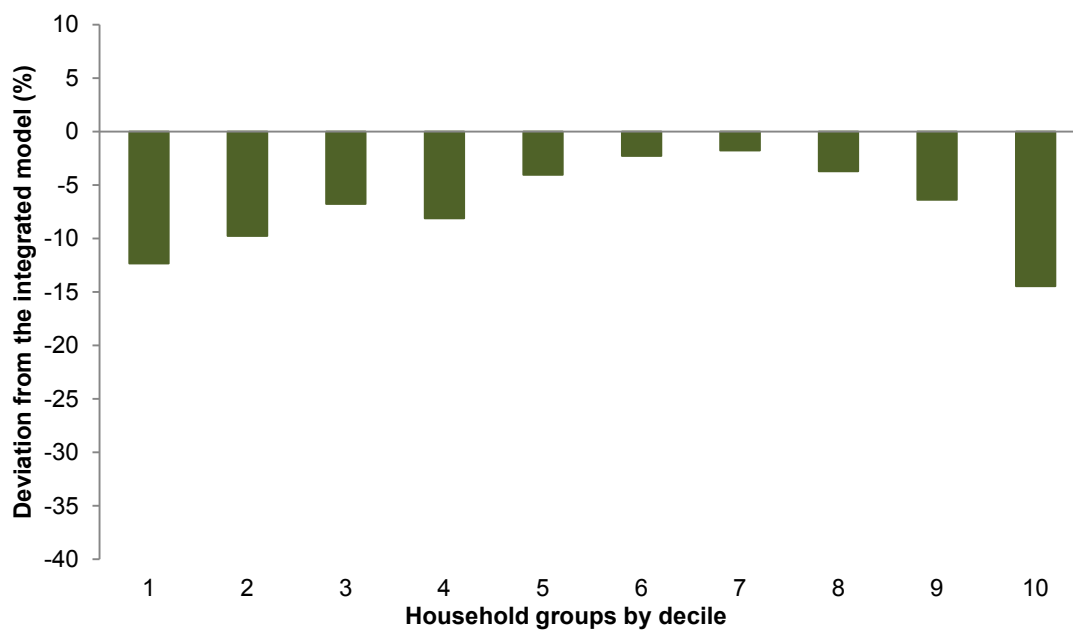
Source: Author simulations.

To show the partial equilibrium errors, the results of the integrated model are compared with that from a two-model approach, in which, the general equilibrium changes in goods and factor prices from a single-household CGE model are imposed onto a MS model to calculate the changes in household real income without any feedback.²⁹ To minimise the possible errors due to data inconsistency, the household data in the two models are scaled to be consistent with each other.

Figure 7 shows results for the real incomes of households by decile group. The differences between the two sets of results can be attributed to the lack of feedback from the household module to the CGE model in the two-model approach. As shown in the figure, the two-model approach underestimates real income changes for households. The differences are larger in lower and higher income decile groups and relatively small in middle income decile groups, which are closer to the single representative household that is used in the two-model approach.

²⁹ This setting is similar to the two-model approach used in Verikios and Zhang (2008).

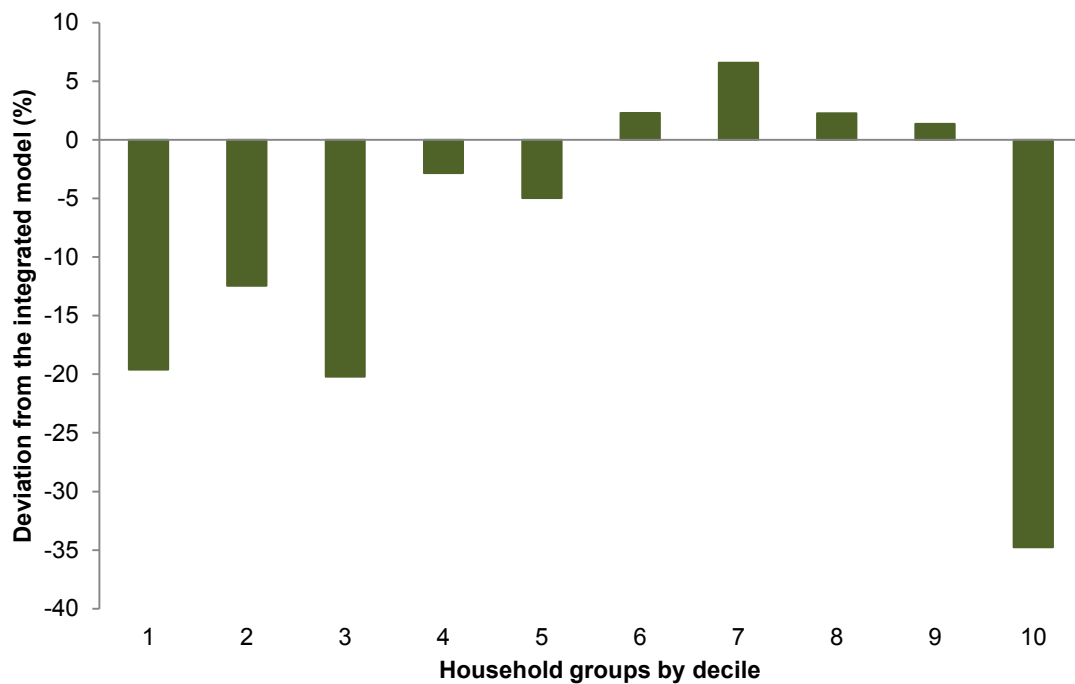
Figure 7 **Partial equilibrium errors: real household income by decile**



Source: Author simulations.

To test for the effect of data inconsistency, we use the original household data in the MS model. Figure 8 compares the results on household real income from the integrated model and from two-model approach. It can be seen in the figure that the two-model approach with unscaled household data shows a similar pattern of errors as that in figure 7, but the error is much larger for every household group. These differences can be attributed to the inconsistencies between the aggregate incomes and expenditures in the two databases.

Figure 8 **Data inconsistency errors: real household income by decile**



Source: Author simulations.

The above comparisons indicate that the use of an inconsistent household survey data set can significantly distort general equilibrium effects. Moreover, a single aggregated household or a limited number of aggregated household groups in a CGE model do not accurately capture the responses of individual households. To introduce general equilibrium effects into household analysis, there seems no substitute to a fully integrated model.

Concluding remarks

The integrated model developed in this paper can analyse the general equilibrium effects of policy changes at the individual household level. This cannot be accomplished with a conventional CGE model with one or few aggregated households. In the literature, integrated models have almost exclusively been used for income distribution issues. In fact, an integrated model can be developed into different versions for a much wider range of applications wherever individual household behaviours are diverse.

For instance, the household behaviour as factor suppliers can be specified to allow for factor supplies to respond to policy changes. As different households are endowed with different factors or receive different types of benefits from the government, they are expected to respond differently to a policy change. The effects of these diverse responses on the economy as a whole are captured in an integrated model.

In practice, to accurately capture the responses of individual households to policy changes may require some innovation in model development. For example, a household member might respond to a policy by increasing his labour supply. The consequent increase in wage income may push income past the threshold for the household to receive a government benefit and, therefore, reduce the total income received by the household. In response to such a possibility, the household member may reduce his labour supply. To model such a system, a level-switching mechanism needs to be introduced.³⁰ As CGE models capture only changes in relative prices, introducing level-switching mechanisms into a CGE framework is a challenge.³¹ However, the solution is only a technical one and, therefore, not unresolvable and the reward, could be enormous. This is because only integrated models can accurately capture the general equilibrium effects on individual households without suffering from the errors and distortions associated with the two-model approach.

With individual household integrated in the model system, a wide range of applications become possible. Income distribution analysis of policies outside of the tax and transfer system is one, which can be carried out using the model structure presented here.

More sophisticated applications can be considered. For example, household survey data contain very detailed information on various government benefits and their distribution. The integrated model makes it possible to single out any benefit and trace precisely the impact that changing such a benefit could have on all households (those that receive it and those that don't), on government's budget (if the changes need to be financed), and the economy as a whole.

In this paper, the household behavioural functions in the integrated model and the original model are assumed to be the same. This was done to make the illustration simple. In fact, with a rich household database, it becomes possible to redefine the household behaviour at more disaggregated levels. For example, different functional forms could be used for different households.³²

Moreover, the same household data that was incorporated in the CGE model, can be used to build a separate behavioural MS model with very detailed household behaviour or a very detailed description of the tax and transfer system. The MS model can then be linked iteratively with the CGE model. The settings in table 3 for swapping from the single household model to the integrated model can be used as the interface between the CGE

³⁰ These mechanisms have been implemented in MS models, because these partial equilibrium models have less constraints and are based on more flexible functional forms. This is part of the reason why some researchers abandon the integrated approach in favour of a two-model approach with iterative links (see, for example, Savard, 2003; Bourguignon and Savard, 2008).

³¹ Alternative solution methods, such as mixed complementarity programming, may prove useful. However, some of these methods could significantly increase the computing power requirements, particularly for models integrated with very large samples of households.

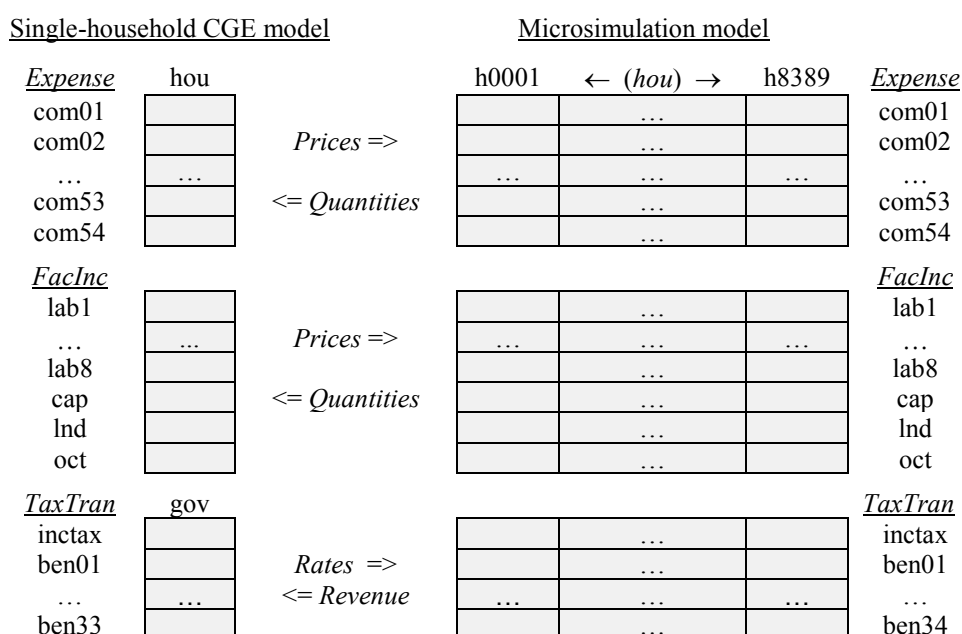
³² Or, with the current functional form, parameters could be re-estimated using household survey data, so that modelled household behaviours follow more closely what is observed in the survey data. That said, choosing a large number of parameters is an onerous task.

model and a separate MS model with individual household behaviours specified on both the goods demand and factor supply sides. In the CGE model, the variable for the demands of the single household for composite goods can be set as exogenous by deleting equation 19, so that they can be determined by the individual households in the MS model. Similarly, the exogenous supplies of factors in the CGE model can also be determined by the household supplies in the MS model.

All prices of goods and factors are determined by the CGE model and supplied to the behavioural MS model, which considers them as exogenous variables. All the quantity variables that relate to the household module, that is, the demands for goods and factor supplies, are determined by the MS, for given prices, and are used to shock the CGE model.

Figure 9 illustrates the links between the CGE model and the MS model. The CGE model generates price changes for the households in the MS model to determine the quantities of their demands for goods and supplies of factors. These quantity changes, in turn, are fed back to the CGE model and determine aggregate household demand for each commodity. These interactions are exactly the same as would be observed in a fully integrated model. The only difference is that the household module is specified independently as a behavioural MS model and is outside the CGE model. Since the databases used by the two models are identical, the iterations between the two models are likely to converge quickly.

Figure 9 Linking the CGE model with a MS model



The integrated model is a very powerful tool for policy analysis, adding a distributional dimension to results, but also, potentially adding to an array of experiments that can be conducted, including differential policy changes across households. This paper provides a CGE model structure, easy to understand and modify, and a simple procedure to reconcile

the household survey and CGE input-output data sets. More work is required to better capture the behaviours of individual households and their relations with government and the rest of the economy. The method proposed in this paper can be seen as a useful practical step in that direction.

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Appendix

This Appendix contains the core equation system for the CGE model (table A2). The core equations are the essential equations required for solving the model's general equilibrium solution. The variables that are used solely for reporting results are not included. It also contains an alternative specification for exports in table A3.

The sets used in the equation system are listed in table A1.

Table A1 Sets used in the model and database

Sets	Definitions
COM(1,...,m)	Commodities (indexed by c)
IND(1,...,m)	Industries (indexed by i)
SRC(dom, imp)	Domestic/import sources of commodities (indexed by s)
FUSR(gov, hou, inv)	Aggregate final users of commodities (indexed by u)
USR1(=IND, gov, hou)	Final users of commodities (indexed by u)
FAC(lab, cap, lnd)	Primary factors of production (indexed by f)
INC(OCC, cap, lab, lnd, oct)	Household primary income sources (indexed by c)

Table A2 The core equations of a national CGE model

1. Industry demand for inputs (1-5)

(1) CES demand of industry i for good c from source s	$Q_{(c,i,s)}^1 = CES(Q_{-s(c,i)}^1, P_{(c,i,s)}^{f1}, P_{-s(c,i)}^{f1})$	$(c \in \text{COM}; i \in \text{IND}; s \in \text{SRC})$
(2) CES price for composite good c for industry i	$P_{-s(c,i)}^{f1} = CES(P_{(c,i,"dom")}^{f1}, P_{(c,i,"imp")}^{f1})$	$(c \in \text{COM}; i \in \text{IND})$
(3) Leontief demand of industry i for composite good c	$Q_{-s(c,i)}^1 = Leontief(Q_{(i)})$	$(c \in \text{COM}; i \in \text{IND})$

(4) Producers price of good c from source s for industry i

$$P_{(c,i,s)}^{r1} = \frac{Q_{(c,i,s)}^1 P_{(c,s)} + \sum_m Q_{(c,i,s,m)}^{mg1} P_{(m,"dom")}}{Q_{(c,i,s)}^1 + \sum_m Q_{(c,i,s,m)}^{mg1}} \quad (c \in \text{COM}; i \in \text{IND}; s \in \text{SRC})$$

(5) Purchaser's price of good c from source s for industry i

$$P_{(c,i,s)}^{f1} = P_{(c,i,s)}^{r1} (1 + \tau_{(c,i,s)}^{c1}) \quad (c \in \text{COM}; i \in \text{IND}; s \in \text{SRC})$$

where $\tau_{(c,i,s)}^{c1}$ is the *ad valorem* rate of tax on good c for industry i from source s .

2. Industry demand for factors (6-10)

(6) CES demand of industry i for labour occupation o	$Q_{(o,i)}^{lab} = CES(Q_{("lab",i)}^{fac}, P_{(o)}^{lab}, P_{("lab",i)}^{fac})$	$(o \in \text{OCC}; i \in \text{IND})$
(7) CES price index for composite factor in industry i	$P_{("lab",i)}^{fac} = CES(P_{(o)}^{lab}, \dots, P_{(o_s)}^{lab})$	$(i \in \text{IND})$

(To be continued)

Table A2 The core equations of a national CGE model (continued)

(8) CES demand of industry i for factor f	$Q_{(f,i)}^{fac} = CES(Q_{(i)}, P_{(f,i)}^{fac}, P_{f(i)}^{fac})$	$(f \in \text{FAC}; i \in \text{IND})$
(9) CES price index for a composite factor in industry i	$P_{f(i)}^{fac} = CES(P_{("cap",i)}^{fac}, P_{("lab",i)}^{fac}, P_{("lnd",i)}^{fac})$	$(i \in \text{IND})$
(10) Leontief demand of industry i for Other Cost Ticket (OCT)	$Q_{(i)}^{oct} = Leontief(Q_{(i)})$	$(i \in \text{IND})$

3. Total demand for and basic price of goods (11-16)

(11) Total demand for good c

$$Q_{(c)} = \begin{cases} \sum_i Q_{(c,i,"dom")}^1 + \sum_u Q_{(c,u,"dom")}^2 + Q_{(c)}^3 & (c \in \text{NCOM}) \\ \sum_i Q_{(c,i,"dom")}^1 + \sum_u Q_{(c,u,"dom")}^2 + Q_{(c)}^3 + Q_{-cis}^{mgn}(c) & (c \in \text{MCOM}) \end{cases}$$

(12) Total demand for margin good m

$$Q_{-cis}^{mgn}(m) = \sum_{c,i,s} Q_{(c,i,s,m)}^{mg1} + \sum_{c,u,s} Q_{(c,u,s,m)}^{mg2} + \sum_c Q_{(c,m)}^{mg3} \quad (m \in \text{MCOM})$$

(13) Leontief demand of industry i for margin good m

$$Q_{(c,i,s,m)}^{mg1} = Leontief(Q_{(c,i,s)}^1) \quad (c \in \text{COM}; i \in \text{IND}; s \in \text{SRC}; m \in \text{MCOM})$$

(14) Leontief demand of user u for margin good m

$$Q_{(c,u,s,m)}^{mg2} = Leontief(Q_{(c,u,s)}^2) \quad (c \in \text{COM}; u \in \text{USR1}; s \in \text{SRC}; m \in \text{MCOM})$$

(15) Leontief demand of exports for margin good m

$$Q_{(c,m)}^{mg3} = Leontief(Q_{(c)}^3) \quad (c \in \text{COM}; m \in \text{MCOM})$$

(16) Basic price of good c from source s

$$P_{(c,s)} = \begin{cases} \frac{\sum_i Q_{-s(i,c)}^1 P_{-s(i,c)}^{t1} + Q_{f(c)}^{fac} P_{f(c)}^{fac} + Q_{(c)}^{oct} P_{(c)}^{oct}}{\sum_i Q_{-s(i,c)}^1 + Q_{f(c)}^{fac} + Q_{(c)}^{oct}} & (c \in \text{COM}; s = \text{dom}) \\ P_{(c)}^{cif} \text{ ex} (1 + t_{(c)}^{imp}) & (c \in \text{COM}; s = \text{imp}) \end{cases}$$

where ex is the nominal exchange rate (domestic currency value per unit of foreign currency), and $t_{(c)}^{imp}$ is the *ad valorem* rate of duty on imported good c .

4. Final user's and foreign demand for goods (17-24)

(17) CES demand of user u for good c from source s	$Q_{(c,u,s)}^2 = CES(Q_{-s(c,u)}^2, P_{(c,u,s)}^{t2}, P_{-s(c,u)}^{t2})$	$(c \in \text{COM}; u \in \text{USR1}; s \in \text{SRC})$
(18) CES price for composite good c for user u	$P_{-s(c,u)}^{t2} = CES(P_{(c,u,"dom")}^{t2}, P_{(c,u,"imp")}^{t2})$	$(c \in \text{COM}; u \in \text{USR1})$
(19) Demand of user u for composite good c	$Q_{-s(c,u)}^2 = \begin{cases} f(Y_{(u)}^{spd}, P_{-s(c,u)}^{t2}) & (c \in \text{COM}; u = \text{hou}, \text{gov}) \\ Leontief(Q_{(u)}^{inv}) & (c \in \text{COM}; u \in \text{IND}) \end{cases}$	

(To be continued)

Table A2 The core equations of a national CGE model (continued)

(20) Producer price of good c from source s for user u

$$P_{(c,u,s)}^{r2} = \frac{Q_{(c,u,s)}^2 P_{(c,s)} + \sum_m Q_{(c,u,s,m)}^{mg2} P_{(m,"dom")}}{Q_{(c,u,s)}^2 + \sum_m Q_{(c,u,s,m)}^{mg2}} \quad (c \in \text{COM}; u \in \text{USR1}; s \in \text{SRC})$$

(21) Purchaser's price of good c from source s for user u

$$P_{(c,u,s)}^{f2} = P_{(c,u,s)}^{r2} (1 + t_{(c,u,s)}^{c2}) \quad (c \in \text{COM}; u \in \text{USR1}; s \in \text{SRC})$$

where $t_{(c,u,s)}^{c2}$ is the *ad valorem* rate of tax on good c for user u from source s .

(22) Foreign demand for export c

$$Q_{(c)}^3 = f\left(\frac{P_{(c)}^{f3}}{ex}\right) \quad (c \in \text{COM})$$

(23) Producer price of export c

$$P_{(c)}^{r3} = \frac{Q_{(c)}^3 P_{(c,"dom")} + \sum_m Q_{(c,m)}^{mg3} P_{(m,"dom")}}{Q_{(c)}^3 + \sum_m Q_{(c,m)}^{mg3}} \quad (c \in \text{COM})$$

(24) Purchaser's price of export c

$$P_{(c)}^{f3} = P_{(c)}^{r3} (1 + t_{(c)}^{c3}) \quad (c \in \text{COM})$$

where $t_{(c)}^{c3}$ is the *ad valorem* rate of tax on export c .

5. Final user's income and expenditure (25-32)

(25) Total demand for imported good c

$$Q_{(c)}^{imp} = \sum_i Q_{(c,i,"imp")}^1 + \sum_u Q_{(c,u,"imp")}^2 \quad (c \in \text{COM})$$

(26) Primary incomes for household and government

$$Y_{(u)}^{pri} = \begin{cases} P^{cap} X_{cap} + \sum_i P_{(i)}^{lnd} X_{(i)}^{lnd} + \sum_o P_{(o)}^{lab} X_{(o)}^{lab} + P_{-cs}^{f2} \sum_i Q_{(i)}^{oct} & (u = \text{hou}) \\ \sum_c \left(\sum_s \left(\sum_i P_{(c,i,s)}^{r1} Q_{(c,i,s)}^1 t_{(c,i,s)}^{c1} + \sum_u P_{(c,u,s)}^{r2} Q_{(c,u,s)}^2 t_{(c,u,s)}^{c2} \right) + P_{(c)}^{r3} Q_{(c)}^3 t_{(c)}^{c3} \right) \\ + P_{(c)}^{cif} ex Q_{(c)}^{imp} t_{(c)}^{imp} & (u = \text{gov}) \end{cases}$$

(27) Disposable incomes for household and government

$$Y_{(u)}^{dis} = \begin{cases} Y_{(u)}^{pri} (1 - tinc) & (u = \text{hou}) \\ Y_{(u)}^{pri} + Y_{("hou")}^{pri} tinc & (u = \text{gov}) \end{cases}$$

where $tinc$ is household income tax rate.

(28) Spending incomes for household, government and investor

$$Y_{(u)}^{spd} = \begin{cases} Y_{(u)}^{dis} (1 - s_{(u)}) & (u = \text{hou}) \\ Y_{(u)}^{dis} (1 - s_{(u)}) & (u = \text{gov}) \\ Y_{("hou")}^{dis} s_{("hou")} + Y_{("gov")}^{dis} s_{("gov")} + P_{-cis}^{f2} Q^{nfi} & (u = \text{inv}) \end{cases}$$

where $s_{("hou")}$ and $s_{("gov")}$ are household and government saving rates ($s_{("hou")}$ is an *undefined* variable), and Q^{nfi} is real net foreign investment inflow.

(To be continued)

Table A2 The core equations of a national CGE model (continued)

(29) Purchaser price index for capital formation

$$P_{-cis}^{f2} = \frac{1}{\sum_{i \in \text{IND}} Q_{(i)}^{inv}} Q_{(i)}^{inv} P_{-cs(i)}^{f2}$$

(30) Purchaser price index for user u

$$P_{-cs(u)}^{f2} = \frac{1}{\sum_c Q_{-s(c,u)}^2} Q_{-s(c,u)}^2 P_{-s(c,u)}^{f2} \quad (u \in \text{USR1})$$

(31) Investment budget constraint (to determine $s_{("hou")}$)

$$Y_{("inv")}^{spd} = \sum_i Q_{(i)}^{inv} P_{-cs(i)}^{f2}$$

(32) Leontief demand of industry i for capital formation

$$Q_{(i)}^{inv} = \text{Leontief} (Q_{("cap",i)}^{fac}) \quad (i \in \text{IND})$$

6. Factor market equilibrium and factor price determination (33-38)

(33) Basic price of capital

$$P_{("cap",i)}^{fac} = \frac{d(P_{("cap",i)}^{fac})}{d(P^{cap})} P^{cap} \quad (i \in \text{IND})$$

where P^{cap} is the general equilibrium rental price of capital used, an *undefined* variable.

(34) Basic price of land

$$P_{("lnd",i)}^{fac} = \frac{d(P_{("lnd",i)}^{fac})}{d(P^{lnd}_{(i)})} P^{lnd}_{(i)} \quad (i \in \text{IND})$$

where $P^{lnd}_{(i)}$ is the general equilibrium rental price of land used in industry i , an *undefined* variable.

(35) Basic price of labour occupation o

$$P_{(o)}^{lab} = \frac{d(P_{(o)}^{lab})}{d(P^{lab}_{(o)})} P^{lab}_{(o)} \quad (o \in \text{OCC})$$

where $P^{lab}_{(o)}$ is the general equilibrium wage rate for occupation o , an *undefined* variable.

(36) National capital market equilibrium (to determine P^{cap})

$$X^{cap} = \sum_i Q_{("cap",i)}^{fac}$$

(37) Industry-specific land market equilibrium (to determine $P^{lnd}_{(i)}$)

$$X^{lnd}_{(i)} = Q_{("lnd",i)}^{fac} \quad (i \in \text{IND})$$

(38) Occupational labour market equilibrium (to determine $P^{lab}_{(o)}$)

$$X^{lab}_{(o)} = \sum_i Q_{(o,i)}^{lab} \quad (o \in \text{OCC})$$

Notes:

1. Variables in red colour are set as exogenous in the basic closure.
2. Variables in blue colour are *undefined*.
3. Equations shaded in yellow are the *market clearing conditions* required to determine the *undefined* variables.

Table A3 Alternative specification: CET supplies of domestic and exported goods

<p>(1) CET supply of good c for domestic market</p> $Q_{(c)}^{dom} = CET(Q_{(c)}, P_{(c)}^{dom}, P_{(c, "dom")}) \quad (c \in \text{COM})$ <p>where $P_{(c)}^{dom}$ is the basic price of domestic good c, an <i>undefined</i> variable.</p>
<p>(2) CET supply of good c for export market (to determine $P_{(c)}^{exp}$)</p> $Q_{(c)}^3 = CET(Q_{(c)}, P_{(c)}^{f3}, P_{(c, "dom")}) \quad (c \in \text{COM})$ <p>(Note: as $Q_{(c)}$ is redefined as a CET composite good, an <i>undefined</i> variable, equation 11 needs to be deleted.)</p>
<p>(3) CET price index for composite good c (to determine $Q_{(c)}$)</p> $P_{(c, "dom")} = CES(P_{(c)}^{dom}, P_{(c)}^{f3}) \quad (c \in \text{COM})$
<p>(4) Total demand for good c for domestic market (to determine $P_{(c)}^{dom}$)</p> $Q_{(c)}^{dom} = \begin{cases} \sum_i Q_{(c,i, "dom")}^1 + \sum_u Q_{(c,u, "dom")}^2 & (c \in \text{NCOM}) \\ \sum_i Q_{(c,i, "dom")}^1 + \sum_u Q_{(c,u, "dom")}^2 + Q_{-cis}^{mgn}(c) & (c \in \text{MCOM}) \end{cases}$
<p>(5) Producer price of export c</p> $P_{(c)}^{r3} = \frac{Q_{(c)}^3 P_{(c)}^{exp} + \sum_m Q_{(c,m)}^{mg3} P_{(m, "dom")}}{Q_{(c)}^3 + \sum_m Q_{(c,m)}^{mg3}} \quad (c \in \text{COM})$ <p>where $P_{(c)}^{exp}$ is the basic price of export c, an <i>undefined</i> variable.</p> <p>(Note: this equation is used to replace equation 23 to redefine $P_{(c)}^{r3}$.)</p>

Notes:

1. Variables in red colour are set as exogenous in the basic closure.
2. Variables in blue colour are *undefined*.
3. Equations shaded in yellow are the *market clearing conditions* or *price equalisation conditions* required to determine the *undefined* variables.

Table A4 The goods and services in the CGE model

1. Agriculture	28. Scientific equipment
2. Mining	29. Other manufactured products
3. Food products	30. Electricity
4. Beverages	31. Gas
5. Tobacco	32. Water
6. Textiles	33. Construction
7. Clothing	34. Wholesale trade
8. Leather	35. Retail trade
9. Footwear	36. Repairs
10. Wood products	37. Hotels
11. Furniture	38. Road transport
12. Paper products	39. Rail transport
13. Printing	40. Water transport
14. Industrial chemicals	41. Air transport
15. Other chemicals	42. Services to transport
16. Petrol	43. Communication
17. Rubber	44. Finance
18. Plastic	45. Insurance
19. Pottery	46. Dwellings
20. Glass	47. Public administration
21. Other non-metallic products	48. Defence
22. Iron and steel	49. Health
23. Non-ferrous metal products	50. Education
24. Metal products	51. Welfare
25. Non-electrical machinery	52. Entertainment
26. Electrical machinery	53. Personal services
27. Transport equipment	54. Other
