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Developing a Partial Equilibrium Model of an Urban Water System

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Preface

Urban water and its management have been the subject of much public debate. The timing and choice of investments to augment water supply, different approaches to water pricing, and the tools of demand management have all been the subject of discussion. Outlined in this paper is a model that can be used to quantify the costs and benefits of policy options to improve outcomes in urban water systems. An earlier version of the paper was presented at the Australian Conference of Economists on 30 September 2009, and was awarded the prize for best contributed paper.

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The authors would like to acknowledge the industry participants who were present at the Commission's workshop in 2009, and provided valuable feedback and suggestions for the modelling. The authors would also like to acknowledge Julia Thomson for her involvement in the early stages of the project.

The views in this paper remain those of the authors and do not necessarily reflect the views of the Productivity Commission, or of the external organisations and people who provided assistance.

OVERVIEW

Key points

- A partial equilibrium model of an urban water system is employed to investigate capacity augmentation decisions, pricing policies and the use of water restrictions in the urban water sector.
- The modelling is based on the solution to a constrained optimisation problem, with the objective to maximise community welfare in the urban water market. The model allows for intertemporal representation of demand and supply; variation in annual inflows to dams; various supply options; and scope to apply policy constraints.
- The model abstracts from the transaction costs of different policies, institutional settings and incentives. Such considerations could in practice have a significant bearing on outcomes and optimal policies.
- To illustrate its use, the model is applied to a hypothetical city, which synthesizes features of Australian capital cities. The results therefore are illustrative only, and cannot be used as a template for assessing actual investment and policies.
  - Several possible new supply sources are considered: desalination; groundwater aquifers; household tanks; new dams; and rural–urban trade.
- The model reinforces the importance of rainfall variability and of making investment decisions regarding new supply sources based on *expected* returns to investment.
  - Actual payoffs to investment depend on future inflows to dams, as prices respond to demand, supply and storages. If future rainfall is plentiful (scarce), returns to investment are likely to be low (high).
  - Guaranteed investment returns lead to inefficient investment and consumption.
  - The amount of water drawn from new investments should be flexible and respond to rainfall patterns (via their impact on water prices).
- Pricing based on the relative scarcity of water was the optimised 'base case' against which a range of illustrative policy applications were evaluated.
  - Constraining prices (including through long-run marginal cost pricing) was found in the model to impose costs on the community. Constrained prices are also likely to require restrictions to ration water during times of scarcity because prices are not able to perform a 'rationing' function.
  - The modelling shows large economic costs from imposing water restrictions, which prevent uses of water that consumers would have been willing to pay for. These costs rise as demand becomes less responsive to price or if inflows to dams become lower in the future.
  - A key feature of scarcity-based pricing is the variability in the price of water over time, depending on rainfall. On average, however, prices are lower under scarcity-based pricing than under the other policy options modelled.
  - Model results also indicate potentially high costs from ruling out access to particular sources of water (for example, relatively low-cost rural–urban trade using pipelines), or from pursuing supply options that are not least cost.
- Potential further work using this modelling framework could include its application to specific urban settings.

Overview

Shortages of water have been commonplace in Australian cities during recent years, as an extended period of low rainfall has reduced inflows to dams. As a result, many jurisdictions have imposed restrictions. These shortages have also triggered debate about appropriate pricing and investment. Studies by governments, academics, industry and environmental groups have suggested that there are potential welfare gains from reforms to water pricing and supply procurement. However, limitations of the existing models available to evaluate urban water policy has hampered quantification of its associated costs and benefits.

In this paper a partial equilibrium model is developed specifically to investigate urban water policy issues. The use of a partial equilibrium model limits the quantification of the impacts to the urban water system, with no ability to include impacts on other sectors of the economy. For example, changes in household spending on water might affect how much income households have left to spend on other goods and services, which could impact on the sectors that supply those goods and services. However, since urban water is a small proportion of household budgets, feedback effects on demand and supply are likely to be minimal. Moreover, adopting a partial approach allows more detail and realistic modelling, for example the inclusion of multiple policy options, binary 'yes/no' investment decisions and stochastic variability of inflows to dams, which are difficult to incorporate into general equilibrium models.

The modelling in this paper is based on the solution to a constrained optimisation problem. Annual inflows to dams vary from year to year and cannot be known with certainty in advance. The model seeks to maximise community welfare in the urban water market, calculated as the expected net present value of welfare from consuming water, less the cost of supplying it (over a simulation period of several years). This is achieved subject to constraints on demand, supply and policy actions, and according to demand and supply decisions that respond to pricing signals. Demand functions for commercial, indoor and outdoor household uses describe the welfare to consumers from water consumption.

As always, the model developed in this study is a simplified or stylised representation of the real world. For example, the model abstracts from the transaction costs of different policies, institutional settings and incentives. Such considerations could have a significant bearing on optimal policies in practice. It nevertheless provides a useful tool for investigation of investment decisions and policy choices.

In the model, scarcity-based pricing is the optimised 'base case' against which other policy options are compared. Scarcity-based pricing is nothing more than economically efficient pricing that maximises consumer plus producer surplus, a microeconomic concept widely understood and applied to evaluate policies. Scarcity-based pricing means that prices paid by consumers to suppliers are able to respond over time to variations in rainfall and storage in dams. When dam levels are low, water is more expensive, reflecting its scarcity. Conversely, when dams are full and additional water would cause them to overflow, water is cheaper. The model allows prices to adjust to equate demand and supply in each year, and across years. The choice of scarcity pricing as the base case is natural in the sense that, in principle, it is the most efficient outcome, and is a useful reference point against which to assess policies that act as a constraint on efficiency and the maximisation of welfare.

Other illustrative policies modelled include regulated pricing based on long-run marginal cost or cost recovery, water use restrictions, and bans on or mandates for particular forms of supply. These policies are modelled by imposing constraints on the model that simulate the operation of pricing and other policies considered by policymakers. By necessity, the modelling is a simplification of actual policies and their implementation. An attempt has been made to minimise the distortions imposed by the policy constraints, in order to obtain a 'lower bound' estimate of the cost of each policy relative to a scarcity-based pricing framework.

The model includes several options for new sources of supply, allowing for investigation of the choice between different options. Additional water is potentially available from desalination, new dams, aquifers, rural–urban trade using pipelines, and household tanks. Each augmentation option has its own characteristics in terms of investment and operating costs, reliability of supply, time to build, and economic life. For example, tanks provide households with a small amount of additional, rainfall-dependant water, at a high construction cost (per unit of water delivered) but with low ongoing costs. Consumers can use water from tanks for outdoor uses, allowing them to compensate to some extent for water restrictions.

Some other supply options have not been modelled, simply because they would introduce significant complexities, not because they are not worth pursuing. For example, wastewater recycling was not incorporated in the model because community concerns about the quality of the water delivered is the main barrier to adoption of this technology. In addition, introducing this option into the model would lead to excessive data and computational difficulties. Similarly, other alternatives that separate water of a different quality — such as dual reticulation systems — were not included as options.

The model has been calibrated on and applied to a hypothetical urban system, representative of large capital cities in Australia. This provides insights generally to the policy issues, but also means the results cannot be used to make policy judgements for any specific city in Australia. Aggregate annual consumption is assumed to be just less than that in Sydney and Melbourne, and greater than that in South–East Queensland, Perth and Adelaide. Variability of inflows to dams has been calibrated to historical Australian data. The results should not be used to infer 'one size fits all' policy prescriptions, as appropriate planning and market outcomes will vary by city. For example, the optimal choice of investments to augment supply varies from city to city, reflecting the characteristics of each city's natural endowment of water resources.

Some illustrative results

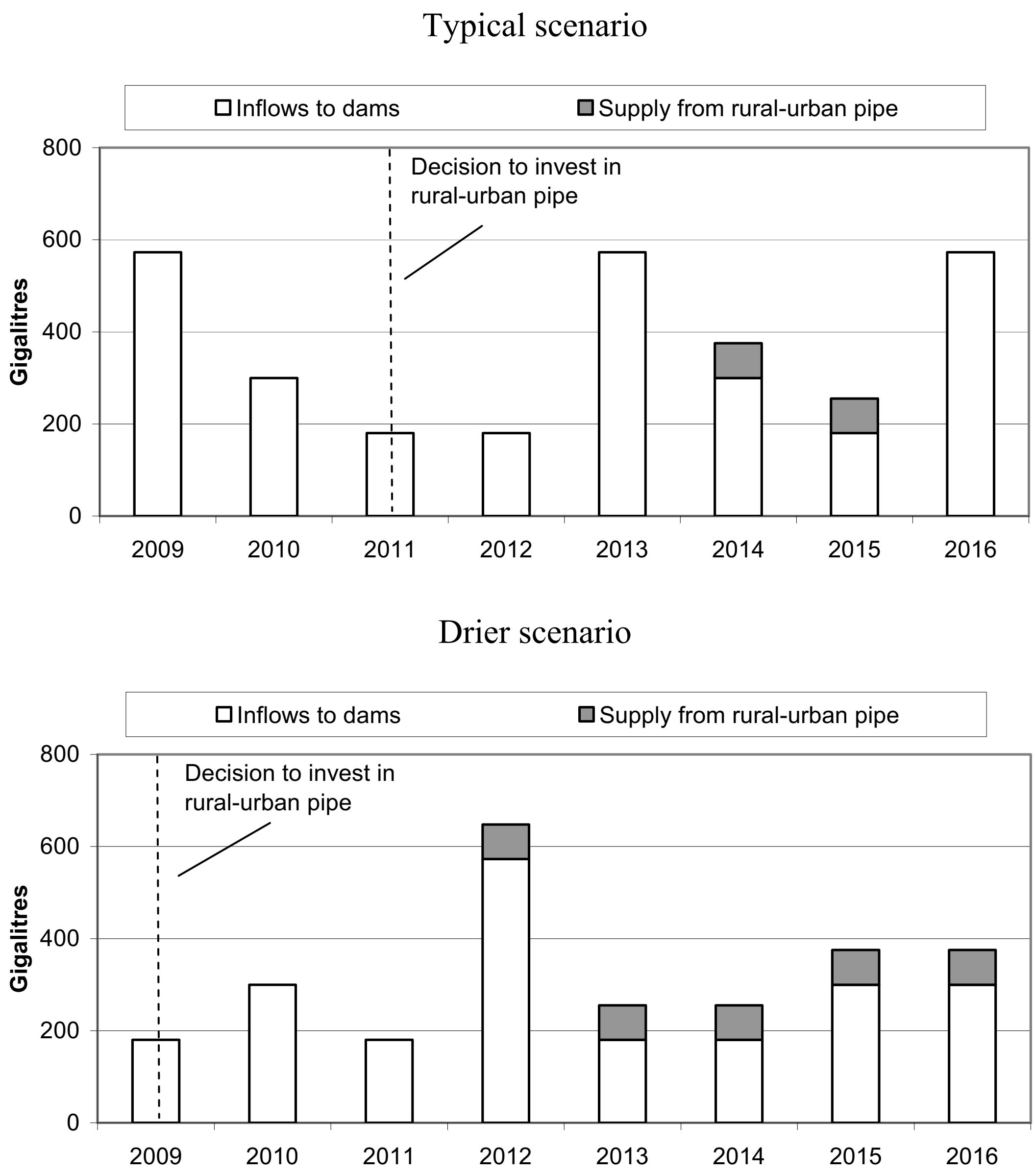
A key insight from this research is the importance of taking into account variability of inflows to dams in assessing supply options and evaluating policy. The extreme variability in streamflows (and thus inflows to dams) in Australia, combined with a reliance on water from dams, means that accounting for variability in project benefits and costs is important to evaluating policy issues in urban water systems. In the model, inflows to dams in each year are either low, medium or high. Results show that pricing (under a range of different pricing policies) and the timing of investment should respond to realised inflows in order to maximise community welfare. For example, examination of the modelling results over a period of years indicates that investing in a pipeline to facilitate rural–urban trade is more likely to be a good decision after a series of dry years, when dam storages are low, than when water is plentiful and dams are full.

Investment in new sources of supply

Investment decisions are made based on expected returns and costs under all possible scenarios for future inflows to dams: investment occurs when the expected benefits from additional supply outweigh the costs. Investment at a point in time also depends on historical inflows to dams and past investment decisions, to the extent that these affect current storages and the future capacity to supply water. As such, investment is likely to be brought forward in dry scenarios (figure 1). In an example 'typical scenario' for rainfall, it is optimal within the model to make a decision to build a pipeline in 2011 to access water through rural–urban trade. After three years for planning and construction, water is available from the pipeline in 2014. In a drier scenario the investment decision is moved forward to 2009. (The model solves for 5000 scenarios simultaneously.)

Figure 1 Timing of investment: the impact of inflows to dams

Water supplied from rural–urban trade under two different inflow scenarios



Data source: Modelling results.

Following investment in capacity to supply water, the decision to use the facility to supply water is made annually. An upfront decision to use the full capacity of the new facility to supply water would reduce operational flexibility and incur economic costs. For example, a wet year in 2016 under the 'typical scenario' illustrated in figure 1 means that it is preferable not to use the rural–urban pipeline, because the price of water is less than the short-run unit cost of water supply from this source.

Whereas investment decisions are based on future expectations, the realised payoffs to investment depend on actual inflows and resultant future revenues and costs after the facility has been commissioned. In the example cited above, a wet year in 2016 reduces investment returns to the rural–urban pipeline. Similarly, building bores to access water from aquifers is likely to offer larger payoffs following investment if it turns out to be dry, compared to another scenario where high rainfall occurs.

Any guarantee of ex post investment returns will carry costs to community welfare. Investment in new sources of water is inherently risky — like investments in many other markets — and returns should ideally reflect this risk. Regulation that fixes ex post returns to investment will distort investment decisions and end-user consumption. For example, where guaranteed investment returns are built into water prices, water might still be supplied from a facility even though the value of water to society is less than the short-run marginal cost of supply.

Modelling results confirm the importance of first accessing water from relatively cheap sources of supply. In the hypothetical model of this study, aquifers and rural–urban trade are the preferred sources of additional water. Rural–urban trade occurs when it is mutually advantageous for buyers and sellers, with water sold at the prevailing price for annual water allocations in rural areas. Given the small size of urban markets relative to rural markets, the price of water in irrigation markets is assumed to be unaffected by the quantity purchased for urban use. However, the price paid for allocations does vary depending on climatic conditions.

Within the model, high costs arise from banning access to particular sources of water. A policy ban on rural–urban trade carries (expected) net present value costs of about $70 million for the hypothetical city modelled. Similarly, there are high costs from choosing inappropriate supply options. A 'one size fits all' approach to new water supply sources could lead to support for inappropriate supply options. For example, there are shown to be significant costs from pursuing desalination where this is not the least cost source of supply available.

Sources of water that are generally more expensive, such as desalination, might be justified where cheaper options are not available, or to avoid large costs from running out of water during extreme dry spells. However, if the reason to construct a desalination plant is to avoid the possibility of ever running out of water, the modelling indicates there might be net community benefits in not running the plant at full capacity all the time. For example, when construction of a desalination plant was imposed in the model, optimal average capacity utilisation over all possible outcomes for dam inflows was just under 75 per cent. This is because when dams are full or near full, it is better not to supply water from a desalination plant than to incur significant operating costs.

Construction of household tanks was found to be worthwhile under some policies, in particular with long-run marginal cost pricing. There are two reasons for this. First, the planning and construction time for tanks is shorter than for larger investments such as desalination and new dams, which allows installation of tanks as required when prices are constrained. Second, household tanks are a means of avoiding the impact of water restrictions, so that outdoor use can be maintained.

Illustrative applications to policy

Modelling results show that water restrictions can impose large economic costs on the community. Restrictions are enforced in the model when storages fall below a threshold level. The cost of restrictions is a consequence of preventing outdoor users from using water that they would have been willing to pay for. Cost estimates are a lower bound, as they do not include additional impacts such as the differential effect of restrictions across households — in reality, some households that are prepared to pay a lot for additional water might have to forgo consumption under restrictions. The finding of high costs from water restrictions is consistent with previous studies, but estimates in this study are somewhat lower, as they represent expected values across many different scenarios for inflows to dams, most of which only rarely require the imposition of water restrictions. This is primarily because many of the scenarios modelled involve rainfall sufficient to avoid the need for restrictions, but also because high prices during times of scarcity encourage users to voluntarily reduce consumption.

Table 1 **Welfare costs of illustrative policies**

Expected net present values over the next eight years, relative to scarcity-based pricing

Policy	*Central estimate*	*Average inflows*		*Demand elasticity*	
		Low (-30%)	High (+30%)	Low (-0.10)	High (-0.50)
Long-run marginal cost	94	241	60	149	117
Restrictions	522	673	267	1 013	401
Restrictions and long-run marginal cost	658	1026	599	1 573	548

Source: Modelling results.

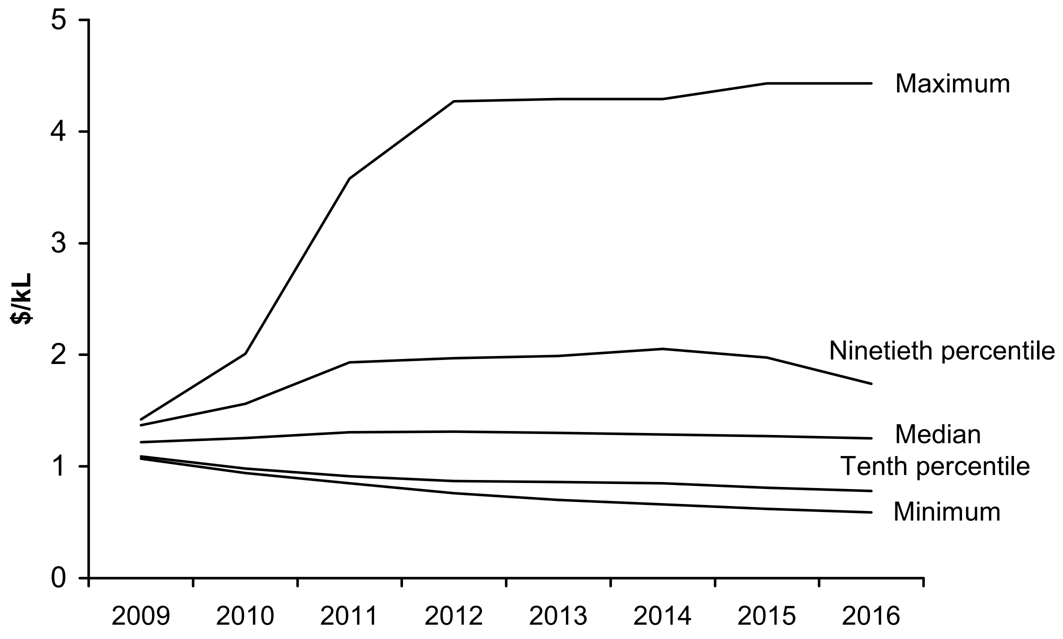
Pricing based on the long-run marginal cost of supply also carries costs relative to scarcity-based pricing. Long-run marginal cost pricing is costly due to 'smoothing' of prices under this type of price regulation: prices are not free to increase or decrease during dry or wet periods. A relatively flexible form of long-run marginal cost pricing was modelled, with the only constraint being that prices are set every four years, and regulators cannot change prices in response to inflows to dams during these four years. More restrictive implementations of long-run marginal cost pricing would carry higher costs. For example, constraining price to be equal to the unit cost of water from the marginal source of supply (including capital costs, distributed over the life of the asset) carries welfare costs that are about three times as large. Average prices paid by consumers, across all rainfall scenarios, were also found to be higher under long-run marginal cost pricing than under scarcity-based pricing.

The cost of long-run marginal cost pricing and restrictions together — about $100 million per year — is greater than the aggregate cost of each of these policies separately. The total cost is more than the sum of the parts because under long-run marginal cost pricing there is less scope for users to voluntarily reduce consumption in response to high prices, so restrictions are required more often, imposing large costs on outdoor water users. The need for restrictions to ration demand implies that the cost estimate for long-run marginal cost pricing and restrictions together is a more relevant cost for this type of regulated pricing.

The key feature of scarcity-based pricing is price volatility. Where consumers are risk averse — as in the modelling — the variability of prices reduces welfare. During extended dry periods, scarcity-based prices increase considerably in the hypothetical example (figure 2). However, prices generally remain below $2 per kilolitre (the model is calibrated to current prices of about $1.20 per kilolitre). On average, prices are lower under scarcity-based pricing than under the other pricing policies modelled. Suppliers can offer low prices in times when water is plentiful, safe in the knowledge that they can increase prices if shortages develop in the future.

Results from sensitivity analysis show that the costs of illustrative policies are higher under low elasticities of demand for water and under lower inflows to dams. Low elasticities of demand are often cited as an argument against scarcity-based pricing, as the price changes required to equate demand and supply would be larger. However, low demand elasticities imply that users are prepared to pay significantly more rather than reduce their consumption of water — that is, they place a high value on their existing uses of water. This means that there are also much higher costs from using water restrictions to curtail the use of water during dry periods (table 1). When average inflows to dams are lower, the costs of not being able to use pricing to ration water are also higher. Further, restrictions are required more often, thus imposing higher expected costs.

Figure 2 **Price variability under scarcity-based pricing**



Data source: Modelling results.

The higher cost of restrictions when less water is available, suggests that if climate change means less water is naturally available for Australian cities in the future, this will increase the potential benefits from using some form of scarcity-based pricing. As mentioned above, modelling results highlight the importance of variability in dam inflows to Australian urban water systems. Thus, it is not surprising that there are benefits from choosing pricing approaches that are flexible enough to respond to this variability. Impacts on people who are least able to afford higher prices during times of drought are important, but there are also potential benefits from lower prices on average under scarcity-based pricing. Any equity concerns could be addressed outside the urban water system, in ways that do not distort consumer decisions. It might also be possible to provide some fixed quantum of 'essential' water to alleviate equity concerns.

The other difficulty associated with scarcity-based pricing is achieving the necessary institutional arrangements to attain the efficient water market pricing embodied in this approach. This is a broader issue that is outside the model developed for this paper: the modelling is useful to investigate the characteristics of an efficient market in an urban water system, but does not specify how such a market could be created. Consumers and suppliers (whether private or government) are assumed not to exploit any market power, which might be difficult to achieve where supply is dominated by a small number of suppliers. Government providers are assumed to act as welfare-optimising social planners. Institutional arrangements for implementing scarcity-based pricing remain an area for further work.

Potential for future work

Future work could build on the modelling framework developed in this paper. For example, it might be possible to better model risk aversion with respect to the possibility of running out of water after a series of exceptionally dry years. There is also potential to extend the analysis of pricing and restrictions policies, including by improving the modelling of long-run marginal cost pricing and by comparing realised returns to investment across different policies. Further, with appropriate data, it would be possible to apply the model to a specific urban water system.

Notwithstanding these areas for further work and the inherent limitations of any model as a simplification of the real world, the modelling framework developed in this paper offers insights into urban water policy issues in Australia that were not directly available from other models of urban water systems. No single model can provide insights into all issues, and the approach presented here complements other models used to analyse urban water systems.

To facilitate further work using this modelling framework, the GAMS model code developed for this paper is available on request.

1 Introduction

Recent drought conditions throughout much of Australia have contributed to significant interest in urban water policy. To inform the policy debate, an economic model has been developed that is suitable for investigating policy choices in urban water.

Set out in the current chapter is the background and motivation for the urban water modelling work undertaken for this study. This includes a description of the existing situation (section 1.1), a summary of some key findings of recent reviews of the urban water system (section 1.2), and a discussion of why there is a need for new modelling work (section 1.3)

1.1 Existing arrangements

Households and businesses in Australia's capital cities and other urban centres are supplied with potable water through centralised supply and piped reticulation systems. Supply is typically the responsibility of state government-owned utilities.

The majority of urban water is used by households for domestic uses. In Melbourne, for example, about two thirds of the water delivered to end users goes to households, with the balance to commercial and industrial uses (Victorian Government 2006). Of total household use, outdoor uses typically account for at least a quarter, but this varies by state (ABS 2007b).

Pricing of water to end users is regulated with the aim of ensuring revenue is sufficient to cover costs and secure adequate supply, including a return on assets. No value is attached to the scarcity of the water resource itself to end users. Regulators typically use long-run marginal cost pricing for price setting, which smooths prices over time. Prices are set periodically (generally every three to five years) with limited scope for prices to vary with the level of inflows to dams during this time. During extended dry periods, this leads to demand for water that is in excess of the supply that can be prudently made available, given the need to secure supplies for the future. Water restrictions are then used to ration supply to end users by proscribing certain outdoor uses of water, for example watering gardens or washing cars. Water restrictions are typically triggered once dam levels fall below some threshold (for example, see DSE 2007).

The supply of water for most urban centres in Australia comes mainly from dams. Among capital cities, the exceptions are Perth (which obtains most of its supplies from groundwater and also has a desalination plant) and Hobart (which sources around 60 per cent of its water from the Derwent River). These dams have high storage capacities, ranging from around four years of use (Canberra and Melbourne) to nearly eight years (Darwin) for the most dam-dependent capitals (PC 2008).

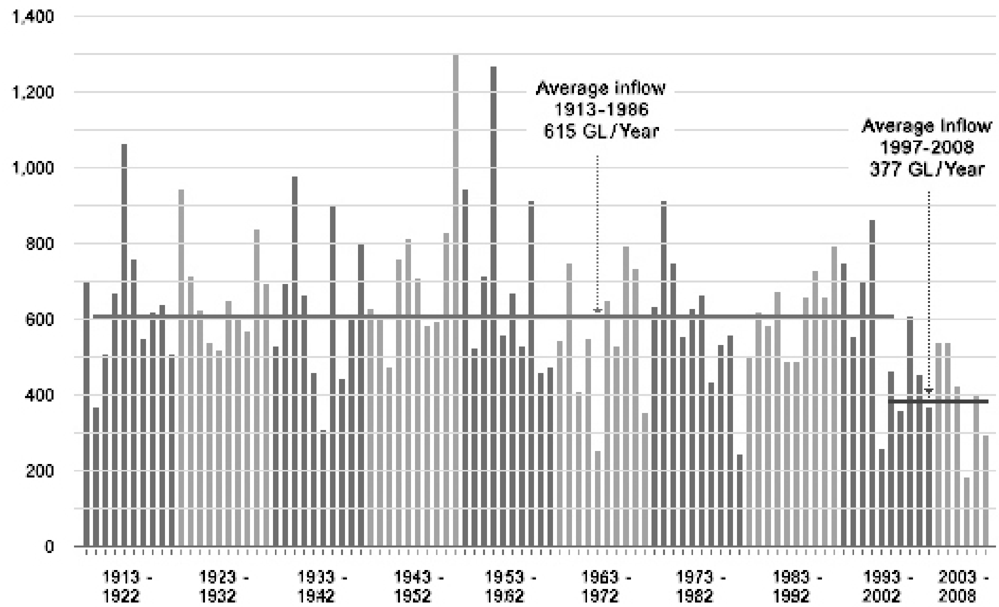
The supply of water from dams is characterised by significant annual variation in inflows, but recent decades have seen variability of supply exacerbated by a decreasing trend in rainfall across much of southern Australia. A reduction in average inflows to dams over periods of a decade or more has been witnessed in Perth and Melbourne (figure 1.1). This has seen extensive reliance on restrictions to curb water use, with approximately 80 per cent of Australia's households subject to water restrictions in 2008 (PC 2008).

Recent shortages have also led to investments to augment water supplies in major centres. These include:

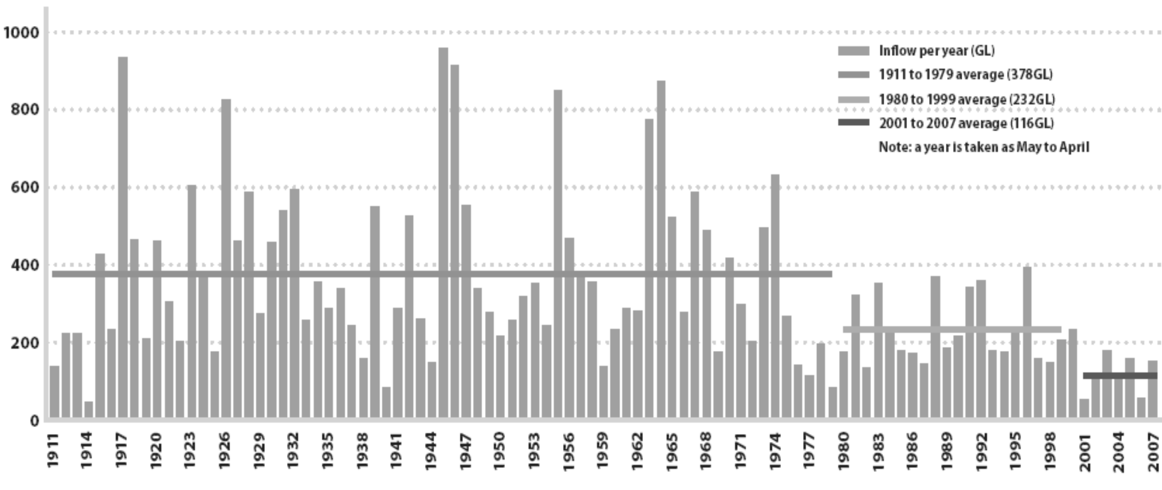
- *Desalination plants*: work is underway on desalination plants to service Sydney, Melbourne, South-East Queensland, Perth and Adelaide. In the case of Perth, the new desalination plant will complement supplies from the existing desalination plant at Kwinana, which has been supplying water since 2006.
- *Water recycling*: most capital cities are pursuing water recycling, typically for non-drinking purposes.
- *Aquifers*: Perth increased its water supply with a series of groundwater bores sunk in 2002 and 2003 (Water Corporation 2009).
- *New dams*: a new dam is under consideration at Tillegra to supply the Lower Hunter region of New South Wales. A proposed dam at Traveston Crossing, to supply South-East Queensland, was vetoed by the Federal Government in November 2009.
- *Sourcing water from rural areas*: a pipeline has been constructed to source water (freed up through improvements in irrigation efficiency) from the Goulburn River to service Melbourne.
- *Household tanks*: an increasing number of households have installed tanks as an alternate source of water to bulk supply, partly as a consequence of government subsidies for tank installation. Where only 7 per cent of households in capital cities had a rainwater tank in 1994 (Australian Government 2004), 12.5 per cent of capital city households had a tank installed by 2007 (ABS 2007a).

Figure 1.1 **Historical inflows to dams in Melbourne and Perth**
GL per year

Melbourne:



Perth:



Sources: Melbourne Water (2009); Water Corporation (2009).

1.2 Reviews of urban water policy

Recent shortages of water in urban areas have triggered a number of calls for reform to the urban water sector. These calls have come from government (Henry 2005; NWC 2007 and 2008; PC 2008), regulators (ERA 2008; ESC 2007; O'Dea and Cooper 2008), industry groups (BCA 2006; WSAA 2005), environmental groups

(ACF 2009), and academics (Crase and Dollery 2006; Edwards 2006; Grafton and Kompas 2006; Quiggin 2007; Sibly 2006). Views differ about the way forward for reform. Nevertheless, three main areas for reform are highlighted in a number of reviews:

- improving methods for price setting
- reducing reliance on water restrictions
- ensuring supply augmentation decisions are made efficiently.

Several reviews have discussed the potential for improving price setting, typically by introducing some form of scarcity-based pricing. Scarcity-based pricing would mean prices increase to ration supply during extended dry periods, but decrease when supplies are plentiful. A move to some form of scarcity-based pricing has been advocated by Grafton and Kompas (2006), Sibly (2006), Crase and Dollery (2006), Hughes et al. (2008) and NWC (2008). Scarcity-based pricing is also discussed in ESC (2007) and O'Dea and Cooper (2008), but these reports are more circumspect about its potential use.

Studies have often been critical of government reliance on water restrictions. By proscribing certain uses of water, restrictions deny households and some businesses the opportunity to choose how to use and/or conserve water. Accordingly, calls for less reliance on water restrictions have come from Edwards (2006), Grafton and Ward (2007), Henry (2005) and the National Water Commission (NWC 2009), among others.

Recent dry conditions and water restrictions in many of Australia's major cities have highlighted potential weaknesses in water supply planning and investment. In many jurisdictions, 'policy bans' on particular forms of investment have been invoked (Wahlquist 2007). The Wentworth Group of Concerned Scientists (2006), the NWC (2006) and Marsden Jacob (2006, for the Department of Prime Minister and Cabinet) have argued that all feasible options should be on the table and considered according to their merits. The NWC has noted that 'urban water shortages in the current drought and the rush to invest in new urban water infrastructure are evidence of planning failure' (NWC 2007, p. 4).

The Productivity Commission's (2008) urban water discussion paper identified potential benefits from reform in all three of these areas — price setting, reducing reliance on water restrictions, and improving decision making on supply augmentation. In particular, the loss of consumer welfare from water restrictions was estimated to impose a multi-billion dollar annual cost for the whole of Australia during the recent drought. One possibility canvassed for improving supply decisions was to allow rural–urban trade in water. The discussion paper also identified potential benefits from structural and institutional reforms.

1.3 A role for new economic modelling?

The reviews cited above have set out a case for reform of urban water markets, particularly in the areas of pricing, restrictions and decisions about new supply sources. However, there has been little quantification of the potential costs and benefits from reform. This makes it difficult to prioritise between different reform options, or to convince policymakers that the benefits from reform outweigh the costs. Economic modelling is a useful means for policy analysis when projection from historical data is not possible due to a lack of historical experience with particular policy options (McCarl and Spreen 1980). This is true of the urban water sector where, for example, there has been little experience with the use of scarcity-based pricing to reduce reliance on restrictions.

Quantification can be achieved using a model that has the following attributes:

1. spatial and intertemporal representation of demand and supply, with both able to respond to price signals
2. stochastic representation of inflows to dams
3. a time horizon sufficient to capture efficient intertemporal pricing of investment and water supply
4. a choice between a range of new supply options to augment supply
5. scope to apply policy constraints to market outcomes, such as regulation of end user prices and water restrictions.

Modelling of urban water systems to date has not had all of these attributes, and as such it is not easy to assess the effects of various urban water policy options using existing models. Much of the existing modelling has used supply-side models, which are focussed on meeting set levels of demand given various engineering constraints (Hughes et al. 2008). Studies by ABARE (Hughes et al. 2008) and Grafton (2008) have incorporated endogenous demand curves so that demand is able to respond to prices. The framework used for these studies is ideally suited to examining optimal investment timing and characteristics of efficient pricing under rainfall uncertainty. However, only a single augmentation option was modelled in any particular simulation and there was no attempt to investigate the effect of policy constraints — such as regulated pricing — on market outcomes. Similarly, ERA (2009) modelling of the short-term value of water is a useful method for setting short-term prices with some regard to the scarcity of water, but is not designed to assess the costs and benefits of a wider range of policy options.

The model presented in this paper is designed specifically to analyse and illustrate various policy options in the urban water market. The model is formulated as a fit-for-purpose tool to examine the efficiency impacts of pricing, demand management and investment policies. To this end, the partial equilibrium model developed displays all five of the attributes listed above. A detailed description of this model is contained in the following chapter. This model enables assessment of costs and benefits of reform to a range of pricing, water use restrictions and supply augmentation decisions.

An additional area of reform identified by the Productivity Commission (2008) — structural and institutional reforms — could also be investigated using the model developed here, but the economic impact would need to be specified exogenously. For example, if there were a productivity improvement from structural and institutional reform that generated cost savings in the supply chain (as considered in Cave 2009), then these cost savings would need to be determined outside the model. The model could then be solved with and without these cost savings to investigate their impact. This has not been pursued for the current study. Similarly, institutional arrangements to achieve the various policy options modelled (in particular, scarcity-based pricing) are outside the scope of the modelling.

2 Partial equilibrium framework

The preceding chapter highlights the important attributes of an urban water model if it is to be useful in quantifying benefits and costs of policy change.

The model presented in this paper is specifically formulated to examine the efficiency impacts of policies relating to pricing, demand management and investment. These issues can be examined using a partial equilibrium (PE) framework.[1] Urban water demand and supply are particularly suited to a partial equilibrium modelling framework, for three principal reasons.

- Water occupies a small share of household budgets (PC 2008). For this reason, the income effects from changes in the urban water market are likely to be small.
- Water has few close substitutes. Therefore, the impact on, and interaction with other markets will be limited. This means that the water market can be considered in isolation from other markets.
- Urban water provision is characterised by a range of interacting policies, as well as several competing supply augmentation options. The detail that can be incorporated into a partial equilibrium framework means that it is well suited to evaluating multiple policies and investment options simultaneously, as well as their impacts on community welfare.

Presented in this paper is a model of a single urban region, ignoring to some extent the possibility of a larger water market. Large capital city markets are typically effectively separate markets because of the high costs of transporting potable water in pipes. As such, a single urban water system can be modelled in isolation from urban water systems in other regions.

The model is designed to estimate the economic impact of the issues outlined in the Commission's (2008) discussion paper using a partial equilibrium approach. It presents the optimal pricing and investment decisions for a risk-neutral policy maker, operating in an environment of variability in (and uncertainty of) dam inflows. In each period, the decision maker faces two key choices: how much water to supply to end users or carry over for future consumption, and whether to invest in

[1] That is, by examining the changes brought about in the urban water market without considering the impact on other markets within the economy.

new sources of supply. The price consumers are willing to pay varies with the quantity of water supplied.

This chapter describes the urban water model used throughout the paper. Section 2.1 briefly outlines the background of the theory underpinning the model, and the advantages of the approach. Section 2.2 describes the extensions of that framework required for the Commission's model, and outlines the model's mathematical structure more fully. Finally, section 2.3 describes our application of the model to an urban water setting, characterising the investment and pricing decisions that drive the model results.

2.1 Introduction to the PE framework

The theoretical framework for the model is based on the spatial and temporal equilibrium framework developed by Takayama and Judge (1971), and first proposed by Samuelson (1952). Labys, Takayama and Uri (1989) further applied the framework for the economic analysis of markets over space and time. There is a wide body of literature applying this framework to various policy environments, from airport regulation, to energy and natural gas transportation, and including examples such as agricultural water and environmental problems (see, for example, Heady and Vocke 1992). This approach allows a market equilibrium to be found, as it incorporates various technologies associated with each supply option (through the use of activity-based linear programming). It allows for competing technologies to be evaluated simultaneously, without any assumptions about which technology will be used. Some technologies may not be used at all.

The market equilibrium is computed by maximising net social welfare in the urban water sector (the sum of Marshallian consumer and producer surplus). That is, it maximises the area under the demand function less the total costs of supply activities. However, the model only maximises welfare in the urban water market. For this reason, welfare in models of this type is often referred to as quasi welfare (Samuelson 1952). It measures only the costs and benefits that result from transactions and investments within the water market. It excludes welfare changes in other markets resulting from income effects (changes in purchasing ability in other markets resulting from price changes in the water market), as well as broader externalities. For the purposes of this report, and bearing these limitations in mind, the sum of the Marshallian consumer and producer surplus is reported as the welfare measure.

At the optimal solution, consumers cannot receive a greater benefit without a more than offsetting increase in the costs. This allows a great deal of flexibility: detailed demand characteristics, supply technologies, and additional constraints can be included to capture the impact of policy constraints on the operation of the market.

A simple, stylised exposition of the framework is presented in box 2.1.

The framework readily incorporates markets temporally: water storages and supply facilities with long economic lives mean that demand and supply are linked temporally. This interaction between storages and investment over time gives value to water in storage (ERA 2005) — stored water has value because it can meet future water demands, reducing the need for investment. This dynamic interaction is of particular importance in urban water provision, and is readily incorporated into partial equilibrium models.

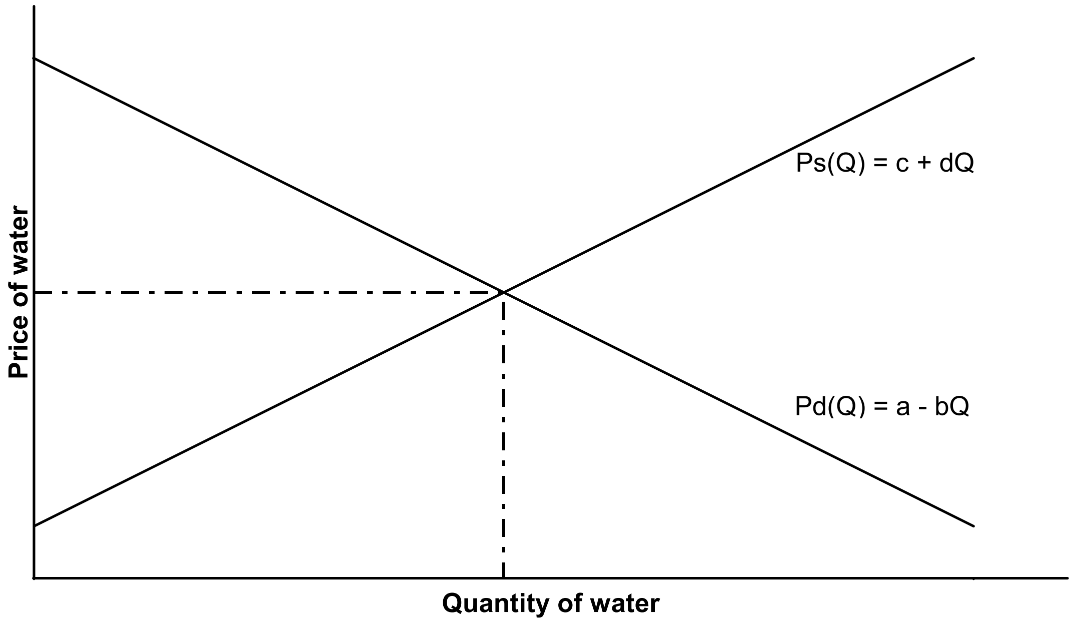
2.2 Stochastic extension: multistage stochastic programming and the probability tree

Urban water supply decisions are complicated by the probabilistic nature of future rainfall. Investment choices are state dependent: past inflows, and the variability associated with future inflows, influence whether to invest in additional supplies, and what form these investments should take. Future rainfall also has important implications for comparison of different augmentation options: relatively expensive rainfall independent sources (such as desalination and recycling) need to be weighed against cheaper rainfall dependent sources (such as rural–urban trade). Alternatively, there is the option to forgo investment altogether if there is sufficient water in storage.

The Commission's model incorporates probabilistic rainfall by embedding the partial equilibrium model within a multistage stochastic programming framework. Two-stage and multistage stochastic programming are well documented approaches to modelling decision making over time with probabilistic expectations of future outcomes (for summaries, see Kall and Wallace 1994 and Birge and Loveaux 1997). Both allow for some decisions to be made subject to rainfall variability while other recourse decisions (quantity of water supplied) can be made after observing rainfall outcomes. Multistage stochastic programming is differentiated from two-stage stochastic programming by allowing for sequences of investment decisions to be made over time as outcomes are observed (Birge and Loveaux 1997).

Box 2.1 **Stylised exposition of the framework**

Maximisation of the area under the demand function minus the area under the supply function is a way of solving for the point at which demand intersects supply.[a]



Maximising net social welfare (NSW):

$$Max\ NSW = \int (a - bQ)dQ - \int (c + dQ)dQ$$

To find the maximum, take the derivative of the net social welfare function with respect to Q, and set it equal to zero:

$$\frac{dNSW}{dQ} = \frac{d}{dQ}\left[\int (a - bQ)dQ - \int (c + dQ)dQ \right] = 0$$

$$(a - bQ) - (c + dQ) = 0$$

$$\Rightarrow Pd = Ps = P*$$

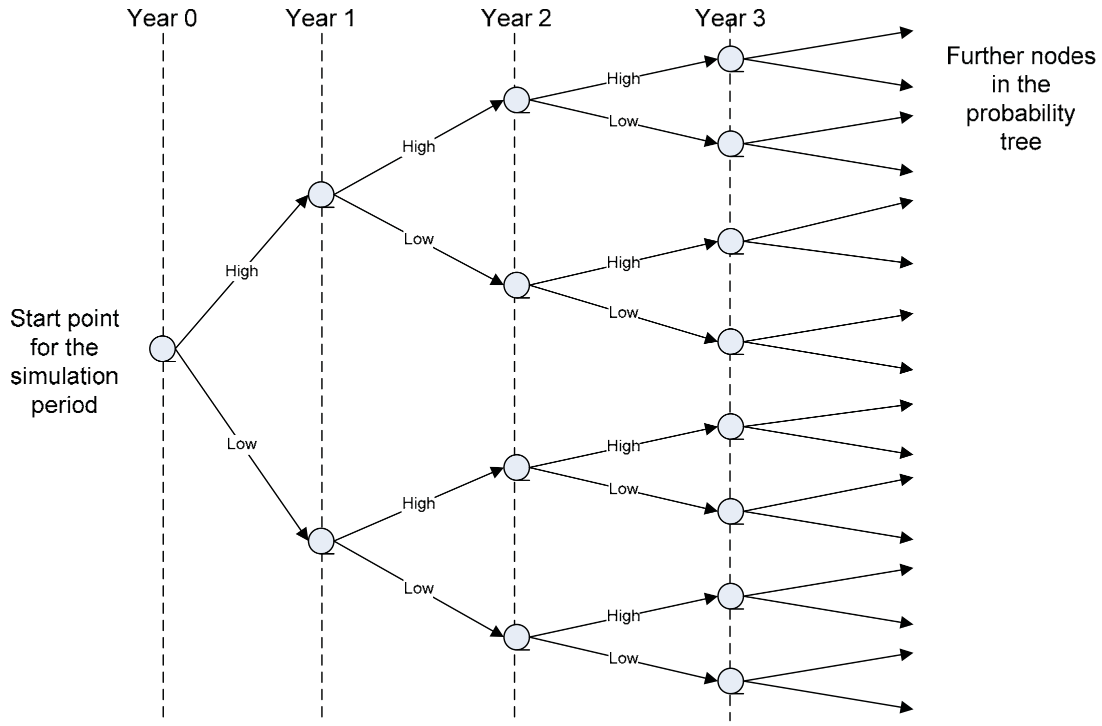
This solves for the value of Q where the demand function intersects the supply function. The value of P is implied from the solution value of Q.

[a] The supply function shown is linear for illustrative purposes. Total supply is represented within the model as the aggregation of the various supply sources, each with their own separate cost profiles. For more detail on the costs of individual investments, see Appendix B. For detail on the full model structure, see Appendix A.

This allows for 'wait and see' decision making: investment decisions can be delayed until storage levels fall below some threshold. All of the investment decisions in the planning period do not need to be made in year one. Rather, investment decisions are made over time as the sequence of inflows, and hence water scarcity, is revealed. This allows decisions to be the best possible given information available at the time, and closely parallels the decision-making constraints faced by real-world policy makers and markets.

Multistage stochastic models are typically solved by approximating the probability distribution of the random variable (inflows) using a discrete probability distribution (Kall and Wallace 1994). The model approximates inflow and rainfall data using three discrete states, each with a corresponding probability: a low, medium and high rainfall scenario for each time period. These discrete inflow states, combined with the inter-temporal decision making, imply an underlying probability tree structure for the model (Figure 2.1).

Figure 2.1 **Illustrative probability tree structure with two inflow states[a]**



a Two inflow states only are shown here for simplicity. The model described in this paper has three inflow states: high, medium and low.

Figure 2.1 shows an illustrative probability tree, containing two inflow states. In the first year, there are only two possible inflow scenarios: high and low. With each additional year, the number of possible cumulative states increases (for example, in the second year, there are high–high, high–low, low–high and low–low combinations). Each of these points in the tree with a cumulative combination of inflow states is called a node. The number of nodes in the probability tree increases with both the number of inflow states and the number of years in the model.

Nodes in the probability tree represent supply, demand, investment and storage decisions at a point in time for a unique scenario of inflows. Each node represents a particular water market at a point in time for a given scenario of inflows to that point in time, and expectations about future inflows over the remainder of the time horizon. In practice, each node is a unique combination of rainfall states over time. Each node contains a snap shot of the levels of all variables relevant to that time period and rainfall state: levels of inflows, demand, supply, and prices. Levels of storage, and investment decisions are passed from one node in the probability tree to the next, with probabilistic knowledge of rainfall states in future nodes.

The model maximises the expected value of discounted net social welfare over time for all possible rainfall states across the entire probability tree (for the full specification of the model, see appendix A). This is analogous to an expected utility function, as set out in von Neumann and Morgenstern (1944). Investment and storage decisions are made based on the expected future welfare they provide.

Investment and storage decisions are made based on their expected future returns throughout the probability tree. In the core market model, an investment will only be built if [the present value of] the expected benefits derived from that investment at least offsets the expected costs. This means that ex ante, any investment that is built will always have a benefit–cost ratio of at least 1. However, after rainfall states are revealed, an investment may, ex post, have a benefit–cost ratio of less than 1. This is because the initial investment decision must be made with only probabilistic knowledge of future rainfall states. Once those rainfall states are realised, the policy maker or investor must live with the earlier investment decision. A priori, this implies that any given decision is unlikely to be optimal for a specific inflow scenario, compared with the situation where the future was known with certainty (Kall and Wallace 1994). For example, an investment in desalination may be made early due to rainfall risk if, on an expected value basis, ex ante the benefit from building the plant outweighs the costs. However, ex post it may be loss-making if inflows to dams turn out to be higher than expected and the value of water from the plant is low. Conversely, if it turned out to be very dry the plant would provide much-needed water, the benefit of which would far outweigh the costs of the plant. More detail on the economic principles behind investment decision making in the model is outlined in appendix C.

Solving the multistage stochastic program

Due to the nature of the probability tree, the model becomes very large, very quickly. For instance, a three inflow state model, with 20 time periods, would have

3^{20} scenarios, resulting in over 200 billion variables and over 100 billion equations, and could not be solved using available computing technology.

The authors worked with academics from Melbourne University (Melbourne Operations Research) to consider methods making solving the model tractable. They investigated several approaches — including nested Benders decomposition with sampling (Infanger 1993), as well as approaches based on stochastic dynamic programming (Ross 1983 and Powell 2007). However, they found that there were no well documented approaches in the literature that suited our model. Any approach would have required substantial investigation, and extensive additional work. The authors also examined another method to reduce the solution times for the model, which involved solving a quadratic programming formulation of our model (McCaulay 1985). This method, however, was proved impractical for the model outlined in this paper.[2]

Considering these limitations, it was determined that the largest model solvable with the desired level of system detail was a 10 time-period model. More specifically, this translates to a model with approximately 2.5 million equations and 5 million variables. Three techniques were used to engineer a model of the desired size: aggregating several years into a single time period for the later years of the simulation, treating investment as a cumulative total, and linearisation.

As mentioned above, given current computing technology, a year-by-year simulation spanning a 20-year time horizon could not be modelled in full. In order to examine investment decisions over such a timeframe, while staying within practical computational limits, aggregate time periods were used for later years in the simulated time horizon. The rationale for this approach is that in early years, accurate price and investment information is required. However, further into the future, outcomes are increasingly uncertain, and precise year-by-year results are less important than the general pattern of prices and investment. In order to model a 20-year time horizon using this approach, the first four time periods were represented as 1-year steps, the next three periods as 2-year steps, the next two periods as 3-year steps, and the final period as a 4-year step[3]. This substantially

2 The method reduces the number of variables in the model, but increases the number of constraints. For a model the size of the urban water model in this paper, this is undesirable. The execution time for a model is approximately related to the cube of the number of equations, while it is relatively unaffected by the number of variables (Hillier and Lieberman 2000). As such, increasing the number of constraints would materially increase solution times.

3 Although aggregation is convenient for many variables, it required assumptions to be made about discounting. For the aggregated 20-year model, discount rates were compounded and applied at the end of each aggregated period. This results in a slight overstatement of the impact of discounting.

reduces the number of equations and variables in the final model, as it is in the later time periods that the probability tree is broadest. This approach was used for longer-run simulations (to analyse investment), while short-run simulations had eight single-year steps (for policy analysis).

In order to make the model solve more easily, investment within the model was treated as a cumulative total, rather than incremental additions in each year. This can be seen in box 2.2, and appendix A. Modelling investment as a cumulative total makes the model solution computationally less onerous. If investment is treated incrementally in the model (i.e. the capacity added in each year) then many equations require summations that include previous incremental additions to capacity (to determine, for example, if desalination plant capacity was added several years earlier, and therefore available for use today). This means that the matrix which must be solved computationally has many more elements. This makes the matrix less 'sparse': there are more values in the matrix that must be solved. This 'sparseness' affects how computationally intensive the model solution is, and, in turn, how long it takes to solve (Hillier and Lieberman, 2000).

Further, linearisation is employed to allow a larger model to be solved. Models using the spatial equilibrium framework can be solved efficiently when in a linear form (Duloy and Norton 1975), and modern linear solver algorithms are much more efficient than non-linear alternatives. The model described in this paper is linearised: non-linearities are approximated using a piecewise linear function. This allows a much larger model to be solved than would be otherwise possible. This implies that any results for the model are an approximation of the true, non-linear solution.

The use of linear programming in a multistage stochastic framework differs from the stochastic dynamic programming approach pursued by Hughes et al. (2008), Grafton (2008) and others to analyse urban water issues. Both approaches have advantages and disadvantages that stem from the way each framework defines its state space — the discrete combinations of states of the world that govern the number of scenarios in the model. Multistage stochastic models define their state space in terms of time. This allows them to include many more investment options (which can be considered simultaneously) while also including continuous inter-temporal dam storage variables. Stochastic dynamic programming, on the other hand, defines its state space in terms of investment options. This allows the models to include more temporal detail (e.g. longer time periods and, seasonality), and maintain a much smaller probability tree. However, all state variables must be discrete (Nandalal and Bogardi 2007), which means that storages cannot be modelled as a continuous variable.

Box 2.2 **The treatment of investment in the model and solution times**

The water model includes a capacity variable for every investment option. This capacity variable appears in two places: linked to investment costs in the objective function and linked to supply in a capacity constraint.

A plant can be built at any point in the simulation period, and can be expanded (up to the maximum capacity) at any point thereafter. At each point in time, a cost of investment is incurred proportional to the level of investment made in that period (the *incremental* level of investment). Further, each investment can provide water equal to the total invested capacity up to that point in time (the *cumulative* level of investment).

It is logical to use the incremental level of investment in all equations. A disadvantage of this formulation is that there are a large number of coefficients in the capacity constraints, which requires significant computing time to generate the model (i.e. for the computer to construct the matrix representing the model for solution). Further, the additional data in the matrix to be solved makes the computational solution process much slower.

The cumulative investment capacity has fewer coefficients. The supply constraint includes only the cumulative variable itself, and the objective function can multiply the investment costs by the change in the cumulative capacity. This also makes the model compilation process faster.

This reduction in coefficients makes the matrix that must be solved more "sparse". Sparse matrixes can be solved much more quickly than dense ones (Hillier and Lieberman 2000).

This can be seen illustratively below with some basic equations. In a simple, three-year model, the incremental investment supply constraint would take the form:

$$Supply_{year=1} \leq Capacity \times Inv_{year=1}^{Increm}$$

$$Supply_{year=2} \leq Capacity \times Inv_{year=1}^{Increm} + Capacity \times Inv_{year=2}^{Increm}$$

$$Supply_{year=3} \leq Capacity \times Inv_{year=1}^{Increm} + Capacity \times Inv_{year=2}^{Increm} + Capacity \times Inv_{year=3}^{Increm}$$

However, for a cumulative specification of investment, there are much fewer instances of each variable in all the constraints:

$$Supply_{year=1} \leq Capacity \times Inv_{year=1}^{Cumulative}$$

$$Supply_{year=2} \leq \quad Capacity \times Inv_{year=2}^{Cumulative}$$

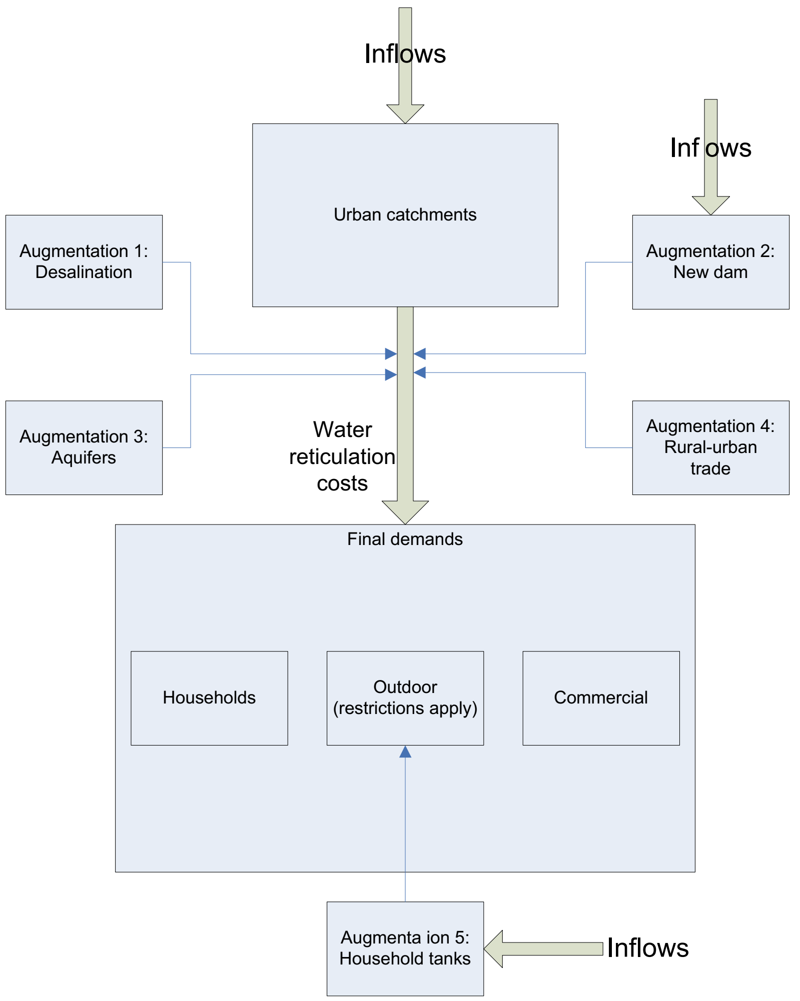
$$Supply_{year=3} \leq \quad Capacity \times Inv_{year=3}^{Cumulative}$$

This change also has implications for the specification of investment costs in the objective function, which can be seen by comparing investment in the objective function in appendix A with the objective function in appendix C. The two objective functions give identical solution values. The change to the objective function has no bearing on solution times.

2.3 Our application

The model describes the behaviour of demand and supply in a hypothetical urban water market (figure 2.2). As discussed in the previous section, the market operates for a time horizon of up to 20 years. Each time period has three possible rainfall states, based on available data (data used for calibration is described in appendix B). The model includes three types of demand, and five augmentation options that are competing with existing dams to supply water. There are several items that have not been included in the model, for varying reasons. These are outlined in box 2.3.

Figure 2.2 **Model of the urban water system**



Box 2.3 **Additional factors not in the model**

Other sources of risk

Different rainfall states are the only source of risk modelled. Other sources of variability — for example, levels of long-term demand growth — are not included in the stochastic programming approach. Variability in other sources of supply (tanks, new dams and rural–urban trade) is assumed to be perfectly correlated with inflows to dams. This is done for computational expediency, avoiding the need for having two or more exploding probability trees.

Correlation between rainfalls

Historically, persistent drought has caused water shortages in many Australian cities. Conversely, there have been periods when catchments have flooded. This suggests that there may be serial correlation between rainfall levels over time: if it is dry today, it is increasingly likely to be dry tomorrow. This can be readily incorporated into the modelling. However, the authors could not find evidence for such correlation in the historical data used (see appendix B), and therefore did not include it in the model results presented in this paper. Future research could potentially shed further light on this issue.

Weather dependent demand

Demands for water may be a function of the weather and rainfall. This is particularly true of outdoor demands: for example, if there are particularly low rainfalls, people are likely to want to water their gardens more. This can be incorporated into the framework described in this paper. However, there are few reliable estimates of the elasticity of outdoor water demand with respect to rainfall. For this reason, the modelling assumes that demands are weather independent.

Storage costs

The model assumes that the marginal cost of dam storage is zero. However, in practice this is unlikely to be the case. In any event, the model includes the annual costs of dam maintenance, and marginal storage costs would likely be small relative to this. The inclusion of marginal dam storage costs would likely have a minimal impact on any modelling results (Tooth 2009).

There are four characteristics of the Commission's model that require further explanation, and that have an important impact on the results:

1. specification of demand
2. supply options included
3. cost characteristics of supply
4. terminal conditions.

Demand

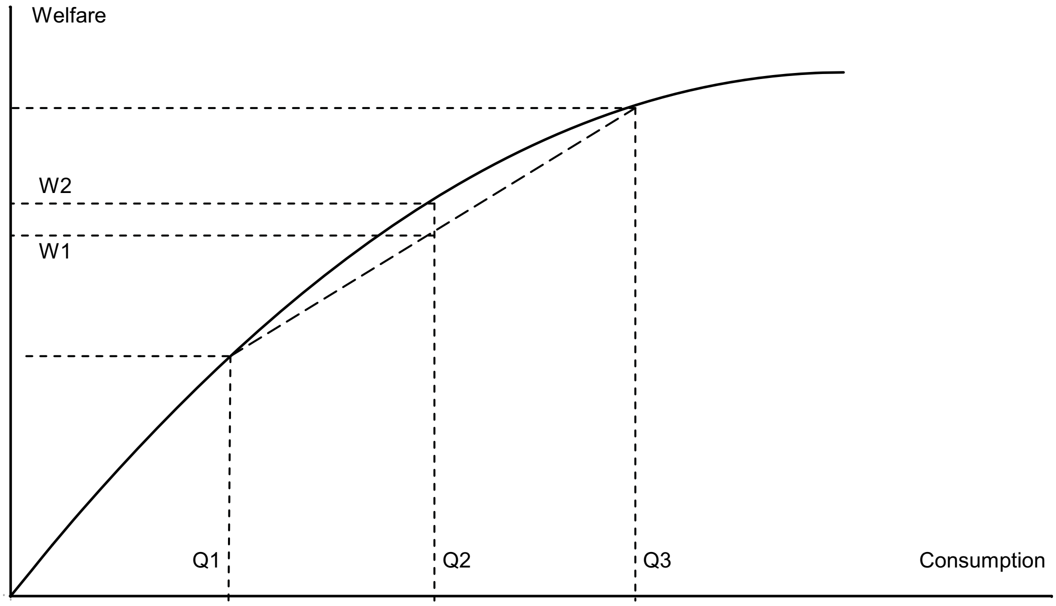
One of the important factors in examining water policies is the relationship between the level of consumption and prices. The model includes three types of demand, all of which are aggregate demands for the urban region. The first is outdoor demand, which is affected by water restrictions. The second is indoor household demand, which — based on previous studies — is assumed to be relatively inelastic (unresponsive) with respect to changes in the price of water. The final demand type is commercial use that is not affected by restrictions and which is relatively more price elastic than indoor household demand for a given price. Each of these demand schedules has a different responsiveness to price, and all are based on a linear demand curve. All three types of demand grow over time, and consumers are risk averse (box 2.4).

Supply options

The supply side of urban water provision in Australia is characterised by numerous competing investment options, as well as the existing dam infrastructure. Existing dams receive an annual inflow of water that varies with the level of rainfall. Residual inflows (after the removal of environmental flows and system losses) can be used for two purposes: meeting demand in the present period, or contributing to storage for future periods. The dams have a maximum storage capacity limiting the amount of water that can be held between any two periods. Any excess storages are lost as spillage. The model represents all catchments in a region using a representative aggregate dam storage. A single aggregate storage model can be used as a satisfactory approximation of a multiple storage system (Perera and Codner 1988), and this approach has been adopted in other economic modelling exercises (for example, Hughes et al. 2008).

Box 2.4 **Risk aversion in the model**

Risk aversion implies a preference for a lower but certain level of consumption over a risky level of consumption. The figure below shows the welfare of a risk-averse consumer for various levels of consumption. If a consumer has an equal chance of a low (Q1) or high (Q3) level of consumption, the welfare of the consumer will be the expected value of welfare resulting from the two possibilities, W1. The consumer would receive greater level of welfare (W2) from a guaranteed mid-range level of consumption (Q2). Therefore, a risk-averse consumer would be willing to pay a premium for certainty.



The welfare function in the model described in this paper has the same 'concave' shape as the graph in the figure above. Since the demand curve has a negative slope (appendix B) and the welfare function is defined as the area under the demand curve less supply costs (which increase at least proportionately with the quantity of water supplied — section 2.1), the welfare function is increasing at a decreasing rate.

The model optimises expected net social welfare in the presence of inflow risk, analogous to an expect utility function (von Neumann and Morgenstern 1944). Consumers make decisions based on expected future levels of consumption, as well as presently available levels of water in storage. A stochastic model can be used to incorporate risk preference into decision making (Hardaker, Huirne and Anderson 1997). The combination of the concave welfare function and a stochastic model can therefore be interpreted as representing partial risk aversion in the consumption of water. As a consequence, there are benefits from 'smoothing' water consumption over time using dam storage — consistent with theoretical work showing benefits from price stabilisation (Massell 1969).

Five potential new supply sources (dams, desalination plants, aquifers, rural–urban interconnection and household tanks[4]) are available, each of which have unique physical characteristics that differentiate them within the model. For example, tanks provide households with a small amount of additional, rainfall dependent water. This water can be used for outdoor uses, permitting households to compensate for water restrictions (up to the amount of water they are able to draw from their tanks). Greater detail about each of the supply sources included in the model is provided in appendix B.

Supply costs

Augmentations of capacity to supply water have three costs: a construction cost; an ongoing, annual fixed maintenance cost; and a marginal cost associated with releasing, delivering or obtaining a unit of water from the supply source. There is also a reticulation cost associated with transporting water from bulk storage to end users, which is uniform across all supply sources. A time lag exists between the decision to invest and commissioning of the facility, which varies between water supply technologies. This time lag influences the optimal investment choice, as some investments can be made more rapidly with higher cost per unit of water delivered (for example, household tanks) while others are slower, but have a lower cost (for example, a rural–urban pipeline). All of these augmentation options are considered together: the model maximises net social welfare by choosing the optimal combination of investments that best meet the willingness of users to pay for water, subject to the costs associated with the investments.

Binary variables introduce significant barriers in terms of the solvable size of the model. However, some investments are best represented with a binary variable. In the model, rural–urban interconnection is represented as a binary variable due to the nature of pipe interconnection investment. Similarly, new dam investments are treated as binary variables. However, the other investment options are continuous (desalination, household tanks, and aquifers). This is because of the significant computational load required for binary variables. The continuous variables are given an upper-bound cap on the total investment possible. However, this representation is not entirely unreasonable for those investments: many augmentation options are highly modular, with opportunities to invest or expand capacity to varying degrees. For example, desalination plants use modular

4 Recycled water has not been included. This is because the properties of recycled water are similar to desalination (weather independent potable water). However, the material barriers to the adoption of such technologies are largely political, or alternatively that the water produced is not perceived to be the 'same' (e.g. quality) as other types of water. This was not included in the model due to the additional data and computational difficulties it introduces.

technology, while household tanks are already an aggregation of many smaller units.

Terminal conditions

In finite period models, an issue arises when a productive asset has a life that extends beyond the time horizon of the model. This is of particular importance for investments made late in the modelling period. While the cost of investment is incurred up front, some benefit likely falls outside the chosen time horizon. In order to ensure investment decisions are not biased by this matter, all investment costs are truncated to reflect the life (and value) of the asset beyond the terminal period of the model. Ideally, the full cost and benefit of all investments would be contained in the simulation period, and since they are not, any formula truncating investment costs must make assumptions about the allocation of the investment costs between the periods within the planning horizon of the model and those beyond the planning horizon. In the model, this truncation is done pro rata: the share of the asset's life that is outside of the simulation period is subtracted from the total investment cost. This presupposes that the cost of an investment is evenly distributed over time. An alternative would be to assume that the cost diminishes with time to reflect the compounding nature of the depreciation schedule. The approach contained in the model creates a bias in favour of long-lived assets, while the latter approach would favour short-lived assets.[5]

Storages in the final period of the model are made endogenous using a terminal condition. This condition attaches value to water in storage in the final period, by imputing a value of future benefits obtained from the final stock of water[6] (McCarl and Spreen 2008). This represents the expected value of the future stream of benefits that would be obtained from the water in storage outside of the simulation period. Without a terminal condition of some kind, dams would empty in the final period, as there is no representation of future value of water in storage. Alternative approaches to the terminal condition could have been used: setting an arbitrary minimum value, or attaching a penalty to depleting storages. The value imputed in the terminal condition is derived from the implicit value attributed to storage based on storage behaviour within the modelled period.

5 As a practical matter, the two approaches give very similar results. For a typical simulation, net social welfare is changed by less than 0.5 per cent, and levels of investment changed by less than 2 per cent.

6 This approach is referred to as a vertical-terminal-line problem (Chiang 2000): the model has a fixed terminal time (the end of the simulation timeframe) at which the final storage must be determined. An alternative specification would have been as a horizontal-terminal-line problem, which would have specified the 'stop' level of storages, as opposed to the 'stop' time.

3 Results for the core market model

This chapter contains results for the core market model. Results are presented to illustrate how the model can be used to draw insights about several interrelated issues: the behaviour of prices, demand and storages over time; the effect of different rainfall patterns; and the characteristics of efficient investment in new supply technologies. The results in this chapter include the implications of investment constraints that are caused by technical or engineering limitations (for example, capacity constraints on aquifers). The chapter concludes with selected results for sensitivity analyses, showing how responsive the core results are to changes in various characteristics beyond the control of policy makers or market participants.

Results for the core market model can be thought of as a scarcity-based price of water: the price of water adjusts to ensure equality between supply and demand. This represents a market price in the absence of market power. In reality, water utilities could have some monopoly power, which could be exploited by commercial operators, or used as a tax-base by a government controlled body. This has not been included in the model, as the degree of market power and the extent to which it is exploited depend on the institutional setting. All policies modelled in this paper are measured against the hypothetical, optimal outcome described by market clearing, scarcity-based prices.

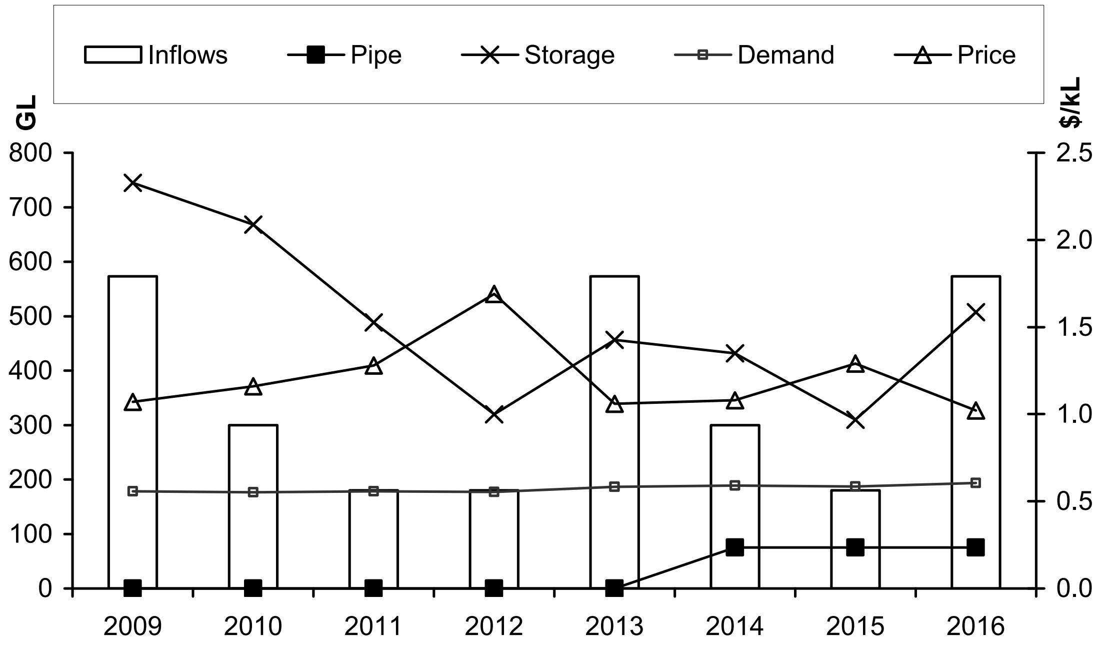
The discussion of efficient investment is generally based on results from solving the model over a time horizon of 20 years, using multiple years for each period in the later stages. This addresses the desirability of considering a planning horizon of about 20 years when making long-term water supply decisions (ATSE 2007). The examination of prices, demand and storage decisions is based on an eight year model. This shorter time period facilitates the additional computational load required to model water restrictions using binary variables (see chapter 4).

3.1 Prices and storage

Each solution of the model gives results for tens of thousands of rainfall scenarios. At each point in time for a given sequence of rainfall events up to that point, decisions about investment and the quantity of water to store are made with probabilistic expectations about future inflows to dams. However, decisions at a point in time also take into account inflows up to that point in time. For example, a dry run of years up to 2011 could cause storages to drop, and prices to increase (figure 3.1). This brings forward investment, which delivers additional water from the rural–urban interconnection from 2014.

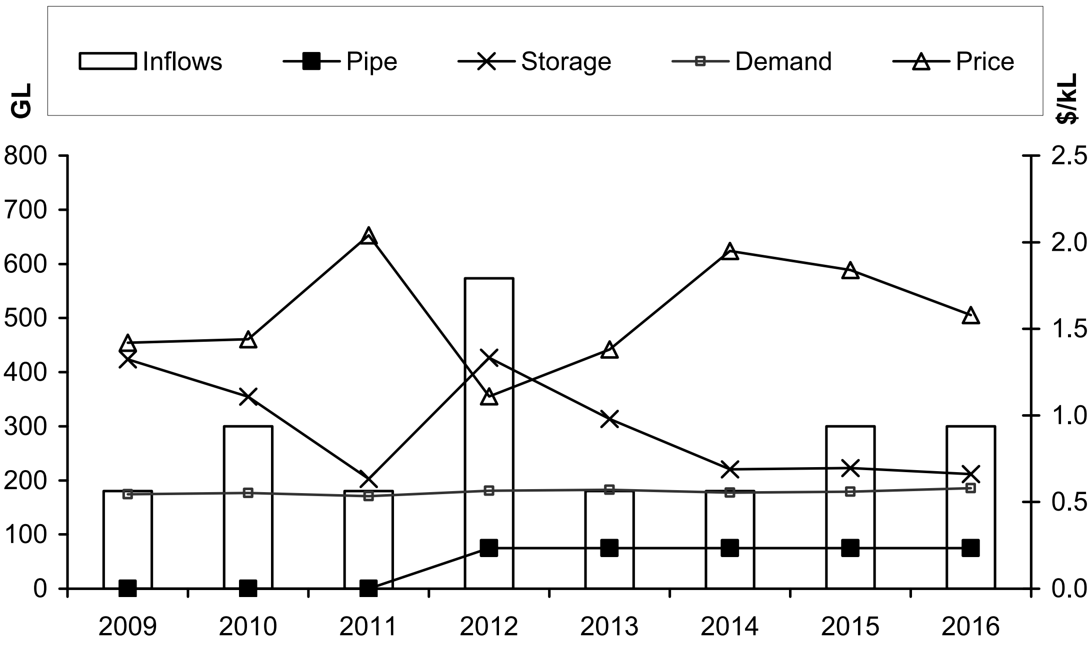
Results from individual scenarios show how prices move up and down according to water availability under scarcity-based pricing (figures 3.1 and 3.2). These charts show results from two illustrative scenarios, with inflows varying from year to year according to the columns in each chart. As in the example noted above, prices are higher during years with low inflows. As a result, in dry years suppliers will do relatively well, while in wet years consumers do relatively well. The quantity of water supplied changes too, as quantity is inversely related to price according to the downward sloping demand functions. Price changes are proportionally larger than changes in the quantity supplied, due to demand for water being inelastic. Changes in quantity supplied are also smaller than changes in inflows, as storage of water in dams enables the quantity of water supplied to be 'smoothed' over different years.

Figure 3.1 **Water price, investment and storage for a typical scenario**
Under scarcity-based pricing



Data source: Modelling results.

Figure 3.2 **Water price, investment and storage for a drier scenario**
Under scarcity-based pricing



Data source: Modelling results.

Across the full range of rainfall scenarios modelled, scarcity-based pricing leads to prices that can diverge significantly depending on rainfall (figure 3.3). During a series of wet years, prices converge to the short-run marginal cost of supplying and distributing water from dams. On the other hand, an extended series of dry years leads to higher prices. Overall, prices remain below $2 per kilolitre 90 per cent of the time.

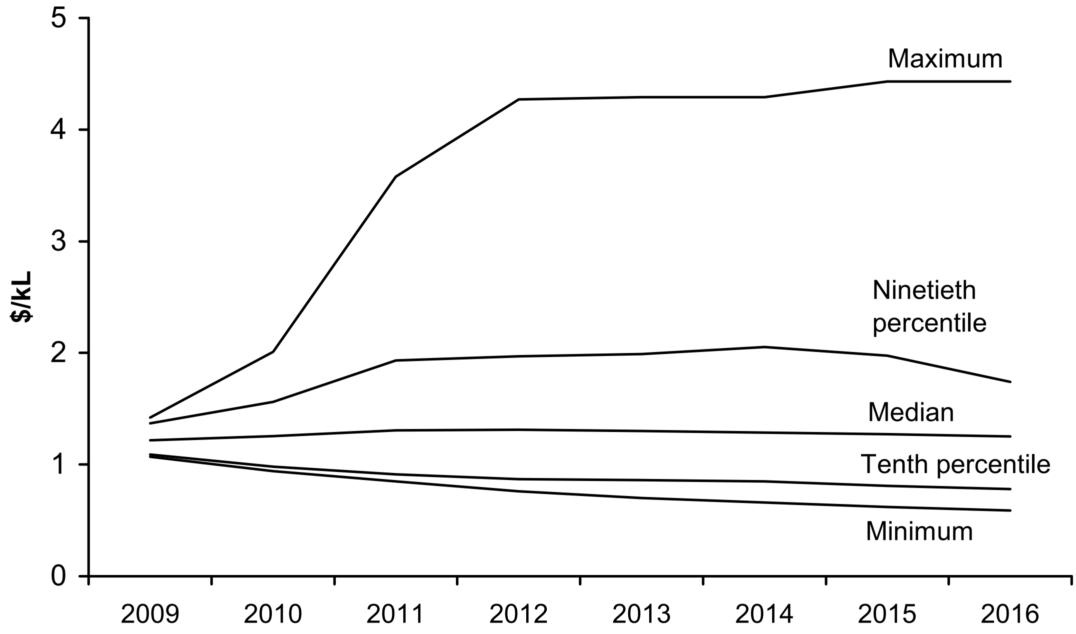
Prices continue to increase beyond $2 per kilolitre only during extreme dry spells. The probability of these extreme scenarios is low. For example, the maximum price path has a probability of only one in 50,000. Under a drier scenario (figure 3.2), prices initially increase. However, the increase in rainfall, and the use of rural–urban trade, reduces prices from 2015. However, in this scenario, such investment does not stop prices from exceeding $4/kL in the exceedingly dry scenarios.[1] There are several reasons for this. First, only a limited quantity of water is available from relatively cheap options for additional supply (aquifers and rural–urban trade), so additional water needs to be supplied from higher cost sources. Second, investment decisions are based on expected values across the range of future rainfall scenarios. Even after several dry years, most future scenarios will involve some periods of

[1] Note that the demand function in the model is calibrated to existing prices of around $1.20/kL (appendix B).

higher future rainfall, reducing the benefits from an investment made at an earlier point in time. This is akin to intertemporal peak-load pricing, whereby incremental capacity costs are recovered from consumption in future dry years. Finally, most new supply options take several years to construct, so the investment decision needs to be made several years in advance, further increasing the cost of augmenting supply.[2]

Figure 3.3 **Prices under scarcity-based pricing**

Across all rainfall scenarios modelled, without water restrictions



Data source: Modelling results.

The amount of water stored for future use also responds to inflows. Less water is stored in dams under a drier rainfall scenario (figure 3.2). Low storage levels are particularly evident during the first few years of the modelled period, as a consequence of low initial storage levels and inflows. Storages are increased in 2012 by higher inflows and the earlier availability of augmented supplies: rural–urban trade is brought to full capacity in 2012 under a drier scenario, compared with 2014 under a wetter scenario. Further discussion about investment in new sources of supply is below.

2 The model does not contain a 'back stop technology', which is a source of water that can be supplied at short notice (e.g. importing water in containers) in the event of extreme shortage. However, given that such a supply option would likely have a price in excess of \$4/kL (appendix B), it would not be selected in a model such as presented here.

The other difficulty associated with scarcity-based pricing is achieving the necessary institutional arrangements to attain the efficient water market pricing embodied in this approach. This is a broader issue that is outside the model developed for this paper: the modelling is useful to investigate the characteristics of an efficient market in an urban water system, but does not specify how such a market could be created. Consumers and suppliers (whether private or government) are assumed not to exploit any market power, which might be difficult to achieve where supply is dominated by a small number of suppliers. Government providers are assumed to act as welfare-optimising social planners. Institutional arrangements for implementing scarcity-based pricing remain an area for further work.

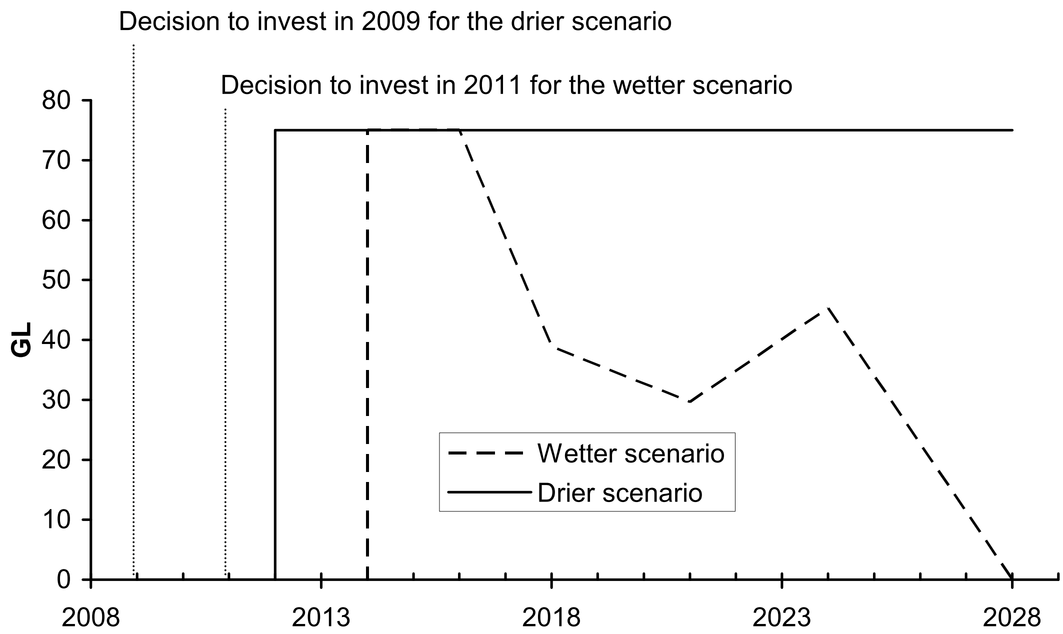
3.2 Source of supply

In all of the modelled scenarios, investment occurs when the expected benefits from additional supply outweigh the costs. Aquifers and rural–urban trade are chosen first within the model's optimisation as new supply options. This is because — based on the parameters used — both provide water at a lower unit cost and at high reliability. While rural–urban trade (through purchasing seasonal allocations) is likely to be a more expensive source of water during dry periods, on average it is a lower cost option than the other options modelled. Aquifers are the lowest incremental cost source of water, so they are used to capacity throughout the modelling period. Further, both aquifers and rural–urban pipelines can be brought online relatively quickly compared to other supply options (for example, new dams). These options might not be available in many jurisdictions, but where they are economically available, the modelling indicates them to be sensible first steps in augmenting supply.

Timing of investment in rural–urban trade is sensitive to inflows, with investment brought forward under a dry scenario (figure 3.4). In a majority of scenarios, rural–urban trade comes online in 2012. Under a wetter rainfall scenario, the supply of water from rural–urban trade is lower in a wet year at the end of the simulation. As supply from dams increases in this wet year, the price falls and supply is drawn from sources with lower variable cost of supply (dams and aquifers). In all scenarios, investment in aquifers takes place in the first year, due to its low investment and unit operating costs.

Figure 3.4 **Timing and utilisation of investment**

Water supplied from rural–urban trade under different inflow scenarios

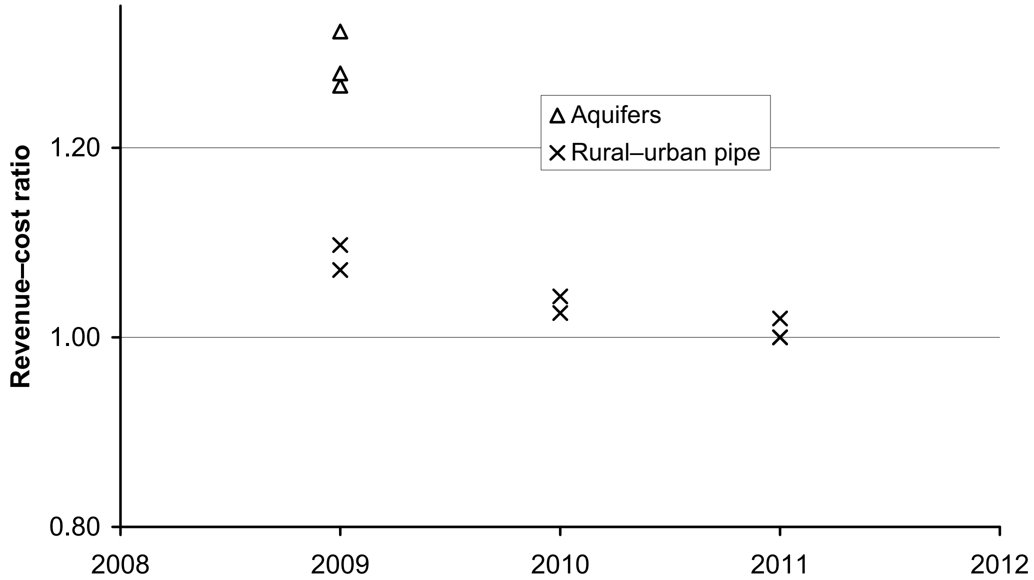


Data source: Modelling results.

Other supply options might offer a trade-off between the cost of water supply and reliability of supply, and might only be worthwhile during particularly dry periods. For example, when inflows were reduced in a sensitivity analysis (section 3.3), desalination investments came online within the modelling solution. While desalination presents a relatively expensive source of water (with a significant time lag between the decision to invest and commencement of operation), in the sensitivity analysis it was an optimal investment choice when inflows were reduced by 30 per cent.

The investment pattern under scarcity-based pricing is consistent with all investment costs being recovered on an ex ante, expected value basis. Investments are paid for by users and are only made where they increase net social welfare, which is measured as an expected value across all possible scenarios subsequent to the investment decision. At each point that an investment is made, the ex ante expected ratio of revenues paid by consumers to costs of supply will always be greater than or equal to one. Each investment decision has its own ex ante expected return (figure 3.5)

Figure 3.5 **Distribution of ex ante returns on investment**

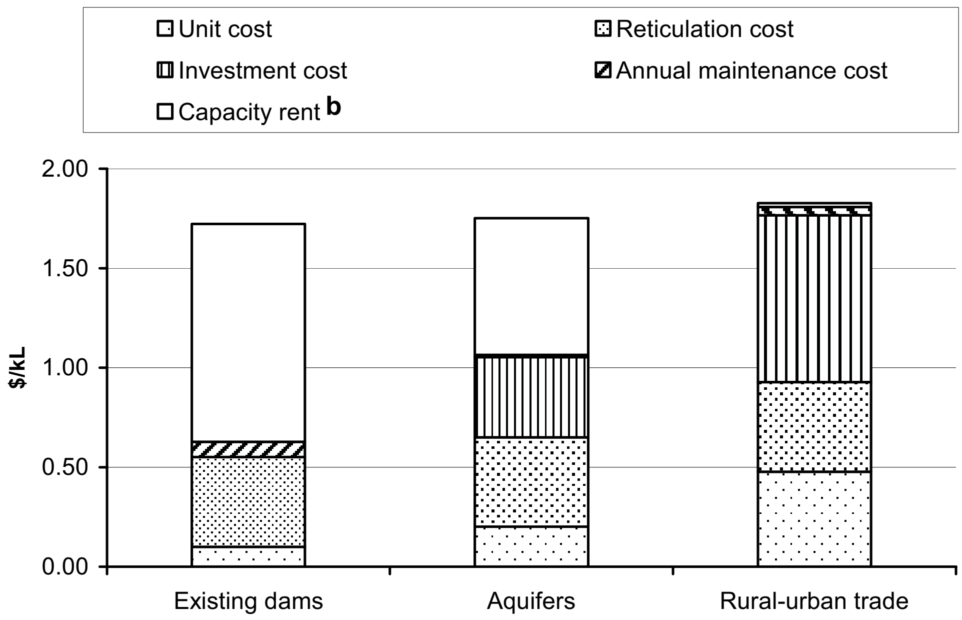


Data source: Modelling results.

In many cases the expected revenue will be in excess of the costs associated with the investment. This is due to capacity rents. Each investment option has a maximum capacity that can be supplied. When capacity is constrained, the price consumers are willing to pay exceeds the cost of supply from that technology and capacity rents accrue. Aquifers are the lowest cost source of supply augmentation, and as modelled, have a limited capacity to supply. This means that all possible aquifer investment options are built in the first period, due to their high expected returns (figure 3.5). Typically, the higher cost option of rural–urban interconnection is also used to augment capacity. Once the capacity of the aquifer is reached, price to consumers increases above the cost of supply from aquifers (reflecting the higher additional unit cost of rural–urban interconnection). Rural–urban pipeline investments come online later in the simulation period, with revenue–cost ratios approaching one as the investment is further delayed.

Rural–urban pipelines do not have revenue–cost ratios as high as aquifers due to their higher investment and unit costs. Further, because many of the rural–urban investments are built later in the simulation period, in scenarios where water scarcity has become acute enough that the costs of the pipeline can be recovered, they are unable to earn significant profits relative to other supply options. Ex ante, this results in a greater per unit scarcity rent accruing to cheaper investments (like aquifers) across the probability tree (figure 3.6).

Figure 3.6 **Recovery of investment costs over a 20 year simulation**
Disaggregation of end-use prices[a] — expected values across all rainfall scenarios



[a] The expected value of prices charged varies by water source because of differences in the timing of water supply from each source and varying prices over time. [b] Pricing above the cost of supply is possible in some or all scenarios because of capacity constraints on the supply of water from each source.

Data source: Modelling results.

Capacity rents are distinct from monopoly rents and have different implications. Monopoly rents arise from exploiting market power, creating costs to community welfare. In the modelling for this study, water suppliers are assumed not to exploit any market power they might have. Capacity rents, on the other hand, accrue to the owners of capacity-constrained resources (such as aquifers), and act to ration limited supply so as to achieve an efficient market equilibrium. Whereas the existence of monopoly rents might mean there is a role for government regulation to address market power, capacity rents should not be regulated away. Where firms make excessive profits as a consequence of capacity rents, this can be addressed more efficiently through resource-rent taxation that does not distort the price of water (Freebairn 2008).

The existence of capacity rents does not mean that all investments necessarily recover their costs on an ex post basis. Where rainfall turns out to be different from expected values, realised returns to investments can vary. For example, low rainfall is likely to result in a higher marginal value of water than expected, delivering higher prices and profits to investments (figure 3.7). On the other hand, if rainfall is plentiful, the investment cost of new supply from aquifers might not be recovered. Water will still be supplied from aquifers whenever the market price exceeds short-run variable costs, but a loss will be made on investment if prices and sales over time do not cover capital costs. However, in these situations where producers suffer an ex post loss, consumers benefit. The high level of inflows in wet scenarios mean that prices are lower, and that water users gain a larger consumer surplus. While producers lose, society as a whole receives a net benefit from the abundant water.

These ex post results stem from the risk associated with future inflows to dams. Because a decision must be made ex ante, the investor does not know the future rainfall with certainty. The investor must make a decision on the best available information (incorporating risk), and then live with the consequences of the decision in terms of ex post realised returns. While the decision was optimal ex ante on an expected value basis, the ex post result can be quite different to what would have optimally been chosen had the future been known with certainty. The ex ante investment decision must reflect the possibility of all future rainfall states. The result is that there will be ex post situations where the realised cash flows from consumers are insufficient to recover costs (including investment costs) faced by suppliers. However, all of the outcomes resulting from any one investment decision will always, on average, yield a break even or positive return[3] (by the very nature of the optimisation solution to the model). For all loss making outcomes, there will be counterbalancing outcomes where profits are made by producers.

The variability of ex post returns highlights the importance of risk when making urban water augmentation decisions, and the importance of the modelling assumption that investors are risk neutral. This assumption means that optimal investment decisions will be made ex ante in the model even if, in reality, a private investor might avoid those investments because project risk associated with rainfall variability is not recognised through higher returns.[4] To encourage investment in risky projects, there might need to be compensatory returns above the risk free rate (represented in the model by the discount rate).

By imposing constraints on ex post outcomes, it would be possible to guarantee ex post returns to investment (using constraints in the model — see chapter 4 for an illustration). However, this will affect ex ante investment choices and prices to consumers, and there would be a loss in net social welfare to achieve this outcome.

---

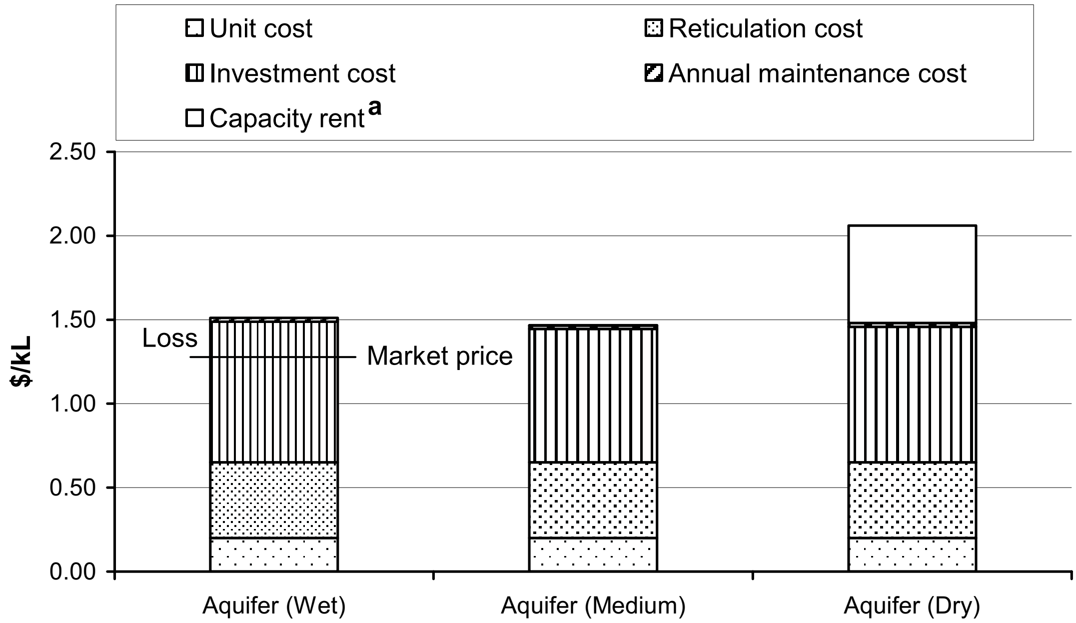
3 Ex ante, an investment will only have a return in excess of investment cost if it is expected to be capacity constrained, on average, after it is constructed.

4 The principles of corporate finance imply that no risk premium is required for unique, project-specific risk, where this is not correlated with market returns and can be diversified away (Brealey and Myers 1984). However, to the extent that investors cannot fully diversify their rainfall-related risks, they might require some risk premium when investing in new urban water supply capacity.

Variability in the returns to investments is part of investment under scarcity-based pricing, just as there can be variable returns to investments in many other markets. More detail on the economic principles underpinning investment timing and decision making in the model is contained in appendix C.

Figure 3.7 **Recovery of investment costs for aquifers over a 20 year simulation**

Disaggregation of end-use prices — expected values across all rainfall scenarios



[a] Pricing above the cost of supply is possible in some or all scenarios because of capacity constraints on the supply of water from each source. Pricing above short-run costs but below the level required to recoup all capital investment is also possible (as per the wet scenario).

Data source: Modelling results.

3.3 Sensitivity analysis

There are numerous parameters in this model that condition the results. Sensitivity analysis has been conducted to examine how results — prices, storage, and investment — are affected by changes to some of these parameters. Several sensitivities were undertaken, including inflow levels, demand elasticities, discount rates and growth rates.

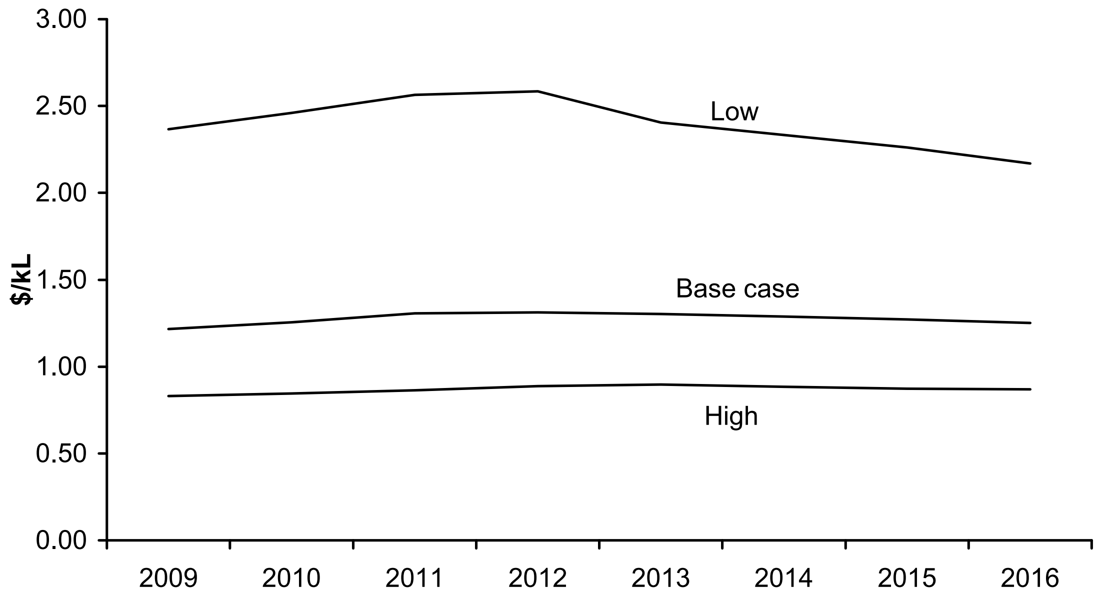
Inflow levels

Inflows to dams are one of the largest sources of water supply for urban water systems in Australia. Consequently, they have an important impact on the model results, in terms of prices, storage and investment. Climate change has the potential to further complicate any forward planning with respect to water supplies. Further, in a multistage stochastic programming framework, the stochastic elements (inflows) have an important impact on the results, and there is a large degree of uncertainty with respect to inflow parameters (appendix B). Considering the importance of inflows, the impact of a 30 per cent change in the mean level of inflows was examined.

A 30 per cent decrease in mean inflows can be interpreted as a dry climate change scenario, while a 30 per cent increase would represent a return to long-term historical averages in cities such as Melbourne and Perth (appendix B). Alternatively, these simulations can be used as a way to represent market outcomes where inflows are reserved for environmental purposes. A 30 per cent decrease in mean inflows thus can give some insight as to what might be required if policy makers were to reallocate 30 per cent of inflows as a contribution to environmental flows.

A reduction in inflows has a much greater impact on model results than an increase. A 30 per cent increase in inflows results in a roughly equivalent reduction in prices and increase in storages, with little impact on investment. Moreover, reducing mean inflows by 30 per cent results in mean storages that are 36 per cent lower, and mean prices that are 88 per cent higher (figure 3.8). This dramatic price rise, brought about by increased water scarcity, induces investment in desalination supply. This investment typically comes online in 2013, tempering the increase in prices. This compares with the base case where no investment in desalination is undertaken, as desalination does not provide a net expected benefit.

Figure 3.8 **Price impact of mean inflows**
Mean prices across probability tree



Data source: Modelling results.

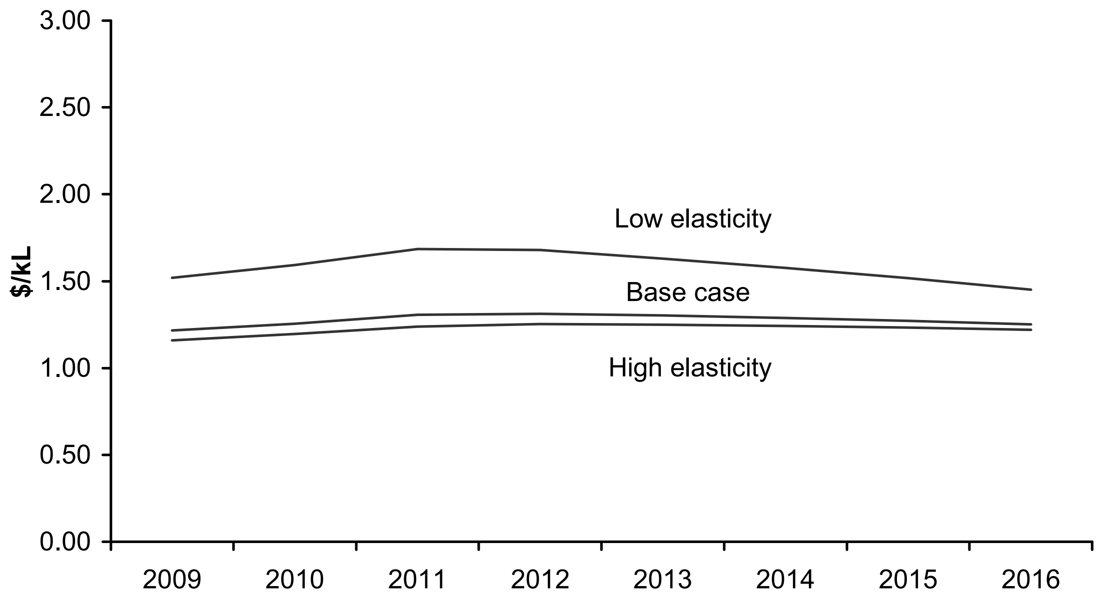
This asymmetry between increased and decreased inflows is also present in the welfare costs. With reduced inflows, the welfare loss (net present value over the next eight years) is $777 million relative to the base case. This is more than double the welfare gain from additional inflows of $338 million. This is because the consequences of running out of water are much more severe than having too much. Excess water can simply be stored or consumed, and once storages are full simply spills from catchments. Water shortages, however, result in large welfare losses in terms of forgone consumption.

Demand elasticities

Demand elasticities for urban water are not known with certainty (appendix B), however they have an important impact on market outcomes. The size of the changes in price needed to ration demand is directly tied to demand elasticities: if elasticities are high, then smaller changes in price have a larger impact on the quantity demanded.

The impact of changing elasticities by two thirds was examined. Lowering the elasticity by two thirds increases prices by 24 per cent on average, while increasing price responsiveness by two thirds only lowers prices by 4 per cent on average (figure 3.9). Prices are approximately twice as variable for low elasticities as for high elasticities. Beyond price, however, the demand elasticities do not have a large impact on other key results. Storages are similar to the base case under both sensitivities, and investment timing and choice are the same. Under the higher elasticity simulations, there is a slightly lower utilisation of rural–urban trade once the investment is made due to the lower price level.

Figure 3.9 **Price impact of demand elasticities**
Mean values



Data source: Modelling results.

Other sensitivity analysis

Further sensitivity analysis involved changes to:

- the distribution of inflows modelled
- the weight attached to low inflow scenarios
- the price and quantity point used for calibration of the demand function
- a different specification (constant elasticity) of the demand function
- growth rates of urban water consumption
- discount rates
- initial storage levels.

These additional simulations were performed to examine how responsive the simulation results were to changes in the assumptions underpinning the model. All parameters tested were changed within feasible real-world bounds (appendix D outlines the exact changes to the parameters). Changes in the distribution of inflows modelled, growth rates of consumption and discount rates have little impact on results. Prices, storages, investment, and the impact of various pricing and restrictions policies (discussed in chapter 4) were not significantly affected when these parameters were changed. The reason these changes had a relatively small impact on the simulation results was because they do not materially change the system's expected ability to supply water, nor the present level of water demand.

Changing to a constant elasticity demand function has only a small impact on results. Average prices and investment decrease slightly, but the maximum price reached is similar. The cost of restrictions is higher (by about 25 per cent) using a constant elasticity demand function. This is because the alternative demand specification has a larger proportionate loss in welfare resulting from low levels of consumption. When restrictions are imposed, outdoor water demand is constrained to a very low level, or in some nodes, drastic steps are taken to avoid imposing restrictions in future.

Increasing the weight attached to low inflow scenarios, increasing the demand quantity point used to calibrate the demand function, and reducing initial storage levels all have a similar effect to decreasing mean inflows. These changes exacerbate water scarcity, increasing prices and bringing forward investment. Inflows and storage provide a large proportion of water supply, and changing the expected value of future inflows (or demand for a given level of inflows) will change the ability of the supply system to meet demand at a given price. However, the magnitude of these effects was muted because each of these changes had a smaller impact on water scarcity than a 30 per cent decline in mean annual inflows.

Systematic reporting of the results from the sensitivity analysis is contained in appendix D.

4 Illustrative applications to policy

The purpose of developing the partial equilibrium model was to examine the welfare, pricing and investment implications of various policies. These are modelled as variations from the 'market' model set out in chapter 3. To quantify their impacts, policies were modelled as constraints on the market outcomes. The five policies modelled were:

- water restrictions
- long-run marginal cost pricing
- cost recovery pricing
- mandatory construction of a desalination plant
- a policy ban on rural–urban trade.

4.1 Description of the illustrative applications

As discussed in chapter 3, the market model can be described as a scarcity-based pricing model of demand and supply for urban water. Prices are allowed to adjust to bring about a market equilibrium that maximises the expected value of net social welfare (Marshallian consumer plus producer surplus).

If one of the above policies is binding, it distorts this market outcome and leads to a reduction in welfare compared with the market reference case. Impacts of different policies on pricing and investment decisions can be examined by constraining the market solution to the model (Pressman 1970). The cost of policy interventions can then be estimated by comparing welfare in the market model with that for the policy constrained model. Further, the partial equilibrium framework attaches a shadow price to every constraint imposed on the model (if it is binding), which provides information about marginal costs of binding policies.

Long-run marginal cost (LRMC) pricing

As noted in chapter 1, regulators in Australia typically use estimates of long-run marginal cost (LRMC) for price setting. LRMC pricing policies ensure that the price of water, at the margin, is equal to the next lowest cost source of additional supply, which tends to 'smooth' prices over time (relative to scarcity-based pricing). There are a variety of approaches used to estimate LRMC prices, with the most prominent being 'average incremental cost' and 'perturbation' methods. Each of these methods require capital expenditure forecasts for a suitable investment planning horizon, typically 20 to 25 years (ESC 2005).

LRMC pricing was approximated in the Commission's modelling as a 'smoothed' pricing policy that applied to prices paid by consumers (box 4.1). An obvious approach to modelling LRMC pricing would be to mimic the perturbation and average incremental cost methodologies used by regulators. However, this was not pursued because these methodologies require capital expenditure forecasts for 20 to 25 years, a timespan that is difficult to model in a multistage stochastic setting.[1] Instead, two somewhat less restrictive requirements were imposed to ensure that prices were smoothed over time.

First, LRMC was modelled as a uniform price to consumers that is set and reset every four years. This four-year period matches regulatory practices in most jurisdictions of Australia, where prices are generally set every 3–5 years (PC 2008).

Second, consumer prices are set in advance with full knowledge about water availability when they are set, but only probabilistic expectations about future inflows. While the regulated prices may differ from year to year, they are not allowed to vary to reflect different inflow states as they are revealed. Within the model, this means that the consumer price is set in advance[2] and is then binding on all subsequent nodes in the probability tree (figure 4.1). Within any regulatory period, all nodes in a given year that are after a price determination will have the same consumer price. At the beginning of the next regulatory period, prices are able to be reset for the remainder of the regulatory period.

1 In addition, endogeneity between pricing and capital expenditure makes it difficult to implement a constraint based on perturbation or average incremental cost methodologies

2 In practice, a constraint was applied to quantities, rather than prices, for the LRMC policy. This was done to suit the primal (quantity) formulation of the model. Fixed quantities imply fixed prices, as consumers use water according to a monotonic demand function.

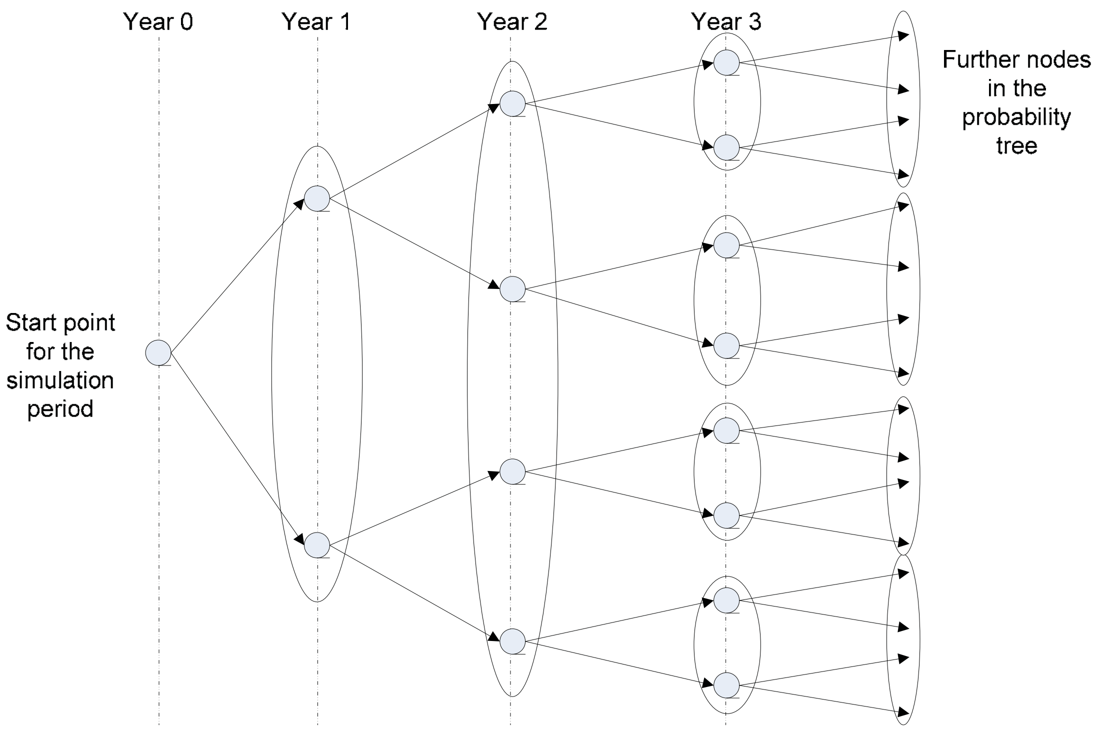
> **Box 4.1 Modelling LRMC as a constraint on consumer prices**
>
> In the modelling for this study, LRMC pricing was assumed to constrain only consumer prices. Investment decisions and supply are optimally determined, subject to the distortion in consumption induced by imposing uniform prices.
>
> This approach captured the key cost of a smoothed pricing regime: consumers do not face a higher price for water during times of scarcity and lower prices when there is abundance of supply. (The exception is at the start of a new regulatory period, when prices can adjust if investment is brought forward during drought or delayed after a wet spell.)
>
> This is consistent with a market equilibrium subject to subsidisation and taxation of consumers. They are 'subsidised' when the consumer price is less than the price of supply. They are 'taxed' when the consumer price exceeds the price of supply. On an expected value basis, the subsidies and taxes cancel out. Alternatively, a planner's problem can be shown to be equivalent to a competitive equilibrium (Hansen and Sargent 1990). As such, the LRMC pricing policy modelled could be interpreted as optimal investment and supply under planned supply decisions (with a goal of maximising net social welfare) subject to a distorted demand price. In either case, there will be a 'gap' between the price of supply and price of demand.
>
> To bring about equality between the uniform demand price and the supply price would involve distortion of investment decisions and additional costs compared with the implementation used here. For example, under LRMC prices a couple of dry years (within a regulatory period) might trigger investment in new, more expensive sources of supply to meet demand. This investment would be optimal as it delivers valuable scarce water. However, this would mean that supply costs increase and diverge from demand prices, as the latter were fixed in advance before rainfall patterns were known. If, instead, investment during times of scarcity is constrained so that supply costs remain equal to fixed demand prices under LRMC pricing, then this will add to the costs of LRMC pricing.

Figure 4.1 shows a representation of the LRMC pricing structure in the model, with the price determined every two years (for illustrative purposes — in the model prices are set every four years). In the first year, a year-1 price and a year-2 price are determined. All nodes in year 1 must have the same consumer price, and all nodes in year 2 must have the same price, although those two prices need not be the same. Regardless of the inflow state in years 1 and 2, the year-2 price is set and 'locked in' in year 1. In year 3, consumer prices are reset. All nodes after year 3 and in the same loop share a single price, determined in year 3.

The welfare impact of this approximation of LRMC-based pricing is a lower bound for two main reasons. First, as the approach used is, in effect, a smoothed scarcity price, the price will still be used to ration demand more than if it were simply set at the cost of the next cheapest form of supply. In the model, the only distortion caused by the policy results from lack of price flexibility. Second, the LRMC constraint is imposed only on the prices charged to consumers (box 4.1). To the extent that LRMC pricing by regulators is built up using an estimate of the incremental cost of new capacity, then used to determine a price for consumers *and* suppliers, this is likely to distort investment decisions, resulting in higher costs than estimated in the modelling.

LRMC pricing was also modelled in conjunction with restrictions. During extended dry periods, LRMC pricing alone is not able to adjust sufficiently to ration demand for scarce water. As such, restrictions are required, and the cost of restrictions are an important part of a pricing policy based on the LRMC of supply.

Figure 4.1 **Representation of LRMC in the model**[a]



[a] In this example, the regulatory period is two years, for ease of diagrammatic exposition. In the model, the regulatory period is four years.

Water restrictions

The use of water restrictions during times of scarcity is an important feature of present management of Australian urban water systems. These enforced restrictions limit the manner in which water may be consumed. For example, during 2009 medium level restrictions in Brisbane forbade: any watering of established lawns; any watering of established gardens between 8:00am and 4:00pm; watering of established gardens using a handheld hose except between 4:00 pm and 4:30pm on each Saturday and each Tuesday for odd numbered premises and between 4:00 pm and 4:30 pm on each Sunday and each Wednesday for even numbered houses (Queensland Water Commission 2009).

Water restrictions are modelled as a constraint on the maximum quantity of water that can be used outdoors. This means that water restrictions only apply to one of the three sources of demand that are modelled.

Water restrictions are triggered in the model when storages fall below a specified threshold level. This is achieved using binary variables: a variable that has a value of 1 when the restriction is triggered, and 0 when it is not.[3] Whether or not a restriction is implemented at a point in time depends on storage levels at the end of the preceding period. This modelling framework approximates stated government policies regarding restrictions (DSE 2007).

Estimates of the welfare impact of restrictions using the model are a lower bound on their costs. When a restriction is binding in the model, it limits the total amount of water that may be consumed in aggregate by all outdoor users — truncating the least valued outdoor uses of water. This aggregate cap means that there is no binding constraint on any single premises, provided that all premises collectively remain below the limit. In practice, a large part of the cost of restrictions comes from the fact that they bind all premises individually (not in aggregate) regardless of the value individual users may attach to the use of water relative to other users. Further, restrictions target certain uses of water (most notably, watering of gardens and lawns) that might not be the least-valued outdoor use of water for many consumers. Finally, only two levels of restrictions were modelled, excluding costs from restrictions that would occur under less severe water shortages (Appendix B).

Cost recovery pricing

An alternative to marginal cost pricing is to set prices at average historical cost. Such an approach — as used historically for price setting — ensures that the total cost of water provision is recovered through variable charges (Baumann, Boland and Hanemann 1998). Some researchers advocate cost recovery pricing to prevent water suppliers from making excess returns on lower-cost, but limited, sources of water, especially existing dams (for example, Dwyer 2005 criticises pricing that allows for returns on assets that have already been ‘paid for’). Instead of allowing the lower-cost source of water to accrue rents up to the cost of the most expensive

3 As noted in chapter 2, the use of binary variables increases markedly the computational requirements of a large model such as this one.

marginal source (to be paid the market price of water), cost recovery pricing averages all prices to ensure revenue and cost equality.

To represent cost recovery pricing, total revenue is constrained to equal total costs in every year, but at an aggregate level across all supply technologies used. Specifically, the sum of total revenue from sales of water to all forms of demand is constrained to be equal to the costs of all supply sources, ensuring that the price charged for water to end users is equal to the average cost of the system. This cost includes a required rate of return on existing assets (based on a weighted average cost of capital and a regulatory depreciation rate, both of which are applied to the total asset base). As for LRMC pricing, cost recovery pricing was modelled as a demand-side constraint. However, in aggregate, it does not provide for capacity rents on constrained sources of supply.

In reality, a cost recovery pricing policy would be likely to require the use of restrictions, but this was not modelled due to computational limitations. Like LRMC pricing, average cost pricing does not allow sufficient flexibility to use price to ration water during extended dry periods, meaning that restrictions are likely to be needed occasionally. However, modelling average cost pricing in conjunction with restrictions would require a primal–dual modelling approach[4] and/or much more complicated nonlinear constraints. This was not feasible given the large size of the multistage stochastic urban water model.

Mandatory desalination

In real urban water settings, decisions about supply options are not necessarily made based purely on the efficient costs of supply. For example, new dams might be ruled out due to concerns about environmental damages, without an investigation of specific options for new dam sites. Other options might be chosen or discarded based on political, rather than economic considerations.

Mandatory construction of a desalination plant was modelled to analyse the potential implications of 'one size fits all' investment strategies, where new supply sources that might be economically justified in one jurisdiction are applied elsewhere without reference to their costs and benefits relative to other available supply options. This was modelled by imposing a constraint that construction of a desalination plant must begin in the first year of the simulation.

4 Under water restrictions, restricted sources of demand are constrained to consume at a point that is not on their demand curve. This means that the calculation of revenue from restricted sources of demand requires access to the shadow price of restrictions, which is not available pre-solution in a primal (quantity) formulation.

This constraint was imposed only on the supply-side of the urban water market. There was no requirement that the costs of the desalination investment must be recovered through prices charged to consumers. If investment costs were recovered through higher prices to consumers, this would require an additional (demand-side) constraint and would inflate the costs of this 'one size fits all' investment strategy.

A ban on rural–urban trade

A ban on rural–urban trade was modelled in order to illustrate the potential costs of policy bans on particular sources of supply. Policy bans on some investment options can force sub-optimal reallocation decisions elsewhere in the system. If least-cost investments are not allowed, then either prices must adjust to ration limited water supplies, or the next least costly investment must be brought on line.

As the partial equilibrium model used is limited in scope to an urban water market, the effects of a ban on rural–urban trade only capture impacts within the urban system. However, the value of water to rural holders of seasonal licenses is captured through the cost of supply of rural water, which varies depending on whether it is a relatively dry or wet year.

4.2 Impact on net social welfare

Compared with scarcity-based pricing, the policy options modelled typically result in a reduction of net social welfare for the hypothetical city of tens to hundreds of millions of dollars over the timeframe modelled (table 4.1). These costs are measured in terms of an expected decrease in discounted net social welfare from the urban water market. The change in net social welfare will vary across different scenarios for inflows to dams, so an expected decrease is used to report a probability-weighted measure of likely costs. These costs must be traded off against price variability under scarcity-based pricing, which is discussed in further detail in chapter 3 and in section 4.3 below.

LRMC pricing

LRMC-based pricing carries costs associated with 'smoothing' prices over time. For the reasons discussed above, estimated costs of LRMC-based pricing are a lower bound estimate of the true costs. Further, during times of extended scarcity, LRMC pricing alone is not flexible enough to ration demand for water, so restrictions are required. As such, the cost estimate for LRMC and restrictions is a more relevant cost for a LRMC-based pricing policy.

Water restrictions

Restrictions impose large costs in forgone consumer surplus, relating to outdoor uses of water that end users would have been willing to pay for. These costs are measured on an expected value basis across a series of scenarios that, for most years, do not have restrictions imposed. This explains much of the difference between the costs estimated here and higher costs estimated for restrictions during an extended drought (Grafton and Ward 2007 and PC 2008). Further, as mentioned previously, the cost estimates here are a lower bound because they do not include additional impacts such as the differential effect of restrictions across households — in the real world, some households that are prepared to pay a lot for additional water might have to forgo consumption under restrictions.

The cost of restrictions is particularly high if they are imposed in conjunction with LRMC pricing. When restrictions on outdoor demand are imposed on a model with scarcity-based pricing, prices are still able to adjust upward during dry years, which reduces indoor and commercial demand. This flexibility means that scarce water can be rationed for most uses through prices instead of restrictions, and thus restrictions are only rarely binding. With restrictions and LRMC pricing in operation together, this pricing flexibility is lost. Restrictions are required during extended dry spells, imposing large costs on outdoor water users.

The cost of the status quo in most Australian jurisdictions — LRMC pricing and restrictions during times of scarcity — is equivalent to about $100 million per year for the hypothetical example modelled, relative to scarcity-based pricing. The hypothetical example is based on large capital cities in Australia (Sydney, Melbourne, Perth, Brisbane and Adelaide) which have an average of 900 000 households. The annual cost estimate is thus equal to approximately $110 per household, or more than 15 per cent of the typical household water bill in large cities ($658 — NWC 2009).

Cost recovery pricing

Cost recovery pricing imposes welfare costs that are higher than LRMC pricing. This is because pricing at average cost is a less efficient means of signalling the price of new sources of supply (Baumann, Boland and Hanemann 1998). Relatively cheap water from dams keep prices low under cost recovery pricing, even as more expensive sources of supply (such as rural–urban trade) are being pursued. Further, the constraint on pricing flexibility under cost recovery pricing means that restrictions are likely to be required during particularly dry years. As discussed above, the combination of cost recovery pricing and restrictions was not modelled due to computational limitations, but allowing for restrictions would increase welfare losses associated with cost recovery pricing.

Table 4.1 **Welfare costs of various policies (expected net present values)**
Probability-weighted average costs relative to scarcity-based pricing with no restrictions, for the next eight years

Policy	*Welfare cost ($m)*
LRMC	94
Cost recovery	153
Restrictions	522
Restrictions and LRMC	658
Mandatory desalination	311
Rural–urban trade ban	69

Source: Modelling results.

Mandatory desalination

Where they are binding on an efficient market, investment mandates distort investment decisions. Mandating the construction of a desalination plant forces a costly investment on the system, resulting in a net present value welfare cost of $311 million. Construction and maintenance costs for the plant total $355 million during the simulation period, with only minor offsetting benefits elsewhere in the urban water system. There is a net transfer of wealth from water suppliers to consumers, who benefit as a consequence of the additional water supply available (and ensuing lower prices — section 4.3). However, consumer benefits are more than outweighed by additional costs to water suppliers.

There may be reasons why a water supply planner might still want to invest in a desalination plant. For example, they might do so as insurance against running out of water in a drier scenario than those modelled. These benefits need to be weighed against the costs. Further, it needs to be demonstrated that a particular augmentation option is the best way to obtain those benefits. In the hypothetical example modelled, aquifers and rural–urban trade are cheaper means to secure more water.

The costs of a mandatory desalination plant would be higher if it were also forced to operate at full capacity from the time of commissioning. Without such constraints, the desalination plant is only used when it is able to recover short-run variable costs associated with its operation.[5] Under the mandatory desalination simulation, average capacity utilisation over all possible outcomes for rainfall is less than

5 Significant annual maintenance costs are still incurred when the desalination plant is not operated (for details of data used for calibration, see appendix B).

75 per cent. This indicates that, when desalination is chosen as a means to insure against extreme low probability drought events, there are benefits from maintaining flexibility about when the desalination plant is actually operated.

On the other hand, when a desalination plant is constructed based on its expected benefits outweighing its costs, flexibility in plant operation does not appear to be so important. Under a scenario where average inflows to dams are 30 per cent lower, modelling results suggest that, in the hypothetical example modelled, desalination is sometimes a worthwhile investment that will recover its costs on an expected value basis (see sensitivity analysis results, appendix D). In this case, lower average inflows to dams mean that water supplies are lower, increasing the demand for water supplied from desalination to the point that average capacity utilisation of desalination (when constructed) is about 96 per cent.

A ban on rural–urban trade

In the Commission's modelling, prohibiting rural–urban trade reduces discounted, probability-weighted welfare (over the following eight years) by $69 million. This cost is incurred by consumers, as higher prices are needed to ration the more limited supply of water in the absence of the supplies drawn from rural–urban interconnection in the base case. A ban on rural–urban trade effects a transfer of wealth from consumers to suppliers, as prices are pushed higher (section 4.3). However, the increase in producer surplus is more than offset by the loss in consumer surplus.

This cost estimate does not include externalities (positive and negative) from rural–urban trade that accrue outside the urban water system. Rural–urban trade provides water in the model through the purchase of seasonal allocations, so rural water users are compensated for the direct cost of the water. However, there might be other social externalities — such as impacts on rural communities — that are not included in the analysis.

Sensitivity analysis: mean inflows and demand elasticities

Welfare costs policies that restrict supply or reduce price flexibility are significantly higher if inflows to dams are lower, or if demand is less price responsive (table 4.2). When inflows are lower, the costs of the alternatives to using prices to ration water are higher. Also, restrictions are required more often, imposing higher costs.

Low demand elasticities are often cited as a reason to avoid scarcity-based pricing, but if demand for water is highly inelastic, then this significantly increases the costs of relying on water restrictions in times of drought. Certainly, as demonstrated in chapter 3, prices will be more variable under scarcity-based pricing if demand elasticities are low, as greater price changes are required to bring about the same reduction in the quantity of water demanded. However, low demand elasticities imply that users are prepared to pay significantly more rather than reduce their consumption of water — that is, they place a high value on their existing uses of water. Restrictions proscribe some of these highly valued uses of water, imposing far higher costs if demand elasticities are lower (table 4.2).

Further detail on the results from sensitivity analysis is available in appendix D.

Table 4.2 **Sensitivity of welfare costs (expected net present values)**

Probability-weighted average costs ($m) relative to scarcity-based pricing, for the next eight years

Policy	*Central estimate*	*Mean inflows*		*Demand elasticity*	
		Low (-30%)	High (+30%)	Low (-0.10)	High (-0.50)
LRMC	94	241	60	149	117
Cost recovery	153	339	137	225	134
Restrictions	522	673	267	1013	401
Restrictions and LRMC	658	1026	599	1573	548

Source: Modelling results.

4.3 Pricing

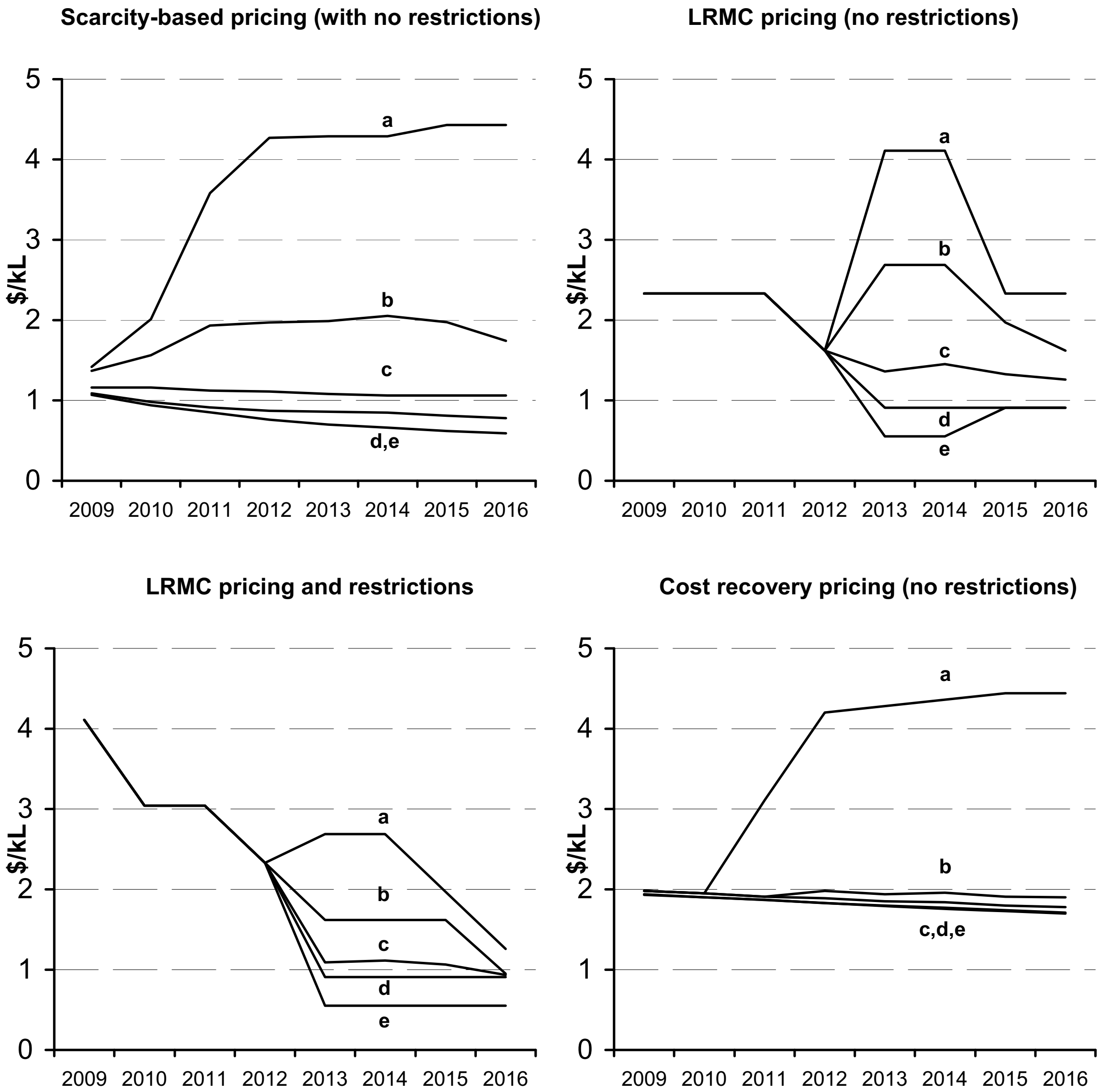
The model can be used to give an indication of pricing behaviour under various policies. However, the change in the price of water will not equal the change in the overall bill that households are likely to receive. This is because fixed access charges (under a two-part tariff) are not included in the modelling undertaken for this study. Fixed access charges could include an amount to cover fixed or common costs of water provision that are unrelated to the specific quantity of water supplied (for example, corporate overheads) and might vary across different pricing policies.

Prices vary with inflows under all the pricing and restrictions policies modelled, but scarcity-based pricing is associated with the greatest variation in prices (figure 4.2). As explained in chapter 2, consumers in the model are risk averse with respect to consumption of water, so variations in price reduce welfare (relative to a price that is fixed at the mean of the variable prices).

Modelled prices under pricing constraints — in particular, LRMC pricing — might vary more than in real-world applications of these policies. To the extent that prices are ‘smoothed’ to a greater extent than in the modelling, this will impose greater efficiency costs than reported above. The price distributions reported in figure 4.2 are the most efficient (that is, they maximise net social welfare) given the constraints imposed. For example, in the case of LRMC pricing, this means these prices are optimal given that prices are set every four years and cannot be adjusted during a regulatory period.

Figure 4.2 **Prices under alternative pricing options**

Across all rainfall scenarios modelled



[a] Maximum price. [b] Ninetieth percentile price. [c] Median price. [d] Tenth percentile price. [e] Minimum price.

Data source: Modelling results.

Relatively high prices ($4 per kilolitre or higher) are still possible under LRMC pricing. This occurs under a dry scenario in later years of the simulation when LRMC is modelled without restrictions, as water shortages at the start of the second regulatory period require high prices until sufficient additional supply capacity (mostly from household tanks) is available. When LRMC pricing is modelled together with restrictions, high prices are used when storages are relatively low at the start of a regulatory period, in order to avoid costly restrictions later in the regulatory period. If, in reality, LRMC pricing does not offer this degree of flexibility, then restrictions are likely to be imposed more frequently and the welfare costs of LRMC and restrictions will be higher.

Using a less flexible representation of LRMC pricing can be shown to carry welfare costs as much as three times those of the lower-bound estimate reported in section 4.2. A pricing regime based on the levelised cost of the marginal investment option (rural–urban trade, then desalination once the rural–urban pipeline has been built) was modelled as part of sensitivity analysis. Pricing under this regime is more stable, but welfare costs are accordingly much higher (appendix D). In general, if pricing rules change from those in figure 4.2 — while still requiring that regulated prices are set every four years and cannot be changed during this time — the welfare costs will increase.

Prices are most stable under cost recovery pricing, but can increase considerably towards the end of the simulation period under a particularly dry scenario. When inflows are low for a prolonged period, investment in more expensive sources of supply is required to meet demand, increasing total costs. Higher prices are then required to continue to maintain revenue equal to these costs.[6] Further, these higher prices will be 'locked in' long into the future under cost recovery pricing, as prices must remain higher to reflect the increased asset base. This is true even if significant inflows mean that the cost of supplying water (including opportunity cost) becomes significantly lower than the cost recovery price.

On average, prices to consumers are actually *lowest* under scarcity-based pricing. The probability-weighted average price (across the whole probability tree) under scarcity-based pricing is about $1.30 per kilolitre, compared with over $1.75 per kilolitre for the alternative pricing scenarios (prices based on the levelised cost of the marginal investment, modelled as part of the sensitivity analysis, are higher again — see appendix D). Although scarcity-based prices are high during extended dry spells, prices are free to adjust downwards as soon as good inflows are

6 As demand for water is inelastic, price increases will be larger (in percentage terms) than the corresponding decrease in quantity demanded, so total revenue will increase with a price increase.

recorded. Also, under scarcity-based pricing, low prices can be offered in times when water is plentiful, safe in the knowledge that if shortages develop in the future then prices can be increased accordingly. As a consequence, median prices under scarcity-based pricing remain below $1.50 per kilolitre throughout the simulation.

Lower average prices under scarcity-based prices suggest that equity concerns — often raised in regard to using prices to ration water — are likely to be overestimated. In any case, there are also other ways to alleviate equity issues from more variable pricing. For example, it might be possible to provide a fixed quantum of low-priced 'essential' water to all households (PC 2008). This would mean that during extended dry periods, when scarcity-based prices would increase, all households would still be able to access sufficient water for essential uses. Equity concerns can also be addressed outside the water market — for example, through the tax–transfer system — in ways that do not distort prices for water and thus the consumption and investment decisions of households and businesses.

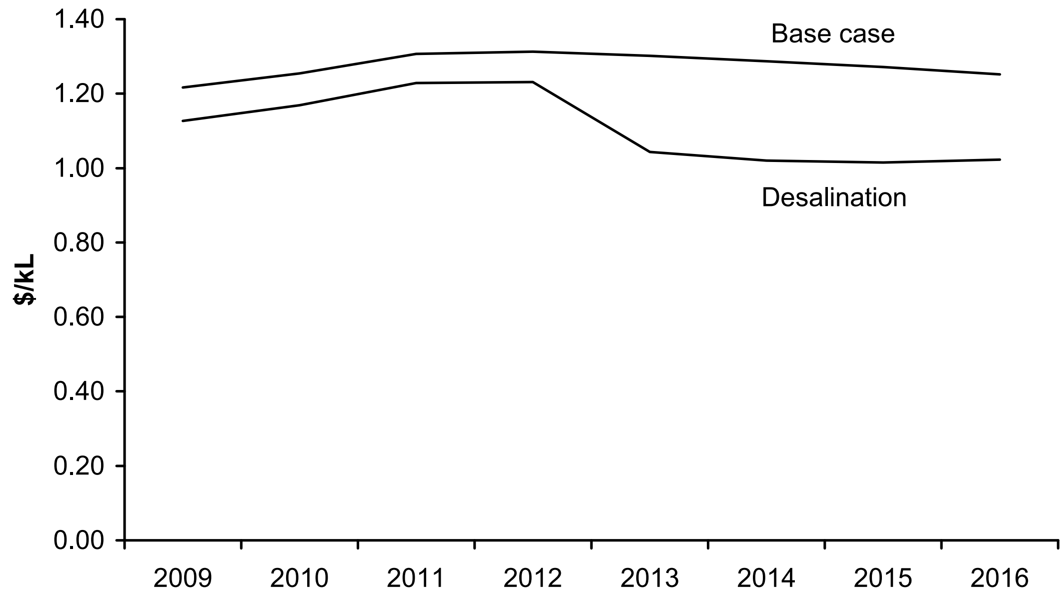
Investment constraints and pricing

Investment mandates and bans have opposite effects on prices: mandating an investment in desalination lowers prices, while prices are higher when supply from rural–urban trade is not available.

Investment in desalination at the start of the simulation period lowers prices by increasing the supply of water. This results in prices that are, on average, 12 per cent lower (figure 4.3), and 13 per cent less variable than the base case. Prices are significantly lower once the desalination plant is completed and able to supply water (after 2012). Prices are also lower before the desalination capacity is available, as less storage is required with the knowledge that desalination will be available to supply water into the future. The reduction in variability comes about because the certain supply of water from desalination reduces the need for rationing in dry periods. However, these lower prices come at the expense of community welfare (as discussed in the previous section), as significant construction and maintenance costs must be incurred to install and maintain the desalination plant.

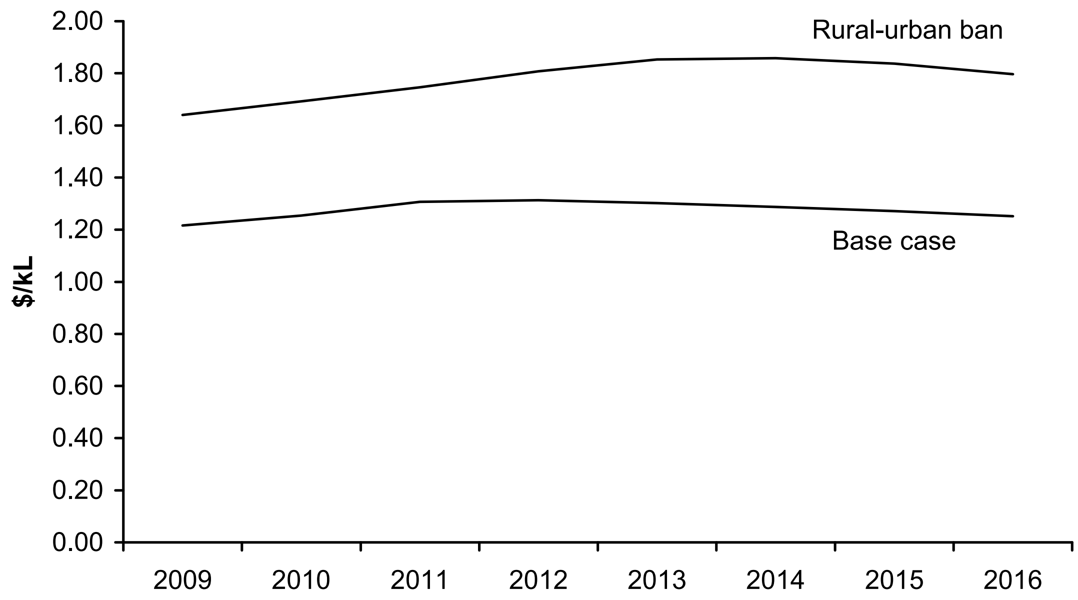
When rural–urban trade is not allowed, prices adjust upward to ration a smaller supply of water available to urban users. On average, prices are 45 per cent higher (and twice as variable) across all rainfall scenarios when rural–urban trade is banned (figure 4.4).

Figure 4.3 **Price impact of mandatory desalination construction**
Mean values



Data source: Modelling results.

Figure 4.4 **Price impact of a prohibition on rural–urban trade**
Mean values



Data source: Modelling results.

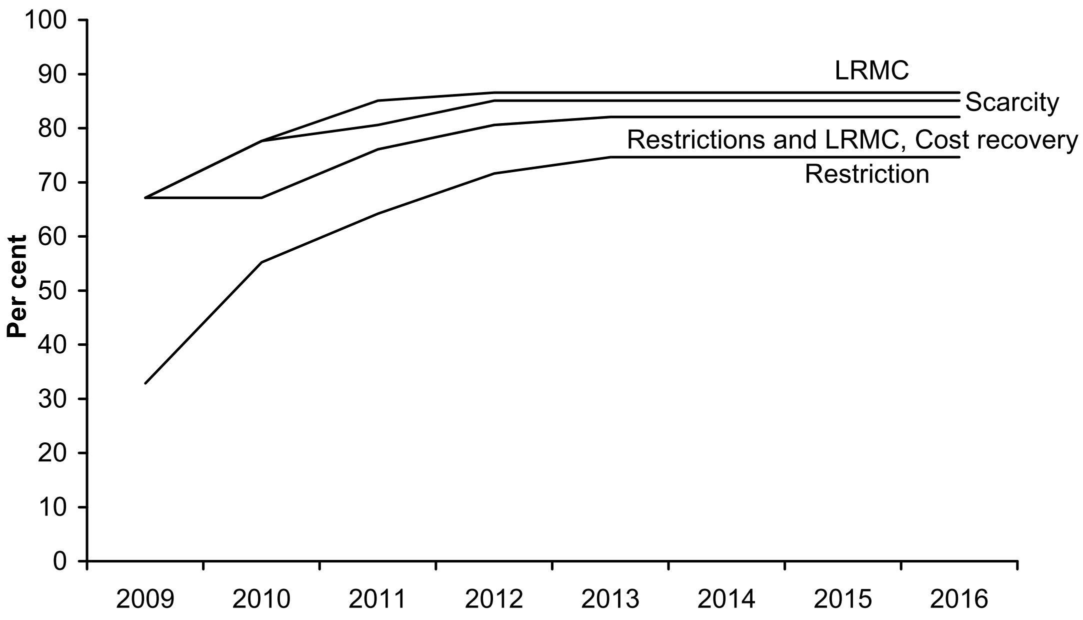
4.4 Investment in new sources of supply

The timing and choice of investments is distorted under different illustrative policies. Although pricing policies are only imposed on consumer demand in the modelling — investment decisions remain optimal — the distortion of demand decisions will flow through to affect investment decisions.

Relative to scarcity-based pricing, more investment is typically undertaken under LRMC pricing, but investment is reduced when restrictions are introduced as well. The decisions to build a pipeline to facilitate rural–urban trade is modelled as a binary 'yes or no' decision. A pipeline is built under some, but not all, scenarios for inflows to dams. A pipeline is built more often under LRMC pricing and less often under LRMC pricing and restrictions (figure 4.5). When restrictions are used in conjunction with LRMC pricing, outdoor demand is constrained during dry periods, reducing the need for new investments. There is also an interaction between high prices to consumers and investment: higher prices when restrictions are used in conjunction with LRMC pricing reduce demand for water and thus the need for new investments.

Figure 4.5 **Timing of investment in rural-urban trade**

Proportion of rainfall scenarios where investment is made

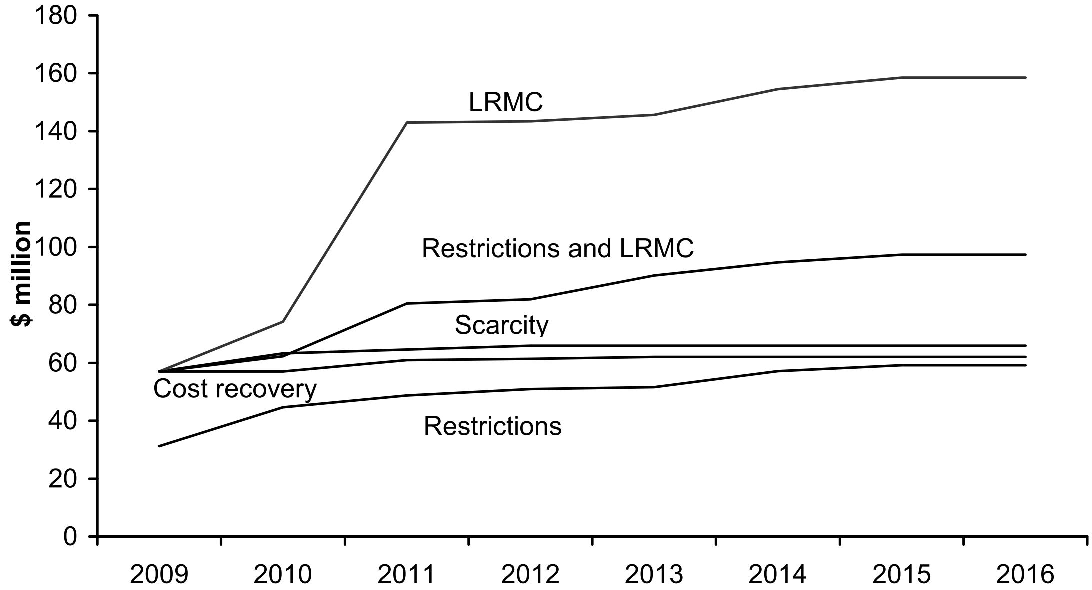


Data source: Modelling results.

Under particularly dry scenarios, investment under LRMC pricing extends to installing household tanks. Without restrictions or scarcity prices to ration demand, more investment is needed to meet demand under LRMC pricing. Household tanks can be installed and commissioned over much shorter timeframes than larger options (such as desalination) so they are a useful way to augment supply towards the end of regulatory periods, when prices are not able to adjust in response to dry conditions. Tanks carry large investment costs per unit of water provided, so there are significant additional investment costs under LRMC pricing (figure 4.6). However, these additional investment costs are still preferable to resorting to restrictions, as evidenced by the net social welfare reported in section 4.2.

Figure 4.6 **Net present value of cumulative investment costs[a]**

Adjusted for the length of the simulation period



[a] Includes the total cost of investment in all new supply sources: aquifers, rural–urban trade and household tanks (new dams and desalination are not pursued in any of the illustrative policy applications).

Data source: Modelling results.

In all the pricing and restriction policies modelled, investment occurs when the expected benefits from additional supply outweigh the costs. As under scarcity-based pricing (chapter 3), this means that, ex ante, investment occurs when the ratio of benefits to costs is greater than one. Thus, investment costs are recovered, on an expected value basis, whenever an investment is made. However, realised benefits from investment in new supply options vary with actual inflows. As under scarcity-based pricing, returns from investments in new sources of supply may or may not turn out to cover their costs, ex post. This issue is difficult to investigate thoroughly in a model as large as the multistage stochastic model used for this study — particularly when policy constraints involve binary variables — and remains an area for further work (chapter 5).

Investment constraints and investment in other supply options

Investment mandates and bans affect investment patterns directly, but can also have indirect effects on the need for alternative forms of investment. For example, mandatory construction of a desalination plant reduces the call on other new sources of supply, particularly rural–urban trade. Across all rainfall scenarios, forcing the construction of a desalination plant reduces the average amount of water drawn from rural–urban trade by 48 per cent (mainly by delaying the construction of the rural–urban pipeline).

5 Opportunities for further work

Presented in this paper is a partial equilibrium model of a hypothetical urban water system, representative of a large urban centre in Australia. The Takayama and Judge (1971) spatial and temporal price equilibrium approach has been extended by embedding it in a multistage stochastic programming framework to incorporate temporal variability in water supplies to dams. This method allows for quantification of the effects of various policy options by imposing constraints on a market model, which had not been possible under previous modelling of urban water systems.

The method adopted in this paper was selected because it can be explicitly used to quantify the benefits and costs of policies relating to pricing and supply augmentation. It provides insights about policy that are not directly available from other models of urban water systems.

Notwithstanding the strengths and desirable aspects of the approach presented in this paper, there are a number of areas that remain for further work. These include:

- examining ex post investment outcomes across different policies
- investigating the scope to model risk aversion with respect to the possibility of running out of water after a series of exceptionally dry years
- investigating the scope to model long-run marginal cost pricing policies (from the supply-side) that better represent the actual practices of regulators
- the application of this modelling framework to actual urban water systems.

As demonstrated in chapter 3, returns to investments are likely to vary according to realised rainfall patterns. This is likely to be true irrespective of the pricing and restrictions policies in place. Comparison of the variation in ex post investment returns across different policies would be useful, but computational constraints have made this difficult in the large scale model used for this study. Being a partial equilibrium model based on the quantity formulation, this requires processing of large amounts of information post-solution. This issue could be pursued using a smaller model to draw out and demonstrate the economic theory in the model.

A key reason for investing in new supply sources is often to avoid very large costs from running out of water under extreme low probability drought events.

Policymakers, suppliers and households are likely to be risk averse with respect to the possibility of taps 'running dry'. The modelling presented in this study does incorporate some degree of risk aversion, but there is much uncertainty about how consumers would respond to very high prices (or, conversely, to very low quantities of water delivered to consumers). Further, exceptionally dry, single years were not modelled (although, as discussed in appendix B, record low inflows for periods of four to five years were modelled). For these reasons, investment decisions that are made to avoid a very low risk of running out of water cannot be analysed well using the model presented in this paper. Instead, the framework is more suited to examining the choice between different technologies for augmenting supply, and the use of these technologies once they are built. Further work in this area would face data and computational challenges: data issues regarding consumer responses to prices higher than those that have been experienced historically; and computational challenges from increasing the size of the probability tree to explicitly model extreme low probability dry years.

The approach used to approximate the pricing policy generally used by regulators in Australia — based on long-run marginal costs — is a lower-bound estimate of the actual costs of such a policy. The approach only required that consumer prices be set once every four years and not be changed in response to water availability during the regulatory period. In practice, long-run marginal cost pricing by regulators is built up from the supply side using an estimate of the incremental cost of new capacity. This is then used to determine a price for consumers *and* suppliers, which is likely to be significantly more constraining on both consumption and supply decisions by comparison with the approach adopted in this paper. A more accurate approximation of long-run marginal cost pricing would allow for estimation of the omitted supply-distorting costs from this pricing approach (relative to scarcity-based pricing).

Finally, and by design, the conclusions do not apply to any particular jurisdiction, so using the results as a template for urban water supply in any jurisdiction should be avoided. If desired and where data are available, it would be possible to apply the model to specific urban settings.

Notwithstanding these areas for further work, the modelling framework developed in this paper offers insights into urban water policy issues in Australia that were not directly available from other models of urban water systems. Nevertheless, no single model can provide insights into all issues and the approach presented here should be considered complementary to the other models used to analyse urban water systems.

A Mathematics of the model

A complete mathematical specification of the urban water model is presented in this appendix. All variables in the model are identified with names that start with lower case letters. All parameters are identified with names that start with an upper case letter (see tables A.2 and A.3).

All variables in the model are positive (i.e. greater than or equal to zero). Three binary variables (cumpipe, vRestr and vRestrTerm) must take on a value of 0 or 1.

Equations for the core market model, representing scarcity-based pricing are presented first (A.1). Policy interventions are modelled by constraining the core market model according to the equations listed in section A.2.

A.1 Core market model

Objective function

$$\text{Max } NW = \tag{A.1}$$

Objective function: area under the linearised demand functions less reticulation costs

$$\sum_{d} \sum_{yrpt(yr,pt)} \sum_{l} Df_{yr}.Prob_{pt}.AreaQd_{d,yrpt(yr,pt),l}.wQd_{d,yrpt,l}$$

$$- \sum_{yrpt(yr,pt)} Df_{yr}.Prob_{pt}.Utcdam.\begin{Bmatrix} qsdesal_{yrpt(yr,pt)} + qsdam_{yrpt(yr,pt)} + qsdam2_{yrpt(yr,pt)} \\ +qspipe_{yrpt(yr,pt)} + qsaqui_{yrpt(yr,pt)} \end{Bmatrix}$$

plus linearised benefit from storage in the terminal period

$$+ \sum_{yrpt(lastyr,lastpt)} \sum_{m} \frac{Df_{yr}}{1+Discount}.Prob_{pt}.AreaSt_{lastpt,m}.wQstdamf_{lastpt,m}$$

less total cost of water supply from desalination

$$- \sum_{yrpt(yr,pt)} Df_{yr}.Prob_{pt}.Vcdesal.qsdesal_{yrpt(yr,pt)}$$

$$- \sum_{yrpt(yr,pt)} \sum_{ids} Df_{yr}.Prob_{pt}.TrDesalInvC_{ids,yr}.\left\{cumdesal_{ids,yrpt(yr,pt)} - cumdesal_{ids,yrpt(yr-1,ptp)}\right\}$$

$$- \sum_{yrpt(yr,pt)} \sum_{ids} Df_{yr}.Prob_{pt}.OperDesal_{ids}.YrIndex_{yr}.cumdesal_{ids,yrpt(yr,pt)}$$

less total supply cost from pre-existing and new dam investments

$$- \sum_{yrpt(yr,pt)} Df_{yr}.Prob_{pt}.Vcdam.qsdam_{yrpt(yr,pt)}$$

$$- \sum_{yrpt(yr,pt)} Df_{yr}.Prob_{pt}.Vcdam2.qsdam2_{yrpt(yr,pt)}$$

$$- \sum_{yrpt(yr,pt)} \sum_{idm} Df_{yr}.Prob_{pt}.TrDamInvC_{idm,yr}.\left\{cumdam_{idm,yrpt(yr,pt)} - cumdam_{idm,yrpt(yr-1,ptp)}\right\}$$

$$- \sum_{yrpt(yr,pt)} \sum_{idm} Df_{yr}.Prob_{pt}.OperDam_{idm}.YrIndex_{yr}.cumdam_{idm,yrpt(yr,pt)}$$

$$-\sum_{yr} OperSystem$$

less total supply cost of piping water from rural to urban regions

$$- \sum_{yrpt(yr,pt)} Df_{yr}.Prob_{pt}.Vcpipe_{pt}.qspipe_{yrpt(yr,pt)}$$

$$- \sum_{yrpt(yr,pt)} \sum_{iru} Df_{yr}.Prob_{pt}.TrPipeInvC_{iru,yr}.\left\{cumpipe_{iru,yrpt(yr,pt)} - cumpipe_{iru,yrpt(yr-1,ptp)}\right\}$$

$$- \sum_{yrpt(yr,pt)} \sum_{iru} Df_{yr}.Prob_{pt}.OperPipe_{iru}.YrIndex_{yr}.cumpipe_{iru,yrpt(yr,pt)}$$

less total supply costs of household tank water supply

$$- \sum_{yrpt(yr,pt)} Df_{yr}.Prob_{pt}.Vctank.qstank_{yrpt(yr,pt)}$$

$$- \sum_{yrpt(yr,pt)} \sum_{itk} Df_{yr}.Prob_{pt}.TrTankInvC_{itk,yr}.\left\{cumtank_{itk,yrpt(yr,pt)} - cumtank_{itk,yrpt(yr-1,ptp)}\right\}$$

$$- \sum_{yrpt(yr,pt)} \sum_{ids} Df_{yr}.Prob_{pt}.OperTank_{itk}.YrIndex_{yr}.cumtank_{itk,yrpt(yr,pt)}$$

less total cost of water supplied from aquifers

$$- \sum_{yrpt(yr,pt)} Df_{yr}.Prob_{pt}.Vcaqui.qsaqui_{yrpt(yr,pt)}$$
$$- \sum_{yrpt(yr,pt)} \sum_{iaq} Df_{yr}.Prob_{pt}.TrAquiInvC_{iaq,yr}.\left\{cumaqui_{iaq,yrpt(yr,pt)} - cumaqui_{iaq,yrpt(yr-1,ptp)}\right\}$$
$$- \sum_{yrpt(yr,pt)} \sum_{iaq} Df_{yr}.Prob_{pt}.OperAqui_{iaq}.YrIndex_{yr}.cumaqui_{iaq,yrpt(yr,pt)}$$

Dam constraints

Maximum dam storage

$$qstdam_{yrpt(yr,pt)} - \sum_{idm} DamStCap_{idm}.cumdam_{idm,yrpt(yr-damlag,ptp)} \le SwMaxS0_{yr} \qquad (A.2)$$

Catchment supply

$$\begin{aligned} & qsdam_{yrpt(yr,pt)} + qstdam_{yrpt(yr,pt)} - qstdam_{yrpt(yr-1,ptp)} \le \\ & Inflow\_state_{pt}.\left\{SwInflows_{yr} - SwEnvFlows_{yr} - SwLossFlows_{yr}\right\} + S0\big|_{firstyr} \end{aligned} \qquad (A.3)$$

Water demand balance

$$\begin{aligned} & \sum_{d} \sum_{l} QtyQd_{d,yrpt(yr,pt),l}.wQd_{d,yrpt(yr,pt),l} - qsdam_{yrpt(yr,pt)} - qsdam2_{yrpt(yr,pt)} \\ & -qsdesal_{yrpt(yr,pt)} - qspipe_{yrpt(yr,pt)} - qstank_{yrpt(yr,pt)} - qsaqui_{yrpt(yr,pt)} \le 0 \end{aligned} \qquad (A.4)$$

Desalination constraints

Maximum desalination supply

$$qsdesal_{yrpt(yr,pt)} - \sum_{ids} SwDesalICap_{ids,yr}.cumdesal_{ids,yrpt(yr-desallag,ptp)} \le 0 \qquad (A.5)$$

Upper bound on installed desalination capacity

$$cumdesal_{ids,yrpt(yr,pt)} \le 1 \qquad (A.6)$$

Cumulative desalination investment

$$cumdesal_{ids,yrpt(yr-1,ptp)} - cumdesal_{ids,yrpt(yr,pt)} \leq 0 \quad \text{(A.7)}$$

New dam constraints

Maximum supply for new dams

$$qsdam_{yrpt(yr,pt)} - \sum_{idm} SwDamICap_{idm,yr}.cumdam_{idm,yrpt(yr-damlag,ptp)} \leq 0 \quad \text{(A.8)}$$

Upper bound new dam capacity

$$cumdam_{idm,yrpt(yr,pt)} \leq 1 \quad \text{(A.9)}$$

Cumulative new dam investment

$$cumdam_{idm,yrpt(yr-1,ptp)} - cumdam_{idm,yrpt(yr,pt)} \leq 0 \quad \text{(A.10)}$$

Rural–urban trade pipeline constraints

Rural–urban trade pipeline supply

$$qspipe_{yrpt(yr,pt)} - \sum_{iru} SwPipeICap_{iru,yr}.cumpipe_{iru,yrpt(yr-pipelag,ptp)} \leq 0 \quad \text{(A.11)}$$

Upper bound rural–urban trade pipeline capacity

$$cumpipe_{iru,yrpt(yr,pt)} \leq 1 \quad \text{(A.12)}$$

Cumulative rural–urban trade pipeline investment

$$cumpipe_{iru,yrpt(yr-1,ptp)} - cumpipe_{iru,yrpt(yr,pt)} \leq 0 \quad \text{(A.13)}$$

Household tank constraints

Tank supply

$$qstank_{yrpt(yr,pt)} - \sum_{itk} SwTankICap_{iru,yr} . \frac{Inflow\_state_{pt} + 1}{2} .cumtank_{itk,yrpt(yr-tanklag,ptp)} \leq 0 \quad \text{(A.14)}$$

Cumulative tank investment

$$cumtank_{itk,yrpt(yr-1,ptp)} - cumtank_{itk,yrpt(yr,pt)} \leq 0 \quad \text{(A.15)}$$

Aquifer constraints

Aquifer supply

$$qsaqui_{yrpt(yr,pt)} - \sum_{iaq} SwAquiICap_{iaq,yr} .cumaqui_{iaq,yrpt(yr-aquilag,ptp)} \leq 0 \quad \text{(A.16)}$$

Upper bound aquifer capacity

$$cumaqui_{iaq,yrpt(yr,pt)} \leq 1 \quad \text{(A.17)}$$

Cumulative aquifer investment

$$cumaqui_{iaq,yrpt(yr-1,ptp)} - cumaqui_{iaq,yrpt(yr,pt)} \leq 0 \quad \text{(A.18)}$$

Constraints related to linearisation

Linearised demand for water

$$\sum_{l} wQd_{d,yrpt(yr,pt),l} = 1 \quad \text{(A.19)}$$

Linearised benefit function for water stored in the terminal period

$$\sum_{m} wQstdamf_{lastpt,m} = 1 \quad \text{(A.20)}$$

Demand for storing water in the terminal period

$$qstdam_{yrpt(lastyr,lastpt)} - \sum_{m} QtyQstdamf_{lastpt,m}.wQstdamf_{pt,m} = 0 \qquad \text{(A.21)}$$

A.2 Policy constraints and variables

Cost recovery (without restrictions)

The cost recovery constraints impose a revenue constraint on suppliers of urban water in each time period (for a particular node). This sets total revenue equal to the total cost of supplying water (including investment costs). An additional constraint is imposed to ensure a uniform price for all users of water (within a year and node), which amounts to average cost pricing.

Defining total annualised costs of supply (setting the revenue requirement)

$$
\begin{aligned}
& costs_{yrpt(yr,pt)} = \left(Vcdam + Utcdam\right).qsdam_{yrpt(yr,pt)} \qquad \text{(A.22)} \\
& + \left(Vcdesal + Utcdam\right).qsdesal_{yrpt(yr,pt)} \\
& + \left(Vcdam2 + Utcdam\right).qsdam2_{yrpt(yr,pt)} + \left(Vcpipe_{pt} + Utcdam\right).qspipe_{yrpt(yr,pt)} \\
& + Vctank.qstank_{yrpt(yr,pt)} + \left(Vcaqui + Utcdam\right).qsaqui_{yrpt(yr,pt)} \\
& + \left(WACC + \left[Depr^{YrIndex_{yr}}\right]\right).assets_{yrpt(yr,pt)} + OperSystem.YrIndex_{yr} \\
& + \sum_{ids} OperDesal_{ids}.YrIndex_{yr}.cumdesal_{ids,yrpt(yr,pt)} \\
& + \sum_{idm} OperDam_{idm}.YrIndex_{yr}.cumdam_{idm,yrpt(yr,pt)} \\
& + \sum_{iru} OperPipe_{iru}.YrIndex_{yr}.cumpipe_{iru,yrpt(yr,pt)} \\
& + \sum_{itk} OperTank_{itk}.YrIndex_{yr}.cumtank_{itk,yrpt(yr,pt)} \\
& + \sum_{iaq} OperAqui_{iaq}.YrIndex_{yr}.cumaqui_{iaq,yrpt(yr,pt)}
\end{aligned}
$$

Written down value of assets in service using a declining balance method

$$assets_{yrpt(yr,pt)} = \left[(1-Depr)^{YrIndex_{yr}} \right].assets_{yrpt(yr-1,ptp)} + Capstock0\big|_{yr=firstyr} \tag{A.23}$$
$$+\sum_{ids} TrDesalInvC_{ids,yr}.\left\{ cumdesal_{ids,yrpt(yr,pt)} - cumdesal_{ids,yrpt(yr-1,ptp)} \right\}$$
$$+\sum_{idm} TrDamInvC_{idm,yr}.\left\{ cumdam_{idm,yrpt(yr,pt)} - cumdam_{idm,yrpt(yr-1,ptp)} \right\}$$
$$+\sum_{iru} TrPipeInvC_{iru,yr}.\left\{ cumpipe_{iru,yrpt(yr,pt)} - cumpipe_{iru,yrpt(yr-1,ptp)} \right\}$$
$$+\sum_{itk} TrTankInvC_{itk,yr}.\left\{ cumtank_{itk,yrpt(yr,pt)} - cumtank_{itk,yrpt(yr-1,ptp)} \right\}$$
$$+\sum_{iaq} TrAquiInvC_{iaq,yr}.\left\{ cumaqui_{iaq,yrpt(yr,pt)} - cumaqui_{iaq,yrpt(yr-1,ptp)} \right\}$$

Total revenue

$$revenue_{yrpt(yr,pt)} = \sum_{d}\sum_{l} \left[PriceQd_{d,yrpt(yr,pt),l}.QtyQd_{d,yrpt(yr,pt),l} \right].wQd_{d,yrpt(yr,pt),l} \tag{A.24}$$

Cost-recovery (total revenue equals total cost requirement)

$$costs_{yrpt(yr,pt)} - revenue_{yrpt(yr,pt)} = 0 \tag{A.25}$$

Uniform price across classes of customers

$$onePrice_{yrpt(yr,pt)} - \sum_{l} PriceQd_{d,yrpt(yr,pt),l}.wQd_{d,yrpt(yr,pt),l} = 0 \tag{A.26}$$

Water restrictions

The water restrictions constraints restrict outdoor water demand when storage is below the trigger level in the preceding year. The restrictions are controlled with the binary variables $vRestr_{yr,pt,stage}$, which have a value of 1 when the restriction is 'on', and 0 when the restriction is off.

Water restrictions

$$vRestr0_{yrpt(yr,pt)} + \sum_{stage} vRestr_{yrpt(yr,pt),stage} = 1 \tag{A.27}$$

Additional term included in the objective function to reflect the cost of restriction after the terminal period

$$-\sum_{yrpt(lastyr,lastpt)}\sum_{m}\left(\frac{Df_{yr}}{1+Discount}.Prob_{pt}.FutStorC_{lastpt}.\sum_{stage}Gf_{yr}.vRestrTerm_{yrpt(lastyr,lastpt),stage}\right)$$

Water restrictions in terminal period

$$vRestrTerm0_{yrpt(lastyr,lastpt)}+\sum_{stage}vRestrTerm_{yrpt(lastyr,lastpt),stage}=1 \qquad \text{(A.28)}$$

Water consumption when restrictions are imposed

$$\sum_{l}QtyQd_{d,yrpt(yr,pt),l}.wQd_{d,yrpt(yr,pt),l}\Big|_{dr(d)}-\sum_{stage}vRestr_{yrpt(yr,pt),stage}.Gf_{yr}.Rest_{d,stage} \qquad \text{(A.29)}$$
$$-vRestr0_{yrpt(yr,pt)}.Gf_{yr}.Rest0_{d}-qstank_{yrpt(yr,pt)}\leq 0$$

Water restrictions triggered when storage is below threshold

$$\sum_{stage}\left(vRestr_{yrpt(yr,pt),stage}.Trig_{stage}\right) \qquad \text{(A.30)}$$
$$+vRestr0_{yrpt(yr,pt)}.Trig0-qstdam_{yrpt(yr-1,ptp)}\leq S0\Big|_{yr=firstyr}$$

Water restrictions triggered when storage is below threshold for terminal period

$$\sum_{stage}\left(vRestrTerm_{yrpt(yr,pt),stage}.Trig_{stage}\right) \qquad \text{(A.31)}$$
$$+vRestrTerm0_{yrpt(yr,pt)}.Trig0-qstdam_{yrpt(yr-1,ptp)}\leq 0$$

Long-run marginal cost pricing (with scope for water restrictions)

The long-run marginal cost policy constraints set a uniform price for all demand classes during the regulatory period (described in figure 4.1). An arbitrarily large term (999) is added to the long-run marginal cost constraints to ensure that when restrictions are active, the level of demand will be determined by restrictions, rather than long-run marginal cost prices.

Setting uniform prices for all classes of demand during each regulatory period — upper bound

$$\left.\sum_{l} QtyQd_{d,yrpt(yr,pt),l}.wQd_{d,yrpt(yr,pt),l}\right|_{PtPrBlk(pt,PrBlk)} \tag{A.32}$$

$$\left.-999.\sum_{stage} vRestr_{yrpt(yr,pt),stage}\right|_{dr(d)} \leq equil_{d,PrBlk}$$

Setting uniform prices for all classes of demand during each regulatory period — lower bound

$$\left.\sum_{l} QtyQd_{d,yrpt(yr,pt),l}.wQd_{d,yrpt(yr,pt),l}\right|_{PtPrBlk(pt,PrBlk)} \tag{A.33}$$

$$\left.+999.\sum_{stage} vRestr_{yrpt(yr,pt),stage}\right|_{dr(d)} \geq equil_{d,PrBlk}$$

Alternative specification of long-run marginal cost pricing with scope for water restrictions

An alternative specification of long-run marginal cost is used as part of the sensitivity analysis (appendix D). This constraint imposes a price of water that is equal to the levelised cost of the next cheapest source of water available to water utilities. Initially rural–urban trade will be the marginal source. Once a pipeline is constructed, desalination is the next cheapest source of water.

Setting prices equal to the marginal source — upper bound

$$\left. \begin{aligned} & Aglin_{d,yr} + Bglin_{d,yr}.\sum_{l}\left(QtyQd_{d,yrpt(yr,pt),l}.wQd_{d,yrpt(yr,pt),l}\right) \leq \\ & Level\_pipe_{yr} + \left\{Level\_desal_{yr} - Level\_pipe_{yr}\right\}.\sum_{iru} cumqpipecap_{iru,yrpt(yr,pt)} \\ & +999.\sum_{stage} vRestr_{yrpt(yr,pt),stage} \end{aligned} \right|_{dr(d)} \tag{A.34}$$

Setting prices equal to the marginal source — lower bound

$$\left. \begin{aligned} & Aglin_{d,yr} + Bglin_{d,yr}.\sum_{l}\left(QtyQd_{d,yrpt(yr,pt),l}.wQd_{d,yrpt(yr,pt),l}\right) \geq \\ & Level\_pipe_{yr} + \left\{Level\_desal_{yr} - Level\_pipe_{yr}\right\}.\sum_{iru} cumqpipecap_{iru,yrpt(yr,pt)} \\ & -999.\sum_{stage} vRestr_{yrpt(yr,pt),stage} \end{aligned} \right|_{dr(d)} \tag{A.35}$$

A.3 Parameters, variables and sets

Table A.1 **Sets in the model**

Name	*Dimensions*	*Description*
aquilag	1	Time lag on aquifer investments
d	1	Classes of demand for water
damlag	1	Time lag on new dam investments
firstyr	1	First year in the simulation period
desallag	1	Time lag on desalination investments
dr	d	Used to apply restrictions to a subset of demands
iaq	1	Aquifer investment options
idm	1	New dam investment options
ids	1	Desalination investment options
iru	1	Rural-urban pipe investment options
itk	1	Household tank investment options
l	1	Linear segments in the demand function
lastpt	1	Nodes associated with the final year in the probability tree
lastyr	1	Final year in the simulation period
m	1	Linear segments in the benefit function for terminal storage
pipelag	1	Time lag on rural-urban pipe investments
PrBlk	1	Long-run marginal cost pricing blocks
pt, ptp	1	Nodes in the decision tree
PtPrBlk	pt,PrBlk	Mapping of node pt to a long-run marginal cost pricing block PrBlk
stage	1	Stage of water restrictions (e.g. stage 3, stage 4)
tanklag	1	Time lag on household tank investments
yr	1	Time period. Single years for the shorter planning horizon model, aggregate years for the larger planning horizon model
yrpt	yr,pt	Mapping each node pt to its matching year in the probability tree

Source: Productivity Commission urban water model.

Table A.2 **Parameters in the model**

Name	*Dimensions*	*Description*
Aglin	d,yr	Constant for the linear demand function d in yr
AreaQd	d,yr,pt,l	Welfare from function curve d, at node pt in yr, for linear segment l
AreaSt	lastpt,m	Welfare from final period storage at node lastpt for linear segment m
Bglin	d,yr	Coefficient for the linear demand function d in yr
Capstock0	scalar	Initial capital stock in the urban water system
DamStCap	idm	Additional dam storage capacity from new dam investment idm
Depr	scalar	Depreciation rate
Df	yr	Discount factor in yr
Discount	scalar	Discount rate
FutStorC	lastpt	Welfare cost of restrictions in the period after final node lastpt
Gf	yr	Growth factor in yr
Inflow_state	pt	Inflow level relative to mean inflows at node pt
Level_pipe	yr	Levelised cost of rural-urban pipe water in yr
Level-desal	yr	Levelised cost of desalination water in yr
OperAqui	iaq	Fixed annual operating cost of aquifer investment iaq
OperDam	idm	Fixed annual operating cost of new dam investment idm
OperDesal	ids	Fixed annual operating cost of desalination investment ids
OperPipe	iru	Fixed annual operating cost of rural-urban pipe investment iru
OperSystem	scalar	Fixed annual operating cost of existing water infrastructure
OperTank	iru	Fixed annual operating cost of household tanks investment itk
PriceQd	d,yr,pt,l	Price of water for users of type d, at node pt in yr, for linear segment l
Prob	pt	Probability of being at node pt
QtyQd	d,yr,pt,l	Demand water quantity of type d, at node pt in yr, for linear segment l
QtyQstdamf	lastpt,m	Final period storage at node lastpt for linear segment m
Rest	d,stage	Restricted maximum demand for type d for stage level restrictions
Rest0	d	Maximum demand for type d for unrestricted demand (999)
S0	scalar	Water in storage at the start of the simulation period
SwAquiICap	iaq,yr	Water capacity of fully constructed aquifer investment iaq in yr
SwDamICap	idm,yr	Water capacity of fully constructed new dam investment idm in yr
SwDamStCap	idm,yr	Additional storage capacity from new dam investment idm
SwDesalICap	ids,yr	Water capacity of fully constructed desalination investment ids in yr
SwEnvFlows	yr	Environmental flows from catchments in yr
SwInflows	yr	Mean inflows in yr
SwLossFlows	yr	System losses in yr
SwMaxS0	yr	Base storage capacity from initial infrastructure in yr
SwPipeICap	iru,yr	Water capacity of fully constructed rural-urban pipe iru in yr
SwTankICap	itk,yr	Water capacity of fully constructed household tanks itk in yr
TrDamInvC	idm,yr	Truncated new dam investment cost for investment idm in yr
TrDesalInvC	ids,yr	Truncated desalination investment cost for investment ids in yr
Trig	stage	Minimum water in storage to trigger stage level restrictions
Trig0	scalar	Minimum water in storage for unrestricted demand

(Continued next page)

Table A.2 (continued)

Name	Dimensions	Description
TrPipeInvC	iru,yr	Truncated rural-urban pipe investment cost for investment iru in yr
TrTankInvC	iru,yr	Truncated household tank investment cost for investment itk in yr
TtrAquiInvC	iaq,yr	Truncated aquifer investment cost for aquifer investment iaq in yr
Utcdam	scalar	Unit transport (reticulation) costs of water
Vcaqui	scalar	Unit cost of water from aquifers
Vcdam	scalar	Unit cost of water from catchments
Vcdam2	scalar	Unit cost of water from new dams
Vcdesal	scalar	Unit cost of water from desalination
Vcpipe	scalar	Unit cost of water from rural-urban trade
Vctank	scalar	Unit cost of water from household tanks
WACC	scalar	Weighted average cost of capital
YrIndex	yr	Number of years in time period yr

Source: Productivity Commission urban water model.

Table A.3 **Variables in the model**

Name	*Dimensions*	*Description*
assets	yr,pt	Total stock of depreciated assets at node pt in yr
costs	yr,pt	Total system costs for cost recovery at node pt in yr
cumaqui	iaq,yr,pt	Cumulative proportion of total aquifer investment iaq made at node pt in yr (greater than or equal to 0)
cumdam	idm,yr,pt	Cumulative proportion of total new dam investment idm made at node pt in yr (between 0 and 1)
cumdesal	ids,yr,pt	Cumulative proportion of total desalination investment ids made at node pt in yr (between 0 and 1)
cumpipe	iru,yr,pt	Cumulative proportion of total rural-urban pipe investment iru made at node pt in yr (binary variable 0 or 1)
cumtank	iru,yr,pt	Cumulative proportion of total household tanks investment itk made at node pt in yr (greater than or equal to 0)
equil	d,PrBlk	Price equilibration for all prices of demand type d in long-run marginal cost pricing block PrBlk
NW	scalar.	Net social quasi-welfare
onePrice	yr,pt	Uniform price for cost recovery
qsaqui	yr,pt	Quantity of water supplied from aquifers at node pt in yr
qsdam	yr,pt	Quantity of water from pre-existing dams at node pt in yr
qsdam2	yr,pt	Quantity of water supplied from new dams at node pt in yr
qsdesal	yr,pt	Quantity of water supplied from desalination at node pt in yr
qspipe	yr,pt	Quantity of water supplied from rural-urban trade at node pt in yr
qstank	yr,pt	Quantity of water supplied from household tanks at node pt in yr
qstdam	yr,pt	Quantity of water stored at node pt in yr
revenue	yr,pt	Total revenues received by the authority for cost recovery at node pt in yr
vRestr	yr,pt,stage	Binary variable determining if restrictions of stage are active in the at node pt in yr
vRestr0	yr,pt	Continuous variable determining if demand is unrestricted at node pt in yr
vRestrTerm	lastyr,lastpt,stage	Binary variable determining if restrictions of stage would be active in the next year after node lastpoint in lastyr
vRestrTerm0	lastyr,lastpt	Continuous variable determining if demand would be unrestricted in the next year after node lastpoint in lastyr
wQd	d,yr,pt,l	Weight or activity level for linear demand segment l at node pt in yr for demand type d
wQstdamf	lastpt,m	Weight or activity level for terminal storage linear segment m in at final-year node lastpt

Source: Productivity Commission urban water model.

B Calibration of the model

The model presented in this paper represents a hypothetical system for providing insights into urban water reform in large cities around Australia. Although the model is hypothetical, there is still a need to calibrate it to be indicative of real world situations. Data for consumption, inflows to dams and new supply options were based on urban locations around Australia, as described below. Specific data is also needed to implement the cost recovery policy option, which is presented at the end of this appendix.

B.1 Consumption

The unrestricted total demand in the model is calibrated to aggregate consumption of 350 GL per annum at a (marginal) price of $1.20 per kilolitre. Annual usage of 350 GL is indicative of large urban water systems in Australia — Sydney, Melbourne, South–East Queensland, Perth and Adelaide — which use in the order of 200–500 GL of water per year[1] (PC 2008). Half of total demand is assumed to be for indoor use by residential customers, with the remainder equally split between outdoor use and indoor commercial use. Consumption is projected to grow at 1.2 per cent per annum, in line with population growth projections for Australian capital cities (ABS 2008).

Response of consumers to prices and restrictions

Consumers are likely to adjust their demand for water in response to changes in prices and any restrictions imposed on water use. However, accurate estimation of the magnitude of these responses is difficult. The relationship between demand and price has been estimated in a large number of studies, and elasticity estimates vary widely across these studies (Worthington and Hoffman 2008). Estimating price elasticities using historical data is challenging, due partly to limited variation in prices over time for urban water and also because of the impact of other demand

[1] Consumption of water in Adelaide during 2008 declined to less than 150 GL, partly due to the imposition of enhanced level 3 water restrictions. Consumption during previous drought conditions in 2002 was just under 200 GL (Maywald 2009).

management measures. Other demand management measures include restrictions, education campaigns and moral suasion. The timing of these measures is often correlated with price changes so that disentangling the impact of price and these other factors on demand is difficult. Alternative methods include surveys to elicit water use plans under different prices, but these suffer from drawbacks too — in particular, stated preferences have often been found to contradict actual (revealed) preferences (Maler and Vincent 2005).

Further complicating matters, demand is likely to be more price responsive over several years than in the short run. Over longer periods of time, consumers are able to modify their behaviour, install water saving technologies and change to less water-intensive gardens in response to water shortages and higher water prices. Incorporating a time-varying elasticity into modelling requires a dynamic representation of demand (for example, along the lines of the partial adjustment model in Phlips 1974). This cannot be easily incorporated into the Takayama and Judge (1971) framework as welfare needs to be separable across different periods to facilitate discounting and this separability is violated under dynamic representation of demand. Linearisation of the demand function would also be complicated by using a non-separable welfare function.

In this model, a single elasticity estimate is used, which should be interpreted as a 'medium term' elasticity somewhere between the immediate response and the eventual, long-term response to prices.

To incorporate the wide range of views regarding price elasticities of demand, sensitivity analysis is undertaken for a range of elasticity estimates (table B.1). The more elastic end of the range reflects the academic literature (as summarised in Worthington and Hoffman 2008) and the less elastic end is based on industry views (for example, as reported in PriceWaterhouseCoopers 2009). The central estimate for household elasticity of demand is slightly lower than that used by Grafton and Ward (2007) and Hughes et al. (2008) for similar modelling work. Outdoor and commercial uses of water are assumed to be more elastic than indoor household use. Demand functions were calibrated to the elasticity figures using an arc elasticity over a representative price range for future prices ($1 to $5 per kilolitre).

The impact of water restrictions in curtailing outdoor demand is calibrated to current (level 3a) and more severe (level 4) restrictions in Melbourne. Modelling only two levels of restrictions excludes costs from restrictions that would occur under less severe water shortages (level 1 and 2). As such, this approach provides a lower bound on the cost of water restrictions.[2]

[2] Including more stage levels for water restrictions requires more binary variables, reducing significantly the size of the model that can be solved.

Table B.1 **Consumer demand characteristics**

Parameter	*Units*	*Central estimate*	*Sensitivity*
Annual water usage			
Total consumption[a]	GL	350	± 35
Outdoor	GL	87.5	± 9
Indoor household	GL	175	± 17
Indoor commercial	GL	87.5	± 9
Delivery system losses	Per cent of total consumption	10	n/a
Growth rate of consumption	Per cent	1.2	±1
Price elasticity of demand			
Aggregate household elasticity	Ratio	- 0.30	± 0.20
Elasticity by demand type			
Outdoor elasticity	Ratio	- 0.60	± 0.40
Indoor household elasticity	Ratio	- 0.20	± 0.13
Indoor commercial elasticity	Ratio	- 0.60	± 0.40
Effect of water restrictions[b]			
Reduction in total water use			
Level 3a	Per cent	- 12.5	**na**
Level 4	Per cent	- 17.5	**na**
Storage trigger level			
Level 3a	Per cent of capacity	36	**na**
Level 4	Per cent of capacity	29	**na**

[a] At a price of $1.20 per kL. [b] Based on restrictions in Melbourne (DSE 2007 and Melbourne Water 2009b). **na** Not applicable.

B.2 Inflows to dams

Median inflows to dams are assumed to be equal to 300 GL per year. This represents a deficit between demand (at a price of $1.20 per kilolitre — see above) and median inflows, as has occurred in Melbourne and Perth in recent years (chapter 1).

Sensitivity analysis is undertaken by modelling inflows to dams that are 30 per cent above and below the base assumption (table B.2). A 30 per cent reduction in inflows is consistent with the lower end (tenth percentile) of CSIRO rainfall projections for 2030 (CSIRO 2007). Reductions in streamflow are generally larger than reductions in rainfall due to evaporation and retention of water in soil, so a 30 per cent decline in inflows can be used to give an indication of what might occur under a dry climate change scenario. A 30 per cent increase would reverse the deficit between demand and median inflows and would represent a return towards historical averages in cities such as Melbourne and Perth.

Table B.2 **Existing dams**

Parameter	*Units*	*Central estimate*	*Sensitivity*
Annual inflows to existing dams			
Median	GL	300	± 90
Storage capacity			
Total capacity	GL	1750	**na**
Initial storage	Per cent of total capacity	35	± 7
Storage not readily available	Per cent of total capacity	10	**na**

na Not applicable.

Storage capacity in existing dams is assumed to be five times the annual consumption of 350 GL per year. This is based on the average across Australian capital cities that rely primarily on water from dams for their water supply (Sydney, Melbourne, South-East Queensland, Adelaide, Canberra and Darwin — PC 2008). The bottom 10 per cent of water in existing dams is assumed to be in deep storage (based on Sydney Catchment Authority 2007) and not readily available for use. Initial dam storages are set at 35 per cent of capacity, based on observed levels during the recent drought in much of Australia. In early 2007, dam levels in Sydney, Melbourne, South-East Queensland, Perth and Canberra all dropped below 35 per cent (ACTEW 2009; Melbourne Water 2009c; Seqwater 2009; Sydney Catchment Authority 2009; Water Corporation 2009c).

Variability of inflows to dams

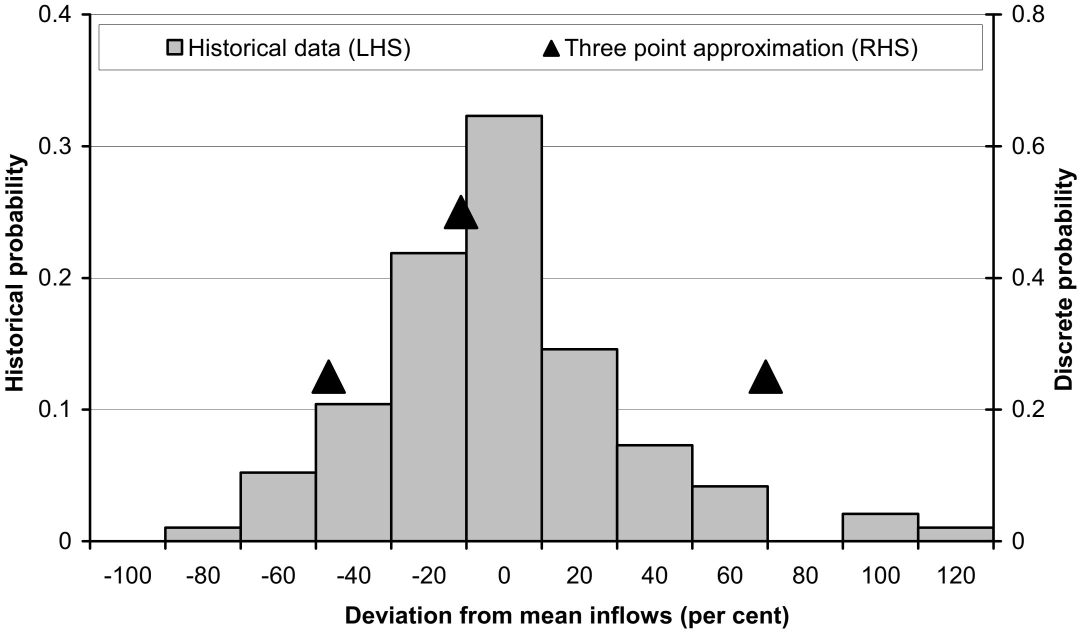
Variability in inflows to dams is represented by a three-point discrete distribution (low, medium and high) for each time period (figure B.1). The three-point distribution was fitted to historical data on inflows captured by dams,[3] providing a coarse approximation of inflows for any one period but a more accurate description of accumulated inflow scenarios over time.

A 25 per cent chance of low or high inflows and a 50 per cent chance of medium inflows have been chosen. Thus, annual inflows outside of a one in four event are not considered. However, the possibility of successive one in four events means that (for example) over a four-year period, one in one hundred year cumulative streamflow events are covered. As a result (in the early years of the modelling period at least) four to five year trends in inflows are described well by the probability tree. This is important because it is these longer-term trends that affect

3 Inflows captured by dams are used instead of gross inflows to adjust for the single storage model used to represent dams. This accounts for the possibility that, under specific inflow conditions, particular dams might be overflowing even though the total system storage is not full.

investment decisions in Australia, where dam storages are typically large enough to hold four to eight years of inflows (PC 2008). In its entirety, the probability tree contains about 60 000 scenarios, each describing a different path for inflows over the 20-year time horizon. The extreme scenarios in this tree describe wet and dry scenarios that exceed any twenty year series in the historical data used for calibration, meaning that the full range of long-term possibilities are canvassed, albeit with very low probabilities[4] (figure B.2).

Figure B.1 **Approximating variability in dam inflows**



Several methods are used to calibrate the three levels of inflows to historical data, each yielding similar estimates. In the first method, the three-point distribution is fitted so as to retain the first three moments (mean, variance and skewness[5]) of the data, following the approach advocated in Hoyland and Wallace (2001). In the second, the Wasserstein distance between the distribution of the existing data and the three-point distribution is minimised, as per Hochreiter and Pflug (2008). In the third, the historical data is divided into three groups, representing low, medium and high inflows. Simple averages of the three groups are used to approximate inflow volumes, similar to an approach presented in Kall and Wallace (1994). Results from

4 The driest and wettest scenarios in the modelling have a probability of about one in a million.

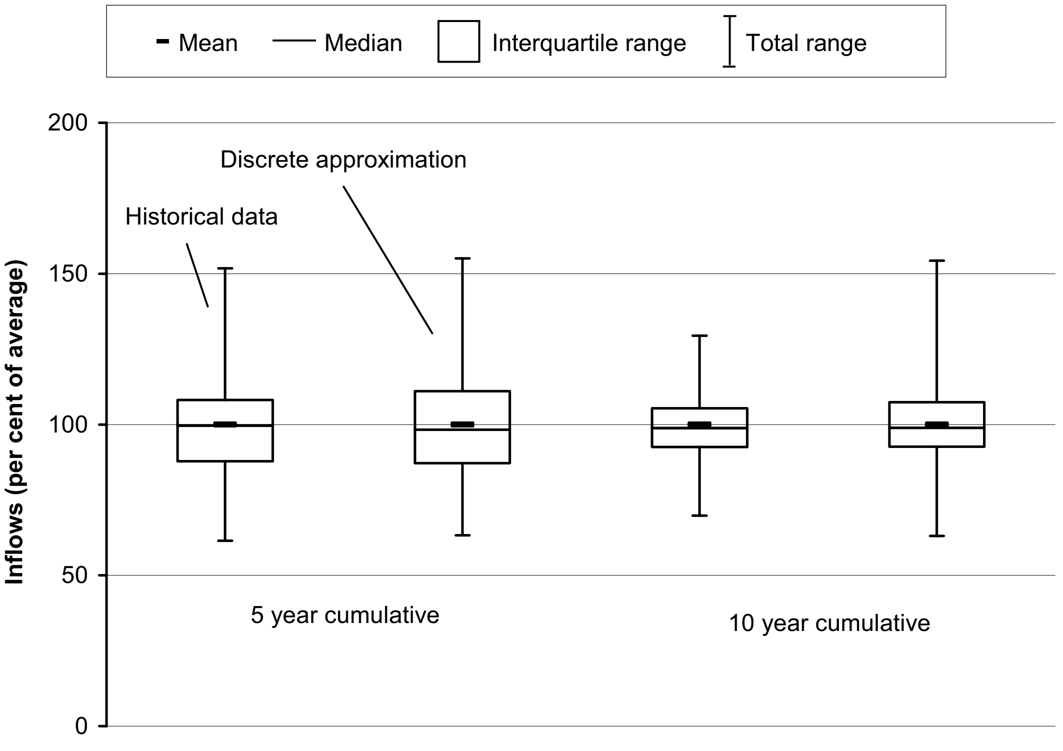
5 It is not possible to also specify the fourth moment (kurtosis) of a three-point distribution with fixed probabilities, as this leads to overspecification, as described in Hoyland and Wallace (2001).

all three approaches are used to make judgements about the discrete approximation applied in the model.

An important feature of variability in supply is the tendency for long periods of dry or wet years, such as during the recent extended drought affecting many of the capital city urban water systems in Australia. This suggests that there is some serial correlation in annual streamflows, which has also been noted in empirical studies of streamflows throughout the world (for example, McMahon et al. 2007). The model allows this effect to be included using a lag 1 autoregressive process. However, captured inflow data used for calibration did not show statistically significant evidence of autocorrelation (at the 5 per cent level). This is likely due to an inverse relationship between the previous year's rainfall and the proportion of inflows that can be captured in smaller dams, as well as additional water supply options — such as pumping to fill dams — that might be pursued during extended dry periods.

Figure B.2 **Approximating inflows over several years**

Historical data compared with discrete approximation



Inflows for time periods of more than one year

A modelling horizon of twenty years is used to investigate efficient investment in new sources of supply. Solving the model for twenty individual one-year periods is not possible under a multistage stochastic programming approach. Instead, twenty-year simulations are based on combining four one-year periods with three two-year periods, two three-year periods and one four-year period (chapter 2). This required parameters to be calibrated to inflows over periods of more than one year.

The calibration of inflows for time periods of more than one year is achieved by matching the mean and spread of historical inflows. Median inflows of 300 GL per annum are maintained, as is the three-point discrete distribution (low with probability 0.25, medium with probability 0.5 and high with probability 0.25). Inflows under low and high inflow scenarios are set with reference to the spread (standard deviation) and mean of multiple year time periods in the historical data record. The relative variation of inflows over multiple time periods is less than that for a single year, as extreme wet and dry years tend to cancel each other out over longer periods of time (table B.3). This approach is also effective in capturing the skewness of inflows in the historical data, which tends to decrease for time periods of longer than one year.

Table B.3 **Inflows over periods of more than one year**

Parameter	*Units*	*Time period*			
		1 year	*2 year*	*3 year*	*4 year*
Inflows to existing dams					
Low (probability = 0.25)	GL	180	400	560	750
Median (probability = 0.5)	GL	300	600	900	1200
High (probability = 0.25)	GL	570	1010	1330	1600

B.3 Characteristics of supply options

The initial supply of water in the hypothetical example comes entirely from dams. These are assumed to have an operating cost of 10 cents per kilolitre of water delivered (ERA 2009) and maintenance costs of $45 million per year.

Each new supply option modelled requires data on three distinct costs: a construction cost; an ongoing, annual fixed maintenance cost; and a marginal cost associated with releasing, delivering or obtaining a unit of water from the supply source. There is also a reticulation cost associated with transporting water from bulk storage to end users, assumed to be 45 cents per kilolitre for all sources (this does not apply to household tanks, which supply water directly to households).

An economic assessment of new supply options should include all relevant costs associated with supplying water from that source, including any environmental costs (where known). Data limitations have meant that, for this study, environmental costs are only incorporated to the extent that they affect costs incurred in building or running the facility. For example, where environmental assessment and remediation is required as part of building a dam, this is included in the cost of the dam. Any remaining environmental impacts are excluded. Similarly, for desalination, additional energy costs required to run the facility using renewable power are included, but any remaining environmental impacts are not.

New supply options are based on those being implemented or considered around Australia (table B.4):

- desalination plants
- new dams
- aquifers
- rural–urban trade
- household tanks.

The list of options considered is not exhaustive. For example, waste water recycling is not modelled because the material barriers to adoption of this technology are largely political, or alternatively that the water produced is not the same quality as other types of water. Introducing these issues into the model would lead to significant data and computational difficulties. Similarly, other alternatives that require water of different quality to be used for different purposes — such as dual reticulation systems — are not modelled. This does not reflect a judgement that these options are not worth pursuing, but rather that their value would be best evaluated through alternative modelling or cost–benefit frameworks.

There is also no 'backstop technology' included in the modelling. A backstop technology is a supply of water that is available at short notice and is perfectly elastic at a certain price. As such, all water demands at or above this price can be met using the backstop supply source, where other supplies are not available. For example, water was trucked in to supply some areas of rural Victoria during 2007, at a cost of about $10 per kilolitre (Goulburn Valley Water 2008). In large cities, supplying water through such a last resort measure is likely to be more difficult, given the quantities of water involved. However, it is not without international precedent. During 2008, water was transported to Barcelona by tanker ships, at a cost of around $5 per kilolitre (Time Magazine 2008). The availability of a backstop technology — at an acceptable price — allows water storages to be operated at a lower level than without such a backstop technology. However, a backstop supply source was not included in the hypothetical example due to the difficulty of supplying a large quantity of water at short notice, and uncertainty about the costs of such a technology given the lack of experience in large cities of Australia.

Omitting a backstop technology does not impact general economic inferences that can be illustrated using this model. Consideration could be given to including backstop technologies if this model were adapted and applied to model a specific urban water system in Australia.

Desalination plants

Desalination offers a source of water that is independent of rainfall. However, obtaining water from desalination involves relatively high per unit costs due to its intensive use of energy. There are also high fixed annual costs to maintain a desalination plant.

New dams

New dams add to the aggregate capacity of the urban region's storage, and also provide additional, rainfall-dependent inflows in each time period. There are likely to be long delays between the decision to build a new dam and the supply of water, as time is needed for planning and environmental approval, construction, and filling of the dam. There are also a diminishing number of sites available for dams, with increasing costs of procurement.

Aquifers

Groundwater supplies from aquifers are a relatively small potential source of water for most jurisdictions. Their low costs and reliable supply of water are based on new groundwater supplies used to augment Perth's water supply during 2002. Aquifers are assumed to provide a fixed and known sustainable yield. In practice, previous yields will have an impact on aquifer yields going forward. A more detailed treatment could also include the potential for interconnection between domestic run-off and recharge of aquifers, allowing for inclusion of any externalities arising from the use of groundwater.

Table B.4 **Characteristics of new sources of supply**

Parameter	*Units*	*Data*	*Source*
Desalination			**Sydney**
Quantity of water available	GL/year	90	WSAA (2008b)
Investment cost[a]	$ million	2 000	Sydney Water (2005)
Annual maintenance cost	$ million/year	37[b]	SMH (2009); MJA (2007a)
Operating costs	$/kL	0.40[b]	SMH (2009); MJA (2007a)
Economic life	years	47	Sydney Water (2007)
Time: inception → supply	years	4[c]	Sydney Water (2007)
Additional dams			**Brisbane**
Quantity of water available	GL/year	70	MJA (2007a)
Additional storage capacity	GL	153	Senate of Australia (2007)
Investment cost[a]	$ million	1 592	MJA (2007a)
Annual maintenance cost	$ million/year	18	MJA (2007a)
Operating costs	$/kL	0.21	MJA (2007a)
Economic life	years	50	QWI (2007)
Time: inception → supply	years	10[d]	Stakeholder consultation
Variability of supply			Same as for existing dams
Aquifers			**Perth**
Quantity of water available	GL/year	21	Water Corporation (2009)
Investment cost[a]	$ million	47	Water Corporation (2009)
Annual maintenance cost	$ million/year	0.5[e]	
Operating costs	$/kL	0.20	ERA (2009)
Economic life	years	50[f]	
Rural–urban trade			**Hypothetical example**
Quantity of water available	GL/year	75	Victorian Government (2008)
Investment cost[a]	$ million	750	Victorian Government (2008)
Annual maintenance cost	$ million/year	7.5[e]	
Operating costs	$/kL	0.25 – 0.70[g]	Waterexchange (2009) and IPA (2008)
Economic life	years	50[f]	
Time: inception → supply	years	3	Victorian Government (2008)
Variability of supply	Addressed by variable operating cost (price of water entitlements)		
Household tanks (per tank, each with 5kL storage capacity)			**Melbourne**
Quantity of water available	kL/year	29	MJA (2007b)
Investment cost[a]	$	2 300	MJA (2007b)
Annual maintenance cost	$/year	20	MJA (2007b)
Operating costs	$/kL	0.05	MJA (2007b)
Economic life	years	30	VCEC (2005)
Variability of supply			Half as much variability as dam inflows

[a] Total investment cost, undiscounted. [b] Based on a $73 million annual cost (SMH 2009) and a 50/50 split between fixed maintenance and variable operating costs (MJA 2007a). [c] Began planning and procurement 2006, supply expected to begin 2010. [d] Includes time for planning, construction and building storage. [e] Estimated at 1 per cent of initial investment cost. [f] As for dams: bulk pipelines are likely to have lifetimes longer than 50 years while pumps have shorter lifetimes. [g] Includes a cost of purchasing water allocations that varies from $0.05/kL during wet years to $0.50/kL during dry years (data from NWC 2008a; Peterson et al. 2004; and Waterexchange 2009) as well as a cost of pumping and treatment of $0.20/kL (IPA 2008).

Rural–urban trade (pipelines)

Rural–urban trade using pipelines allows urban water to be obtained by purchasing water rights from irrigation regions and delivering it to urban centres. This is modelled as an opportunity for urban regions to purchase annual water allocations from rural markets. Given the small size of urban markets relative to rural markets (PC 2008), the price of water in irrigation markets is assumed to be unaffected by the quantity purchased for urban use. This assumption is made to limit the size of model by avoiding the need to linearise the supply function of water from irrigation regions. However, the unit price of water is assumed to vary with rainfall patterns: in dry years, rural water is expensive, while in wet years it is relatively cheap.

Household tanks

Tanks provide households with additional water at a relatively low per-unit cost, but involve substantial capital costs per unit of water delivered. Supply from tanks is rainfall dependent, but like rainfall itself, yields from tanks do not vary as much as inflows to dams (since dams need significant rainfall just to saturate the soil and begin the runoff process — MJA 2007b). Annual yields from tanks are assumed to be half as variable as inflows to dams, based on the observed relationship between rainfall variability and dam inflows in Melbourne (BOM 2009 and Melbourne Water 2009a). Their chief advantage over other supply options is their scope to supply water that can be used outdoors at times when water restrictions are enforced. Also, unlike other supply options, in the model there is no limit imposed on the total amount of water that can be supplied from tanks.[6]

B.4 Cost recovery pricing policy

Modelling the cost recovery pricing policy requires additional data. The cost recovery constraint requires annual revenue to equal costs at every node (chapter 4). The method applied is similar to that used by regulators in applying the building blocks method (Howe and Rasmussen 1982). The revenue requirement is made up of:

- operating expenses
- depreciation

[6] In practice, roof area is likely to constrain the amount of water that can be supplied from tanks in any particular city. However, this would only be an issue after a vast number of tanks had been installed throughout the city, which does not occur in the modelling results.

- a return on the asset base (written down value of assets) using an appropriate rate of return (the weighted average cost of capital).

Estimating annual costs requires converting the written down value of assets to an annuity, using a regulatory weighted average cost of capital and a regulatory depreciation rate (table B.5). Depreciation is calculated using the declining balance method. Overhead operating costs, such as head office and information systems costs, also need to be considered in cost recovery pricing.

Table B.5 **Parameters used for recovery of capital and fixed operating expenditure**

Parameter	*Units*	*Central estimate*	*Sensitivity*
Discount rate[a]	Per cent	6.0	± 4
Weighted average cost of capital (rate of return)	Per cent	6.0	**na**
Regulatory rate of depreciation	Per cent per annum	1.5	**na**
Regulatory asset base (rate base)	$million	4 000	**na**
Annual overhead operating costs[b]	$million/year	100	**na**

[a] A discount rate of 6 per cent was used throughout the simulations to calculate discounted net social welfare in the objective function. A sensitivity of ±4 per cent was used based on Harrison (2007). [b] Costs that are unrelated to the quantity of water delivered, for example head office and information systems costs. **na** Not applicable.

C Investment and supply decisions in a simplified model

Specific economic principles govern investment and supply decisions in the urban water model developed for this paper. However, the partial equilibrium model used here is formulated using the quantity formulation and the economic principles are not immediately obvious or observable from the mathematical description of the model in appendix A. The price information in this model is embedded in the Lagrange multipliers and the economic principles for the market equilibrium are derived by applying the Kuhn-Tucker conditions for optimality (Takayama and Judge 1971; Martin 1981).

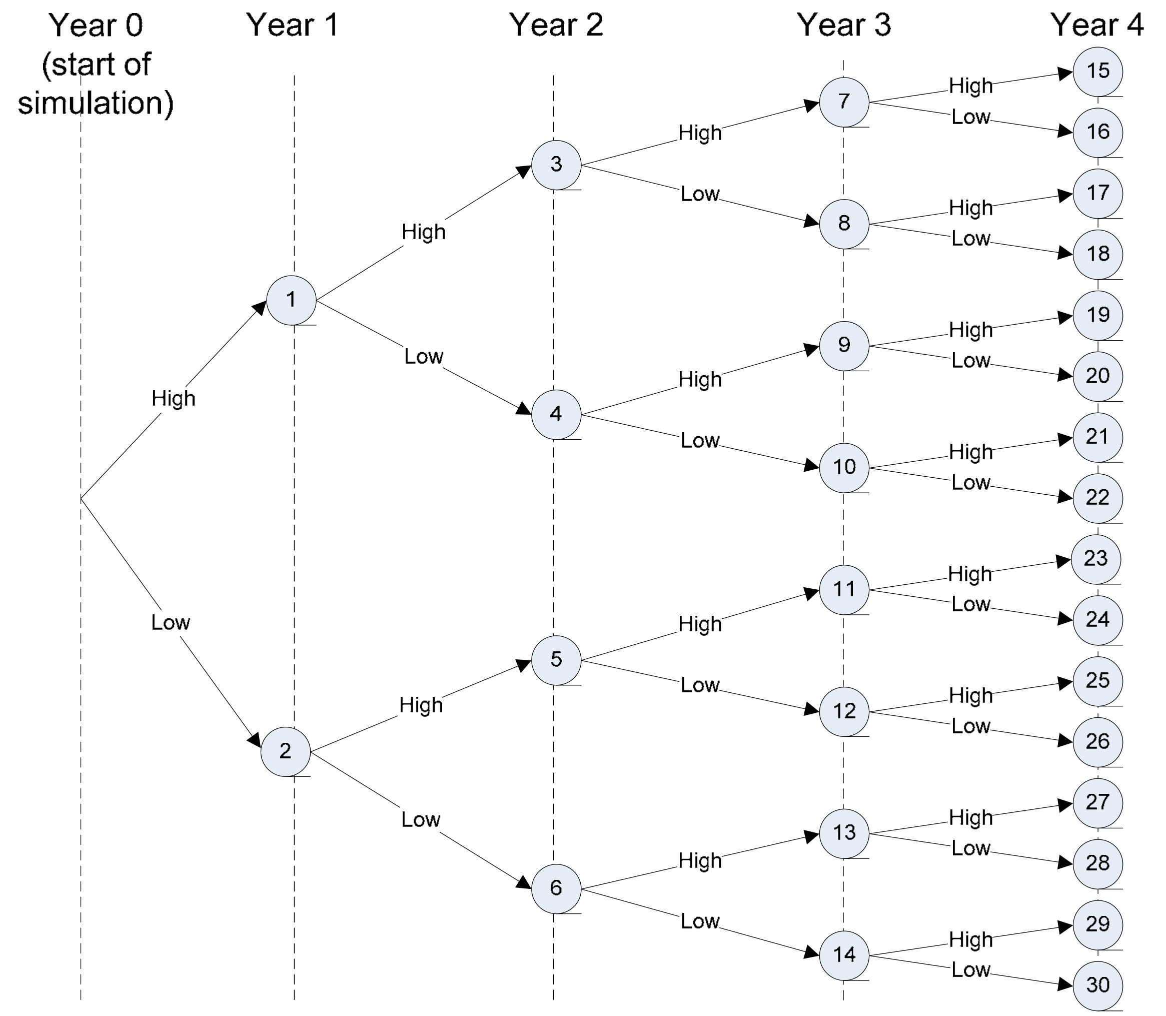
The complexity of the model presented in this paper makes a straightforward exposition of these principles difficult. There are more than 5000 scenarios for the smallest version of the model solved, and decisions at the start of the simulation must take all of these different possible outcomes into account. The range of competing options for new supply, lags between investment decisions and commissioning, and modelling of regulatory constraints all make it challenging to mathematically write out the principles in a way that is easily understood. Instead, a simplified version of the model is presented in this appendix to outline the economic principles underlying the competitive economic equilibrium in the model. The first order conditions, together with results from this simplified model, are used to illustrate the principles that underlie the competitive equilibrium in the larger model, with a greater emphasis on explaining investment.

The model utilised in this appendix is simpler on many fronts (table C.1) and is used for illustrative purposes only. The full probability tree for the simplified model contains only 30 nodes (figure C.1).

Table C.1 **Differences between the core model and simplified model**

Core model	*Simplified model used in this appendix*
Simulation period of 8–20 years	Simulation period of 4 years
3 outcomes for annual inflows to dams (high/medium/low)	2 outcomes for annual inflows to dams (high/low)
5 new supply options (dam, desalination, aquifers, rural–urban trade, household tanks)	1 new supply option (desalination)
Costs of supply options include a fixed annual cost, independent of the quantity supplied	No fixed annual costs
Multiple year lags between investment decisions and supply of water, to account for planning and construction	Water is available from new investments in the year following investment
Investment in new supply measured using cumulative capacity built up to a point in time (box 2.2)	Investment measured using the incremental amount of investment in any period in any node

Figure C.1 **Probability tree for the simplified model**



C.1 A simplified urban water model

All variables in the model have names that start with lower case letters. All fixed parameters have names that start with an upper case letter (for definitions, see tables A.1, A.2 and A.3 in Appendix A). Some additional notation is required for this appendix in order to model investment incrementally, rather than cumulatively as in the large model[1]:

- $incrdesal_{yrpt(yr,pt)}$: Additional ('incremental') investment in desalination capacity at each node in each year.
- anc(pt,ptp): Mapping of node pt to all preceding 'ancestor' nodes, from which current supply is available from earlier investments in desalination capacity. Alternatively, the transpose of anc(pt,ptp) can be used to map an investment to supply in all subsequent nodes looking down the probability tree.
- suc(pt,ptp): Mapping of node pt to 'successor' nodes that immediately follow node pt in the next year.

Objective function

$$\text{Max } NW = \tag{C.1}$$

Objective function: area under the demand function less reticulation costs

$$\sum_{d} \sum_{yrpt(yr,pt)} \sum_{l} Df_{yr}.Prob_{pt}.AreaQd_{d,yrpt(yr,pt),l}.wQd_{d,yrpt,l}$$

$$-\sum_{yrpt(yr,pt)} Df_{yr}.Prob_{pt}.Utcdam.\left\{qsdesal_{yrpt(yr,pt)} + qsdam_{yrpt(yr,pt)}\right\}$$

plus: benefit from storage in terminal period

$$+\sum_{yrpt(lastyr,lastpt)} \sum_{m} \frac{Df_{yr}}{1+Discount}.Prob_{pt}.AreaSt_{lastpt,m}.wQstdamf_{lastpt,m}$$

less: variable cost of water supply from the existing dam

$$-\sum_{yrpt(yr,pt)} Df_{yr}.Prob_{pt}.Vcdam.qsdam_{yrpt(yr,pt)}$$

1 See box 2.2 for a description of the equivalence between these two approaches

less: variable and investment costs of water supply from desalination

$$-\sum_{yrpt(yr,pt)} Df_{yr}.Prob_{pt}.Vcdesal.qsdesal_{yrpt(yr,pt)}$$
$$-\sum_{yrpt(yr,pt)} Df_{yr}.Prob_{pt}.TrDesalInvC_{yr}.incrdesal_{yrpt(yr,pt)}$$

Constraints

Maximum dam storage

$$qstdam_{yrpt(yr,pt)} \leq SwMaxS0_{yr} \quad \text{for all yrpt} \tag{C.2}$$

Dam supply

$$\begin{aligned} &qsdam_{yrpt(yr,pt)} + qstdam_{yrpt(yr,pt)} - qstdam_{yrpt(yr-1,ptp)} \leq \\ &Inflow\_state_{pt}.\left\{SwInflows_{yr} - SwEnvFlows_{yr} - SwLossFlows_{yr}\right\} + S0 \end{aligned} \quad \text{for all yrpt} \tag{C.3}$$

Water demand balance

$$\sum_{d}\sum_{l}\begin{pmatrix} QtyQd_{d,yrpt(yr,pt),l}.wQd_{d,yrpt(yr,pt),l} \\ -qsdam_{yrpt(yr,pt)} - qsdesal_{yrpt(yr,pt)} \end{pmatrix} \leq 0 \quad \text{for all yrpt} \tag{C.4}$$

Desalination constraints

Supply balance from desalination

$$\begin{aligned} &qsdesal_{yrpt(yr,pt)} \\ &-\sum_{yrpt(yrp,ptp)}^{yr-1} SwDesalICap_{yrp}.incrdesal_{yrpt(yrp,ptp)}\Big|_{anc(pt,ptp)} \leq 0 \end{aligned} \quad \text{for all yrpt} \tag{C.5}$$

Upper bound on total desalination capacity

$$\sum_{yrpt(yrp,ptp)}^{yr-1} incrdesal_{yrpt(yrp,ptp)}\Big|_{anc(pt,ptp)} \leq 1 \quad \text{for all yrpt} \tag{C.6}$$

Constraints related to linearisation

Convexity of linearised demand

$$\sum_{l} wQd_{d,yrpt(yr,pt),l} \leq 1 \quad \text{for all yrpt and all d} \tag{C.7}$$

Convexity of linear benefit function for water stored in final period

$$\sum_{m} wQstdamf_{lastpt,m} \leq 1 \quad \text{for all lastpt} \qquad \text{(C.8)}$$

Demand balance for storage in the terminal period

$$\sum_{m} QtyQstdamf_{lastpt,m}.wQstdamf_{lastpt,m} - qstdam_{yrpt(lastyr,lastpt)} \leq 0 \quad \text{for all lastpt} \qquad \text{(C.9)}$$

C.2 Kuhn–Tucker conditions for optimisation

The urban water model is a constrained optimisation problem with inequality constraints. The necessary conditions for a solution to such a problem are given by the Kuhn–Tucker conditions (Chow 1997; Lambert 1993; Intriligator 1971). Applying the Kuhn–Tucker conditions to a 'Lagrangean' expression yields first order and complementary slackness conditions, which reveal the market equilibrium conditions imbedded in the model.

The Lagrangean and Kuhn-Tucker conditions for the simplified model are listed below, along with a brief discussion of the economic meaning of each.

Lagrangean

$$L = \qquad \text{(C.10)}$$

$$\begin{aligned}
&\sum_{d} \sum_{yrpt(yr,pt)} \sum_{l} Df_{yr}.Prob_{pt}.AreaQd_{d,yrpt(yr,pt),l}.wQd_{d,yrpt,l} \\
&- \sum_{yrpt(yr,pt)} Df_{yr}.Prob_{pt}.Utcdam.\left\{qsdesal_{yrpt(yr,pt)} + qsdam_{yrpt(yr,pt)}\right\} \\
&+ \sum_{yrpt(yrlast,lastpt)} \sum_{m} \frac{Df_{yr}}{1+Discount}.Prob_{pt}.AreaSt_{lastpt,m}.wQstdamf_{lastpt,m} \\
&- \sum_{yrpt(yr,pt)} Df_{yr}.Prob_{pt}.Vcdam.qsdam_{yrpt(yr,pt)} - \sum_{yrpt(yr,pt)} Df_{yr}.Prob_{pt}.Vcdesal.qsdesal_{yrpt(yr,pt)} \\
&- \sum_{yrpt(yr,pt)} Df_{yr}.Prob_{pt}.TrDesalInvC_{yr}\ incrdesal_{yrpt(yr,pt)} - \sum_{yr} OperSystem \\
&+ \sum_{yrpt(yr,pt)} \lambda^{QST}_{yrpt(yr,pt)} \left(SwMaxS0_{yr} - qstdam_{yrpt(yr,pt)}\right) \\
&+ \sum_{yrpt(yr,pt)} \lambda^{QSDAM}_{yrpt(yr,pt)} \begin{pmatrix} Inflow_state_{pt}.\left\{SwInflows_{yr} - SwEnvFlows_{yr} - SwLossFlows_{yr}\right\} + S0 \\ -qsdam_{yrpt(yr,pt)} - qstdam_{yrpt(yr,pt)} + qstdam_{yrpt(yr-1,ptp)} \end{pmatrix} \\
&+ \sum_{yrpt(yr,pt)} \lambda^{QD}_{yrpt(yr,pt)} \left(qsdam_{yrpt(yr,pt)} + qsdesal_{yrpt(yr,pt)} - \sum_{d} \sum_{l} QtyQd_{d,yrpt(yr,pt),l}.wQd_{d,yrpt(yr,pt),l}\right)
\end{aligned}$$

$$+\sum_{yrpt(yr,pt)} \lambda^{QSDESAL}_{yrpt(yr,pt)} \left(\sum_{yrpt(yrp,ptp)}^{yr-1} \left(SwDesalICap_{yrp}.incrdesal_{yrpt(yrp,ptp)} \right) \Big|_{anc(pt,ptp)} - qsdesal_{yrpt(yr,pt)} \right)$$

$$+\sum_{yrpt(yr,pt)} \lambda^{DESALCAP}_{yrpt(yr,pt)} \left(1 - \sum_{yrpt(yrp,ptp)}^{yr-1} incrdesal_{yrpt(yrp,ptp)} \Big|_{anc(pt,ptp)} \right)$$

$$+\sum_{d} \sum_{yrpt(yr,pt)} \lambda^{QDLIN}_{d,yrpt(yr,pt)} \left(1 - \sum_{l} wQd_{d,yrpt(yr,pt),l} \right) + \sum_{yrpt(lastyr,lastpt)} \lambda^{QSTLIN}_{lastpt} \left(1 - \sum_{m} wQstdamf_{lastpt,m} \right)$$

$$+\sum_{yrpt(lastyr,lastpt)} \lambda^{QSTLAST}_{lastpt} \left(qstdam_{yrpt(lastyr,lastpt)} - \sum_{m} QtyQstdamf_{lastpt,m}.wQstdamf_{lastpt,m} \right)$$

Table C.2 defines each Lagrange multiplier. More detail is provided in the following sections.

Table C.2 **Lagrange multipliers and their interpretation**

Multiplier	*Description*
$\lambda^{QST}_{yrpt(yr,pt)}$	The probability weighted, discounted price of dam storage capacity. When storage is not constrained by dam storage capacity, the value is zero.
$\lambda^{QSDAM}_{yrpt(yr,pt)}$	The probability weighted, discounted shadow price of drawing down dam storages.
$\lambda^{QD}_{yrpt(yr,pt)}$	The probability weighted, discounted demand price of water to consumers.
$\lambda^{QSDESAL}_{yrpt(yr,pt)}$	The probability weighted, discounted return to or payment for the use of desalination assets at each node in each year. This represents a price per unit of water to recover investment costs and any capacity rent if capacity is binding.
$\lambda^{DESALCAP}_{yrpt(yr,pt)}$	The probability weighted, discounted shadow price of additional desalination capacity. If desalination is not at capacity, this is zero. If desalination is at capacity, this equals an implied price for additional capacity.
$\lambda^{QDLIN}_{d,yrpt(yr,pt)}$	The probability weighted, discounted value of consumer surplus.
$\lambda^{QSTLIN}_{lastpt}$	The probability weighted, discounted value of consumer surplus derived from terminal storage.
$\lambda^{QSTLAST}_{lastpt}$	The probability weighted, discounted price for storing water in the terminal period.

First condition — price for (linearised) demand for water

$$\frac{\partial L}{\partial wQd_{d,yrpt(yr,pt),l}} \leq 0$$

$$Df_{yr}.Prob_{pt}.AreaQd_{d,yrpt(yr,pt),l} - \lambda^{QD}_{yrpt(yr,pt)} QtyQd_{d,yrpt(yr,pt),l} - \lambda^{QDLIN}_{d,yrpt(yr,pt)} \leq 0 \quad \text{(FOC 1)}$$

$$\left(\frac{\partial L}{\partial wQd_{d,yrpt(yr,pt),l}} \right) wQd_{d,yrpt(yr,pt),l} = 0$$

$$\begin{pmatrix} Df_{yr}.Prob_{pt}.AreaQd_{d,yrpt(yr,pt),l} \\ -\lambda^{QD}_{yrpt(yr,pt)} QtyQd_{d,yrpt(yr,pt),l} - \lambda^{QDLIN}_{d,yrpt(yr,pt)} \end{pmatrix} .wQd_{d,yrpt(yr,pt),l} = 0 \quad \text{(CS 1)}$$

The first complementary slackness condition (CS 1) indicates that the condition FOC 1 holds with equality whenever a positive quantity of water is supplied to consumers. Condition FOC 1 states that the area under the demand function must be less than or equal to total revenue from sales to consumers and consumer surplus. $\lambda^{QD}_{yrpt(yr,pt)}$ is the unit price paid by consumers and $\lambda^{QDLIN}_{d,yrpt(yr,pt)}$ is total consumer surplus.

Second condition — price of (linearised) terminal storage

$$\frac{\partial L}{\partial wQstdamf_{lastpt,m}} \leq 0$$

$$\frac{Df_{yr}}{1+Discount}.Prob_{pt}.AreaSt_{lastpt,m} - \lambda^{QSTLIN}_{lastpt} - \lambda^{QSTLAST}_{lastpt} QtyQstdamf_{lastpt,m} \leq 0 \quad \text{(FOC 2)}$$

$$\left(\frac{\partial L}{\partial wQstdamf_{lastpt,m}} \right) wQstdamf_{lastpt,m} = 0$$

$$\begin{pmatrix} \frac{Df_{yr}}{1+Discount}.Prob_{pt}.AreaSt_{lastpt,m} \\ -\lambda^{QSTLAST}_{lastpt} QtyQstdamf_{lastpt,m} - \lambda^{QSTLIN}_{lastpt} \end{pmatrix} wQstdamf_{lastpt,m} = 0 \quad \text{(CS 2)}$$

Condition FOC 2 is analogous to that for FOC 1. The benefit function for the storage of water in the terminal period can be thought of as a demand for storing water in the terminal period. FOC 2 states that the area under the benefit function must be less than or equal to the imputed value of revenue from storage and the imputed consumer surplus.

The slackness condition CS 2 states that if water stored in the terminal period is positive, then FOC 2 applies as an equality.

Third condition — supply price from dam

$$\frac{\partial L}{\partial qsdam_{yrpt(yr,pt)}} \le 0$$

$$\lambda^{QD}_{yrpt(yr,pt)} - Df_{yr}.Prob_{pt}.\left(Utcdam + Vcdam\right) - \lambda^{QSDAM}_{yrpt(yr,pt)} \le 0 \qquad \text{(FOC 3)}$$

$$\left(\frac{\partial L}{\partial qsdam_{yrpt(yr,pt)}}\right) qsdam_{yrpt(yr,pt)} = 0$$

$$\left(\lambda^{QD}_{yrpt(yr,pt)} - Df_{yr}.Prob_{pt}.\left(Utcdam + Vcdam\right) - \lambda^{QSDAM}_{yrpt(yr,pt)}\right) qsdam_{yrpt(yr,pt)} = 0 \qquad \text{(CS 3)}$$

Conditions FOC 3 and CS 3 state that if water is supplied from dams to consumers then the consumer price is equal to the sum of:

- the unit cost of reticulation
- the variable cost per unit of water supply from dams
- a shadow price representing the opportunity cost of drawing on dam supplies at the current node, which is made up of two components, discussed in FOC 4 and CS 4.

If water is not supplied from dams, then the consumer price can be less than the price of water supplied from dams. That is, at that price, it is uneconomic for dams to supply water.

Fourth condition — price on dam storage

$$\frac{\partial L}{\partial qstdam_{yrpt(yr,pt)}} \le 0$$

$$\lambda^{QSTLAST}_{lastpt}\Big|_{yr \in lastyr} + \lambda^{QSDAM}_{yrpt(yr,pt)} - \sum_{yrpt(yrp,ptp)} \lambda^{QSDAM}_{yrpt(yrp,ptp)}\Big|_{suc(pt,ptp)} - \lambda^{QST}_{yrpt(yr,pt)} \le 0 \qquad \text{(FOC 4)}$$

$$\left(\frac{\partial L}{\partial qstdam_{yrpt(yr,pt)}}\right) qstdam_{yrpt(yr,pt)} = 0$$

$$\begin{pmatrix} \lambda^{QSTLAST}_{lastpt}\Big|_{yr \in lastyr} - \lambda^{QSDAM}_{yrpt(yr,pt)} \\ + \sum_{yrpt(yrp,ptp)} \lambda^{QSDAM}_{yrpt(yrp,ptp)}\Big|_{suc(pt,ptp)} - \lambda^{QST}_{yrpt(yr,pt)} \end{pmatrix} qstdam_{yrpt(yr,pt)} = 0 \qquad \text{(CS 4)}$$

Conditions FOC 4 and CS 4 define the shadow price (opportunity cost) of supply from dams at a node in terms of:

- the sum of the shadow prices of water held in storage in subsequent periods (looking down the probability tree) for each state of inflow, and the value of holding water in the terminal period
- an imputed rent on dam capacity if storages are being constrained by the storage capacity of dams.

If the storage is positive, FOC 4 holds with equality. If the storage is zero, then the shadow price for using storages at the current price might be zero or less than the sum of the shadow price in the subsequent period.

Fifth condition — supply price of water from desalination

$$\frac{\partial L}{\partial qsdesal_{yrpt(yr,pt)}} \leq 0 \qquad \text{(FOC 5)}$$

$$\lambda^{QD}_{yrpt(yr,pt)} - Df_{yr}.Prob_{pt}.\left(Utcdam + Vcdesal\right) - \lambda^{QSDESAL}_{yrpt(yr,pt)} \leq 0$$

$$\left(\frac{\partial L}{\partial qsdesal_{yrpt(yr,pt)}}\right) qsdesal_{yrpt(yr,pt)} = 0 \qquad \text{(CS 5)}$$

$$\left(\lambda^{QD}_{yrpt(yr,pt)} - Df_{yr}.Prob_{pt}.\left(Utcdam + Vcdesal\right) - \lambda^{QSDESAL}_{yrpt(yr,pt)}\right) qsdesal_{yrpt(yr,pt)} = 0$$

Conditions FOC 5 and CS 5 state that if water is supplied from desalination to consumers, then the consumer price is equal to the sum of:

- the unit cost of reticulation
- the variable cost per unit of water supplied from desalination
- a shadow price per unit of water supply representing a contribution to the recovery of the capital costs of installing desalination plants plus an imputed rent if desalination capacity is constrained to its upper bound (FOC 6 and CS 6)
- If water is not supplied from desalination, then consumer price is less than the cost of supply from desalination — it is uneconomic to use desalination at that node.

Sixth condition — price on desalination capacity

$$\frac{\partial L}{\partial incrdesal_{yrpt(yr,pt)}} \leq 0$$

$$\begin{pmatrix} SwDesalICap_{yrp} \sum_{yrpt(yrp,ptp)} \lambda^{QSDESAL}_{yrpt(yrp,ptp)} \Big|_{anc(ptp,pt)} \\ -Df_{yr}.Prob_{pt}.TrDesalInvC_{yr} - \sum_{yrpt(yrp,ptp)} \lambda^{DESALCAP}_{yrpt(yrp,ptp)} \Big|_{anc(ptp,pt)} \end{pmatrix} \leq 0 \qquad \text{(FOC 6)}$$

$$\left(\frac{\partial L}{\partial incrdesal_{yrpt(yr,pt)}} \right) incrdesal_{yrpt(yr,pt)} = 0$$

$$\begin{pmatrix} SwDesalICap_{yrp} \sum_{yrpt(yrp,ptp)} \lambda^{QSDESAL}_{yrpt(yrp,ptp)} \Big|_{anc(ptp,pt)} \\ -Df_{yr}.Prob_{pt}.TrDesalInvC_{yr} \\ - \sum_{yrpt(yrp,ptp)} \lambda^{DESALCAP}_{yrpt(yrp,ptp)} \Big|_{anc(ptp,pt)} \end{pmatrix} incrdesal_{yrpt(yr,pt)} = 0 \qquad \text{(CS 6)}$$

The decision to invest in desalination capacity (at a node) is based on the expected returns looking down the probability tree. For an investment to be economic, the expected returns must equal to or exceed:

- a charge (per unit of water supplied) to recover the capital cost of the investment
- an imputed rent if the installed capacity is being constrained by the maximum allowable capacity.[2]

If the capacity constraint is binding, then the expected net present value (NPV) of revenue from sales of water from the plant can exceed the investment costs. If capacity is not constrained, the expected NPV of revenue is equal to the investment cost. An investment is only made if the expected NPV of revenue is at least equal to the investment cost.

2 Note that the transpose of the set anc is being used to look down the tree rather than up the tree.

Seventh condition — maximum dam storage constraint

$$\frac{\partial L}{\partial \lambda^{QST}_{yrpt(yr,pt)}} \le 0$$
$$qstdam_{yrpt(yr,pt)} - swMaxS0_{yr} \le 0 \qquad \text{(FOC 7)}$$

$$\left(\frac{\partial L}{\partial \lambda^{QST}_{yrpt(yr,pt)}}\right)\lambda^{QST}_{yrpt(yr,pt)} = 0$$
$$\left(qstdam_{yrpt(yr,pt)} - SwMaxS0_{yr}\right)\lambda^{QST}_{yrpt(yr,pt)} = 0 \qquad \text{(CS 7)}$$

FOC 7 is simply a restatement of the model constraint C 2. When dam storage is not at capacity, the price on storage capacity is zero.

Eighth condition — dam supply constraint

$$\frac{\partial L}{\partial \lambda^{QSDAM}_{yrpt(yr,pt)}} \le 0$$
$$\begin{pmatrix} qsdam_{yrpt(yr,pt)} + qstdam_{yrpt(yr,pt)} - qstdam_{yrpt(yr-1,ptp)} - S0 \\ -Inflow\_state_{pt}.\left\{SwInflows_{yr} - SwEnvFlows_{yr} - SwLossFlows_{yr}\right\} \end{pmatrix} \le 0 \qquad \text{(FOC 8)}$$

$$\left(\frac{\partial L}{\partial \lambda^{QSDAM}_{yrpt(yr,pt)}}\right)\lambda^{QSDAM}_{yrpt(yr,pt)} = 0$$
$$\begin{pmatrix} qsdam_{yrpt(yr,pt)} + qstdam_{yrpt(yr,pt)} - qstdam_{yrpt(yr-1,ptp)} - S0 \\ -Inflow\_state_{pt}\left\{SwInflows_{yr} - SwEnvFlows_{yr} - SwLossFlows_{yr}\right\} \end{pmatrix}\lambda^{QSDAM}_{yrpt(yr,pt)} = 0 \qquad \text{(CS 8)}$$

FOC 8 is a restatement of constraint C 3. When this constraint is binding, which it invariably is because there is a loss of potential consumer welfare resulting from having ‘slack’ or unused water, the shadow price (opportunity cost) of using water in storage is positive.

Ninth condition — water demand balance constraint

$$\frac{\partial L}{\partial \lambda^{QD}_{yrpt(yr,pt)}} \leq 0$$

$$\sum_d \sum_l QtyQd_{d,yrpt(yr,pt),l}.wQd_{d,yrpt(yr,pt),l} - qsdam_{yrpt(yr,pt)} - qsdesal_{yrpt(yr,pt)} \leq 0 \tag{FOC 9}$$

$$\left(\frac{\partial L}{\partial \lambda^{QD}_{yrpt(yr,pt)}}\right)\lambda^{QD}_{yrpt(yr,pt)} = 0$$

$$\left(\sum_d \sum_l \begin{matrix} QtyQd_{d,yrpt(yr,pt),l}.wQd_{d,yrpt(yr,pt),l} \\ -qsdam_{yrpt(yr,pt)} - qsdesal_{yrpt(yr,pt)} \end{matrix}\right)\lambda^{QD}_{yrpt(yr,pt)} = 0 \tag{CS 9}$$

FOC 9 is a restatement of constraint C 4. The demand price of water for consumers is given by $\lambda^{QD}_{yrpt(yr,pt)}$.

Tenth condition — maximum desalination supply constraint

$$\frac{\partial L}{\partial \lambda^{QSDESAL}_{yrpt(yr,pt)}} \leq 0$$

$$qsdesal_{yrpt(yr,pt)} - \sum_{yrpt(yrp,ptp)}^{yr-1} SwDesalICap_{yrp}.incrdesal_{yrpt(yrp,ptp)}\Big|_{anc(pt,ptp)} \leq 0 \tag{FOC 10}$$

$$\left(\frac{\partial L}{\partial \lambda^{QSDESAL}_{yrpt(yr,pt)}}\right)\lambda^{QSDESAL}_{yrpt(yr,pt)} = 0$$

$$\left(\begin{matrix} qsdesal_{yrpt(yr,pt)} \\ -\sum_{yrpt(yrp,ptp)}^{yr-1} SwDesalICap_{yrp}.incrdesal_{yrpt(yrp,ptp)}\Big|_{anc(pt,ptp)} \end{matrix}\right)\lambda^{QSDESAL}_{yrpt(yr,pt)} = 0 \tag{CS 10}$$

FOC 10 is a restatement of constraint C 5. $\lambda^{QSDESAL}_{yrpt(yr,pt)}$ is the price per unit of water supplied from desalination, used to recover the investment costs and any capacity rents if capacity is binding.

Eleventh condition — upper bound desalination capacity constraint

$$\frac{\partial L}{\partial \lambda^{QDESALCAP}_{yrpt(yr,pt)}} \leq 0$$
$$\sum_{yrpt(yrp,ptp)}^{yr-1} incrdesal_{yrpt(yrp,ptp)}\Big|_{anc(pt,ptp)} - 1 \leq 0 \qquad \text{(FOC 11)}$$

$$\left(\frac{\partial L}{\partial \lambda^{QDESALCAP}_{yrpt(yr,pt)}}\right)\lambda^{QDESALCAP}_{yrpt(yr,pt)} = 0$$
$$\left(\sum_{yrpt(yrp,ptp)}^{yr-1} incrdesal_{yrpt(yrp,ptp)}\Big|_{anc(pt,ptp)} - 1\right)\lambda^{QDESALCAP}_{yrpt(yr,pt)} = 0 \qquad \text{(CS 11)}$$

FOC 11 is a restatement of constraint C 6. When investment in desalination has reached maximum allowed capacity there is an imputed rent on constrained capacity.

Twelfth condition — demand convexity constraint

$$\frac{\partial L}{\partial \lambda^{QDLIN}_{yrpt(yr,pt)}} \leq 0$$
$$\sum_{l} wQd_{d,yrpt(yr,pt),l} - 1 \leq 0 \qquad \text{(FOC 12)}$$

$$\left(\frac{\partial L}{\partial \lambda^{QDLIN}_{yrpt(yr,pt)}}\right)\lambda^{QDLIN}_{yrpt(yr,pt)} = 0$$
$$\left(\sum_{l} wQd_{d,yrpt(yr,pt),l} - 1\right)\lambda^{QDLIN}_{yrpt(yr,pt)} = 0 \qquad \text{(CS 12)}$$

FOC 12 is a restatement of constraint C 6. Consumer surplus from that level of demand is given by $\lambda^{QDLIN}_{d,yrpt(yr,pt)}$.

Thirteenth condition — demand for storage in final period convexity constraint

$$\frac{\partial L}{\partial \lambda_{lastpt}^{QSTLIN}} \le 0$$

$$\sum_{m} wQstdamf_{lastpt,m} - 1 \le 0 \qquad \text{(FOC 13)}$$

$$\left(\frac{\partial L}{\partial \lambda_{lastpt}^{QSTLIN}} \right) \lambda_{lastpt}^{QSTLIN} = 0$$

$$\left(\sum_{m} wQstdamf_{lastpt,m} - 1 \right) \lambda_{lastpt}^{QSTLIN} = 0 \qquad \text{(CS 13)}$$

FOC 13 is a restatement of constraint C 7. $\lambda_{lastpt}^{QSTLIN}$ is interpreted as the 'consumer surplus' for the derived demand for storage in the terminal period.

Fourteenth condition — linking constraint for the linearised final period's storage

$$\frac{\partial L}{\partial \lambda_{lastpt}^{QSTLAST}} \le 0$$

$$\sum_{m} QtyQstdamf_{lastpt,m}.wQstdamf_{lastpt,m} - qstdam_{yrpt(lastyr,lastpt)} \le 0 \qquad \text{(FOC 14)}$$

$$\left(\frac{\partial L}{\partial \lambda_{lastpt}^{QSTLAST}} \right) \lambda_{lastpt}^{QSTLAST} = 0$$

$$\left(\sum_{m} QtyQstdamf_{lastpt,m}.wQstdamf_{lastpt,m} - qstdam_{yrpt(lastyr,lastpt)} \right) \lambda_{lastpt}^{QSTLAST} = 0 \qquad \text{(CS 14)}$$

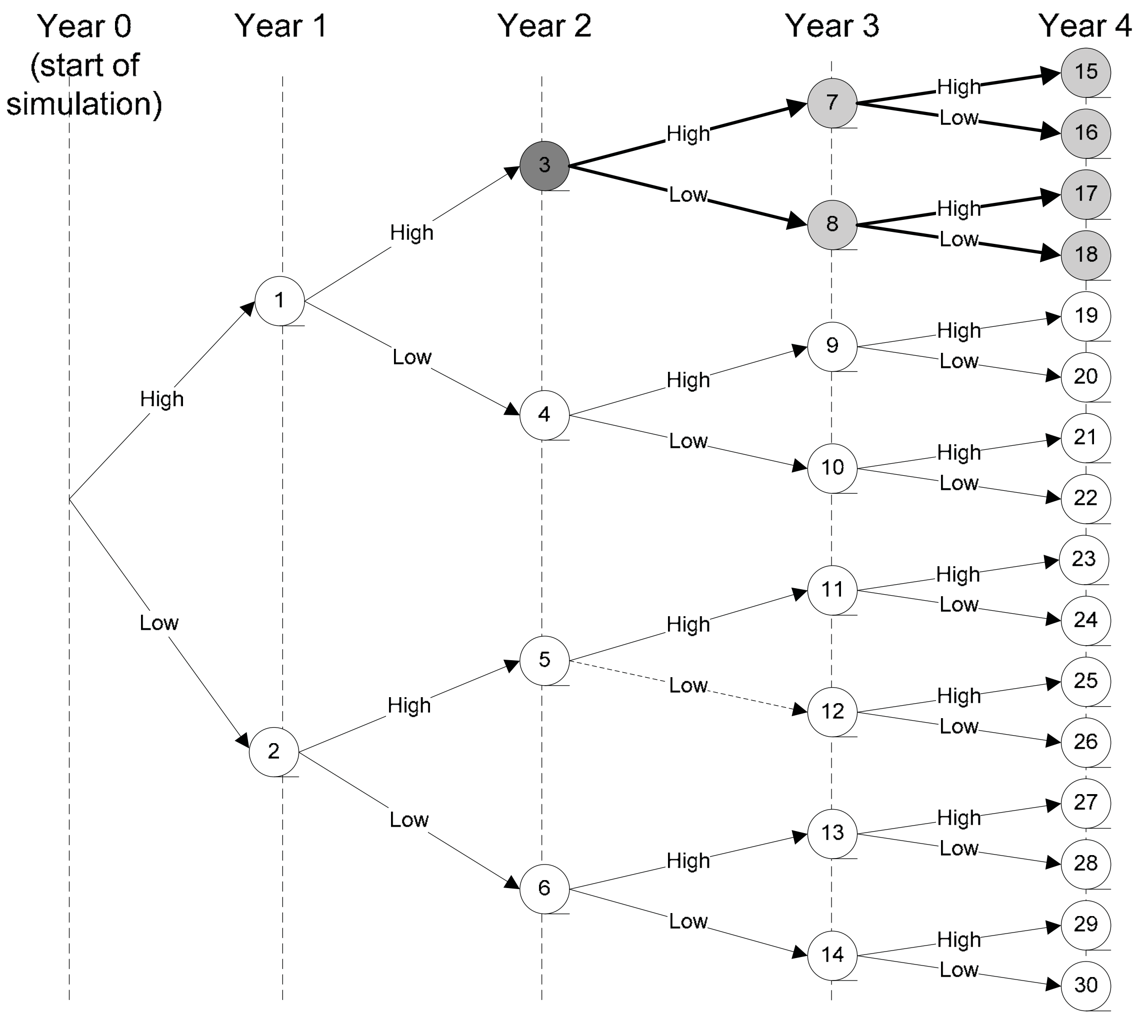
FOC 14 is a restatement of constraint C 8. $\lambda_{lastpt}^{QSTLAST}$ is the imputed price (opportunity cost) of storage in the terminal period based on the derived demand for storage in the terminal period.

Insights into the investment and supply decisions

The first order conditions outline the conditions for a market equilibrium, when the Takayama and Judge approach is embedded in a multistage stochastic framework. A decision to build a desalination plant is made with reference to expected inflows to dams in the future (figure C.2). Decisions also depend on what has happened until the decision point, to the extent that this influences the level of storages or how much investment has been made in new capacity. For example, if dams are full, or if large investments have already been made, then investment in new supply capacity is unlikely. But, aside from affecting how much water and existing capacity is available, decisions are otherwise independent of parts of the tree that do not follow from the decision point. Consider node 3 in figure C.2. The decision to invest in desalination is affected by inflows, consumption and storage outcomes in the tree up to node 3. It also reflects the expectations of inflows, consumption, storage and supply of water from desalination looking down the tree (future possible realisations) in nodes (7,8) and nodes (15-18).

Figure C.2 **Investment decision at node 3**

Decision is made 'looking down the tree'[a]



[a] An investment decision at node 3 is made based on expected returns to that investment across subsequent nodes. Preceding nodes (node 1) and nodes on other branches of the tree (nodes 4–6, 9–14 and 19–30) only affect the investment decision at node 3 indirectly, through the amount of supply capacity and water in storage brought forward to node 3.

C.3 Results from the simplified model

Solving the simplified model — calibrated so that an investment in desalination is occasionally made — yields results (table C.3) that illustrate those of the larger model. The quantity of water supplied from dams responds to inflows, and is lowest after several years of low inflows (for example, nodes 14 and 30). Similarly, prices

Table C.3 **Results by node**

Year	*Node*	*Water supplied (GL)*		*Demand price ($/kL)*	*Storage (GL)*	*Desalination investment (GL)*
		Dam	Desalination			
yr	*pt*	$qsdam$	$qsdesal$	$\frac{\lambda^{QD}}{\left(Df_{yr}.Prob_{pt}\right)}$	$qstdam$	$SwDesalICap_{yrp} \times incrdesal$
1	1	333		1.76	786	
1	2	313		2.05	572	
2	3	346		1.64	947	
2	4	319		2.02	739	
2	5	322		1.99	757	
2	6	302		2.30	544	56
3	7	365		1.42	1088	
3	8	326		1.98	893	
3	9	330		1.92	917	
3	10	305		2.29	707	90
3	11	332		1.88	932	
3	12	305		2.27	725	82
3	13	255	56	2.20	796	
3	14	228	56	2.60	589	34
4	15	391		1.13	1204	
4	16	341		1.83	1020	
4	17	351		1.70	1050	
4	18	297		2.44	870	
4	19	354		1.64	1069	
4	20	303		2.35	887	
4	21	253	71	2.07	961	
4	22	188	90	2.73	792	
4	23	358		1.58	1081	
4	24	306		2.33	899	
4	25	271	53	2.07	961	
4	26	200	82	2.68	798	
4	27	327		2.04	976	
4	28	235	56	2.56	834	
4	29	213	90	2.38	883	
4	30	164	90	3.07	698	

Source: Modelling results from the simplified model.

are highest during extended periods of low inflows. Dam storages vary across different scenarios, with storages falling during dry years. However, the results from the simple model are only illustrative, as they ignore many factors that are important in real-world urban water systems.

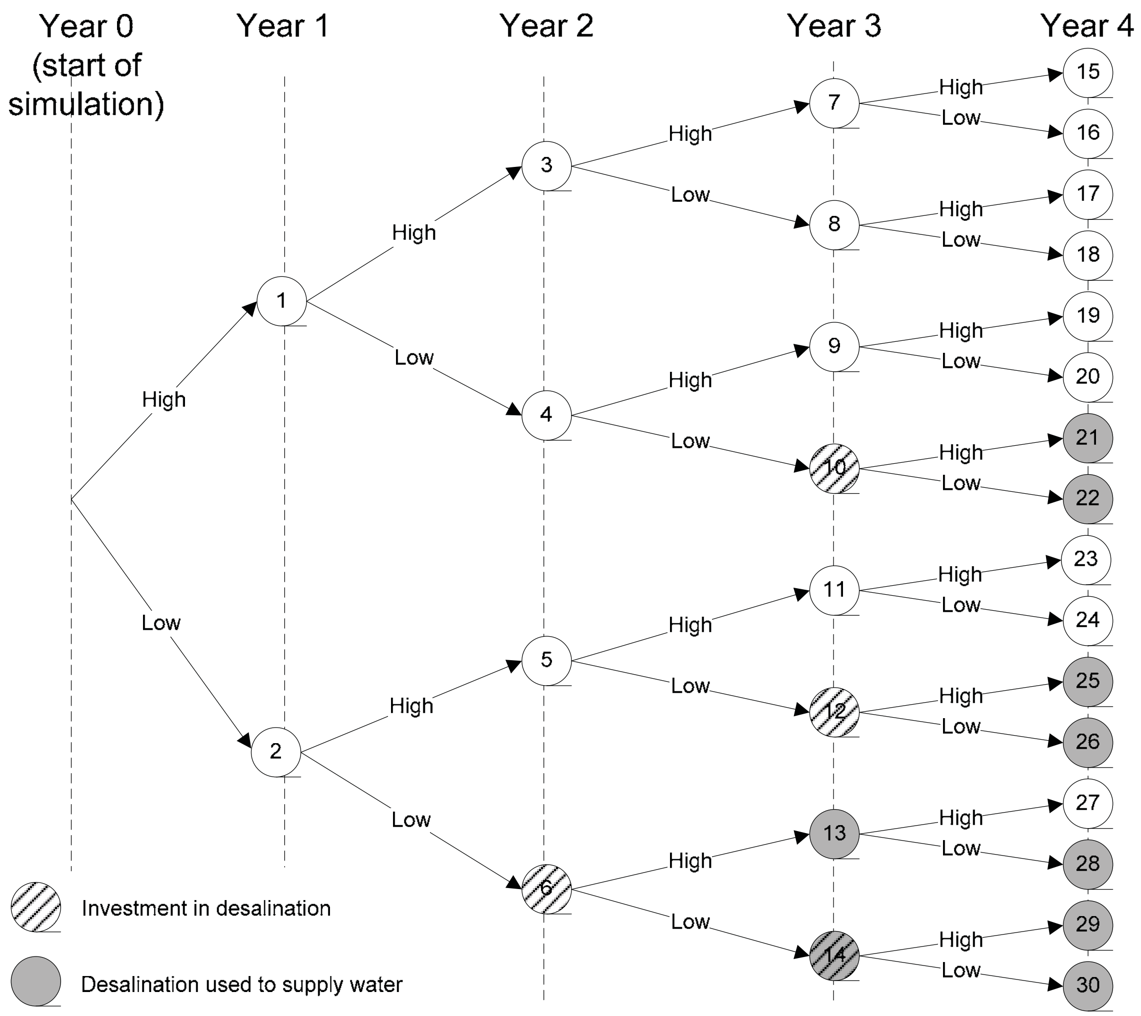
Decisions to invest in desalination are made based on current conditions and looking at potential returns down the tree. Investments are made when prices are highest: demand prices are above $2.25 each time a decision is made to invest in desalination (table C.3). High current period prices indicate that expected future prices are higher, as dam storages are used to link expected prices over time (demonstrated by FOC 4 in the previous section). So, when the dam is relatively empty, an investor is more likely to choose to build a desalination plant as they are more likely to make an acceptable return on their investment, based on the probability that water will remain relatively scarce.

Once an investment decision is made, a new plant is not necessarily used at full capacity, or indeed used at all. For example, at node 27 in year 4 there is desalination capacity available (from investment in year 2) but no water is provided from desalination because prices are not high enough to cover variable costs of desalination (figure C.3). If a decision to build new supply capacity is followed by unexpectedly wet weather, it is uneconomic to use the new supply capacity because water supply from dams is available at lower cost.

A desalination plant might be built but not used all of the time because investment decisions are made before actual inflows are known, based on whether expected returns (looking down the probability tree) cover investment costs. An investment is made whenever expected net revenue from the additional water supplied is greater than the investment cost (table C.4).

In some cases, expected investment returns include some capacity rent and thus exceed the investment cost. In this simplified example, as in the larger model, desalination capacity is limited to 90 GL. This means that in year 3, at node 10, net returns to investment more than cover costs, but no additional capacity can be built. This means that (on an expected value basis) a capacity rent accrues to the owner of the desalination plant. On the other hand, at node 12, expected returns to desalination only just cover costs, so construction stops at below maximum capacity and there are no capacity rents.

Figure C.3 **Results: investment in desalination and its use**



Data source: Modelling results from the simplified model.

In year 2, expected investment returns exceed costs, but desalination is not built to maximum capacity due to the ‘option value’ of holding off on investment. The investment decision in year 2 is more complex than investment in year 3, because it is not the last year in which investment can be made. When there are capacity rents looking down the tree, expected returns from desalination supply will need to exceed investment costs in order for an investment to be made (CS 6 from the first order conditions). As such, the optimal choice is to build some desalination capacity in year 2 — for which expected returns will exceed costs — and then ‘wait and see’ what happens to inflows in year 3 before building any more capacity. This captures the ‘option value’ referred to in real options analysis of investment decisions (Dixit and Pindyk 2001; WSAA 2008a), demonstrating that concepts from real options underlie decision making in the partial equilibrium model developed for this paper.

Table C.4 **Expected investment returns (ex ante)**

Year	*Node*	*Desalination investment (GL)*	*Investment cost ($M)*[a]	*Expected net revenue ($M)*	*Revenue/ cost ratio*
yr	*pt*	$SwDesalICap_{yrp} \times incrdesal$	$TrDesalInvC_{yr} \times incrdesal$	$\left(\lambda^{QD}_{yrpt(yr,pt)} - utcdam - vcdesal\right) \times \sum_{yrpt(yrp,ptp)} qsdesal\Big\vert_{anc(ptp,pt)}$	
2	6	56	31.1	39.9	1.28
3	10	90	25.7	27.9	1.09
3	12	82	23.3	23.3	1.00
3	14	34	9.7	21.1	2.17

[a] Investment cost is truncated to account for the proportion of the asset's life that is within the modelled timeframe.

Source: Modelling results from the simplified model.

Actual investment returns depend on realised rainfall after the investment decision has been made (table C.5). When it turns out to be wet, investment in a desalination plant is unlikely to pay off. For example, when a desalination investment is made in year 3, at node 10, then in the following wet year (node 21), revenue from desalination sales will only be sufficient to cover variable costs. In fact, the desalination plant is not even run at capacity (table C.2) as revenue from any additional desalination sales would not cover variable costs. The decision to build a desalination plant in year 3, at node 10, is based on the expected returns. If the following year turns out to be dry (node 22), then net revenue is more than twice that required to cover investment costs. The investment decision was made looking down the tree, taking the possibility of wet and dry outcomes into account. When realised investment returns (the revenue/cost ratio in table C.5) are probability weighted, this yields the expected payoffs at the time of investment (the revenue/cost ratio in table C.4) used to illustrate the original investment decision. The ex ante benefit–cost ratio must be at least one for investment to take place (table C.4). However, ex post it is possible to realise a benefit–cost ratio less than one (table C.5).

Table C.5 **Actual investment returns (ex post)**

Year	*Node*	*Desalination investment (GL)*	*Investment cost ($M)[a]*	*Scenario: final node*	*Realised net revenue ($M)*	*Revenue/ cost ratio*
yr	*pt*	$SwDesalICap_{yrp} \times incrdesal$	$TrDesalInvC_{yr} \times incrdesal$	*pt*	$\begin{pmatrix} \lambda^{QD}_{yrpt(yr,pt)} \\ -utcdam \\ -vcdesal \end{pmatrix} \times qsdesal$	
2	6	56	31.1	27	6.8	0.22
2	6	56	31.1	28	31.0	1.00
2	6	56	31.1	29	43.9	1.41
2	6	56	31.1	30	78.1	2.51
3	10	90	25.7	21	0.0	0.00
3	10	90	25.7	22	55.8	2.17
3	12	82	23.3	25	0.0	0.00
3	12	82	23.3	26	46.7	2.00
3	14	34	9.7	29	10.1	1.04
3	14	34	9.7	30	32.1	3.30

a Investment cost is truncated to account for the proportion of the asset's life that is within the modelled timeframe.

Source: Modelling results from the simplified model.

C.4 Relevance to the larger urban water model

The analysis contained in this appendix is not feasible for a larger model. A much larger probability tree, numerous investments with varying lags and modelling of regulatory constraints makes such an exercise difficult to follow at best, and at worst, intractable. However, the concepts demonstrated are important for understanding investment decisions in real urban-water systems and continue to be relevant in a larger, more complex version of the model.

The relationships and decision-making framework exposed in this appendix continue to hold in the larger model used for the rest of this paper. Investment and supply decisions continue to be based on expected values, looking down the probability tree. Results continue to vary with future rainfall, but distortions caused by regulated pricing and restrictions further complicate the story.

D Sensitivity analysis

Sensitivity analysis was undertaken to test the impact of variations to key parameters in the model. The parameters varied are:

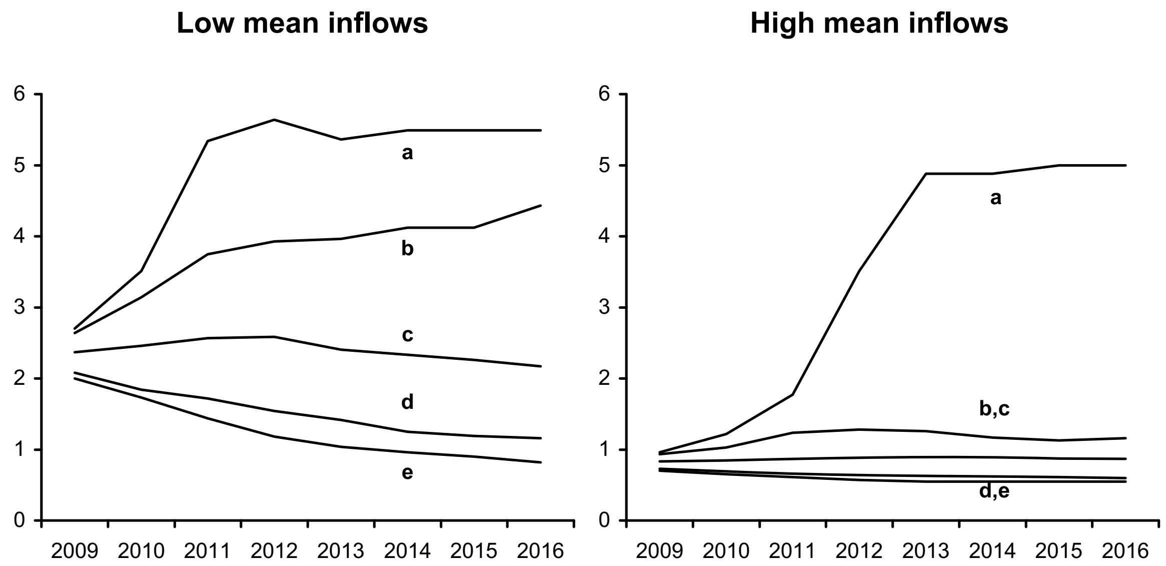
- mean inflows to dams (section D.1)
- the distribution of inflows modelled (section D.2)
- the weight attached to the low inflow scenarios (section D.3)
- price elasticities of demand (section D.4)
- the price and quantity point used to calibrate the linear demand function (section D.5)
- an alternative specification (constant price elasticity) of the demand function (section D.6)
- growth rates of urban water consumption (section D.7)
- discount rates (section D.8)
- initial storage levels (section D.9)
- an alternative specification of long-run marginal cost (LRMC) pricing (section D.10).

The effects of changes in these parameters on key results — prices, storage, investment, and the impact of various pricing and restrictions policies — are presented in this appendix. The results are compared to the 'central estimates': the simulation results presented in chapter 3 and 4, using all the default parameter values (as described in appendix B).

D.1 Mean inflows to dams

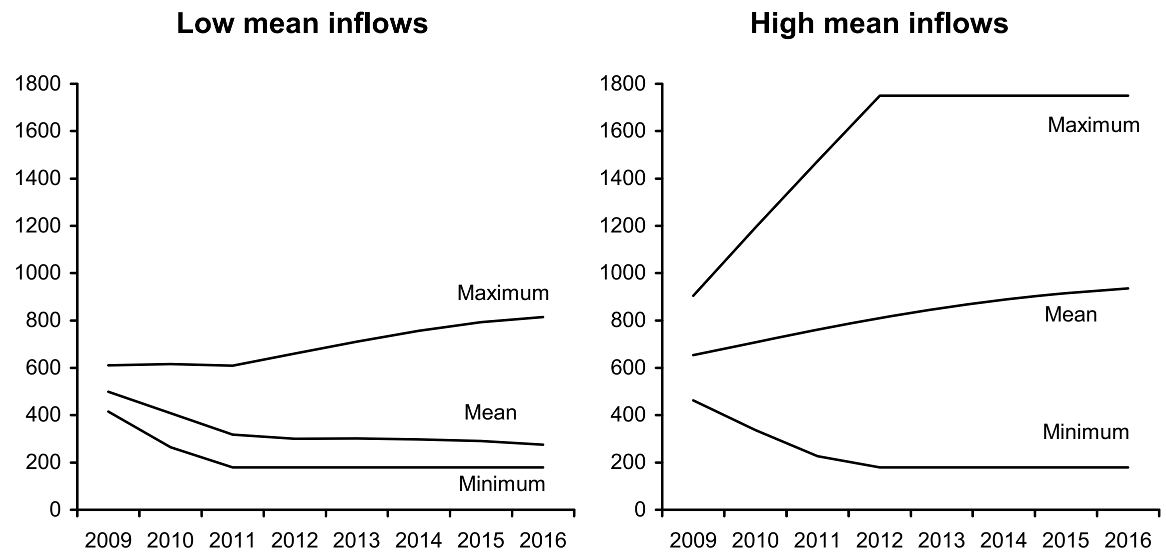
This sensitivity test involved running simulations with mean inflows 30 per cent higher (390 GL per year), and 30 per cent lower (210 GL per year) than the base case of 300 GL per year. Results for prices are included in figure D.1, storage in figure D.2, investment in figure D.3, and welfare results are in table D.1.

Figure D.1 **Price distributions for different mean inflow assumptions**
Under scarcity-based pricing ($/kL)



a Maximum price. **b** Ninetieth percentile price. **c** Median price. **d** Tenth percentile price. **e** Minimum price.
Data source: Modelling results.

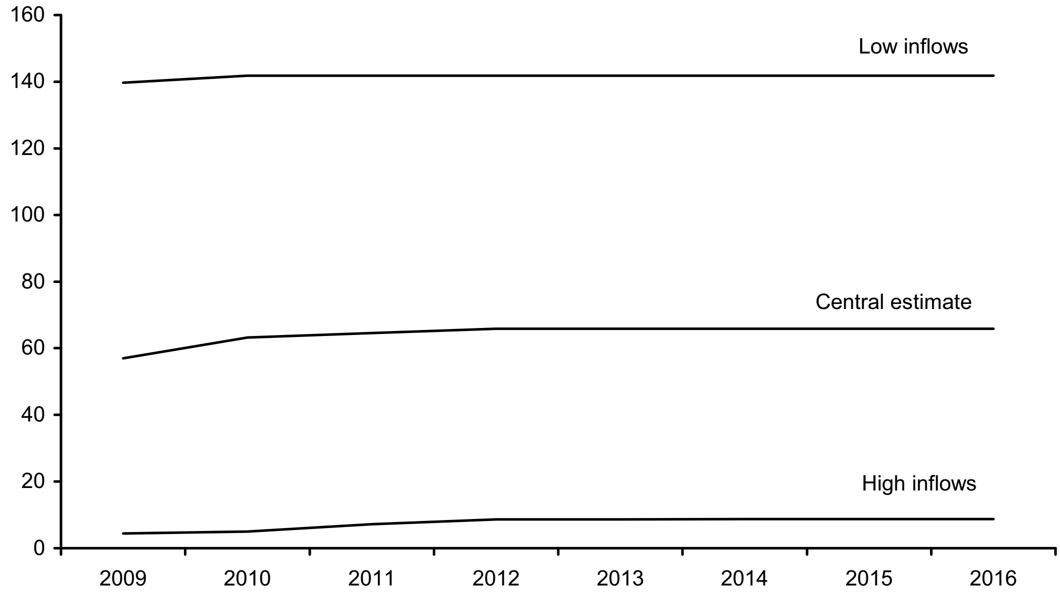
Figure D.2 **Water in storage for different mean inflow assumptions**
Under scarcity-based pricing (GL)



Data source: Modelling results.

Figure D.3 **Mean discounted, truncated investment expenditure for different mean inflow assumptions**[a]

Under scarcity-based pricing ($ million)



[a] Includes the total cost of investment in all new supply sources.

Data source: Modelling results.

Table D.1 **Net social welfare costs of policy constraints for different mean inflow assumptions**

Expected, net present value of costs ($ million) relative to scarcity-based pricing, for the next eight years

Policy	*Central estimate*	*Mean inflows*	
		Low (-30%)	High (+30%)
Restrictions	522	673	267
Restrictions and LRMC	658	1 026	599
LRMC	94	241	60
Cost recovery	153	339	137

Source: Modelling results.

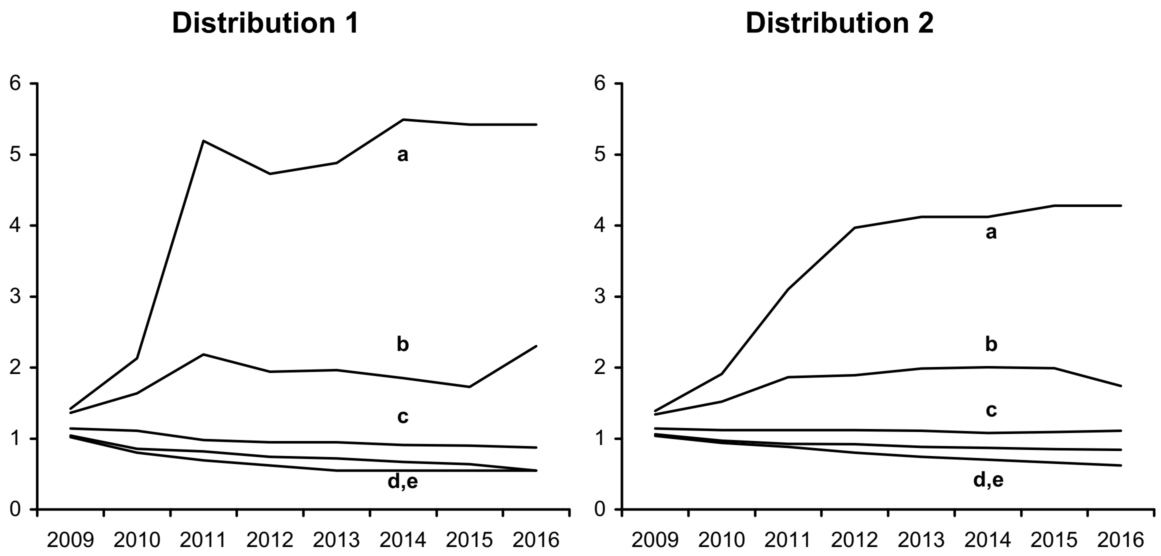
D.2 Distribution of inflows

This sensitivity test involved running simulations with alternative inflow distribution assumptions. These were constructed by changing the probabilities attached to low, medium and high inflows in the three-point discrete distribution of inflows. Inflows to dams under the alternate distributions were set by maintaining the median, mean and standard deviation of inflows (table D.2). Results are shown in figures D.4, D.5 and D.6, and in table D.3.

Table D.2 **Distribution of annual inflows for sensitivity testing**

	Base case		*Distribution 1*		*Distribution 2*	
	Probability	Inflows (GL)	Probability	Inflows (GL)	Probability	Inflows (GL)
Low	0.25	180	0.10	156	0.33	189
Medium	0.50	300	0.80	300	0.33	300
High	0.25	573	0.10	744	0.33	537

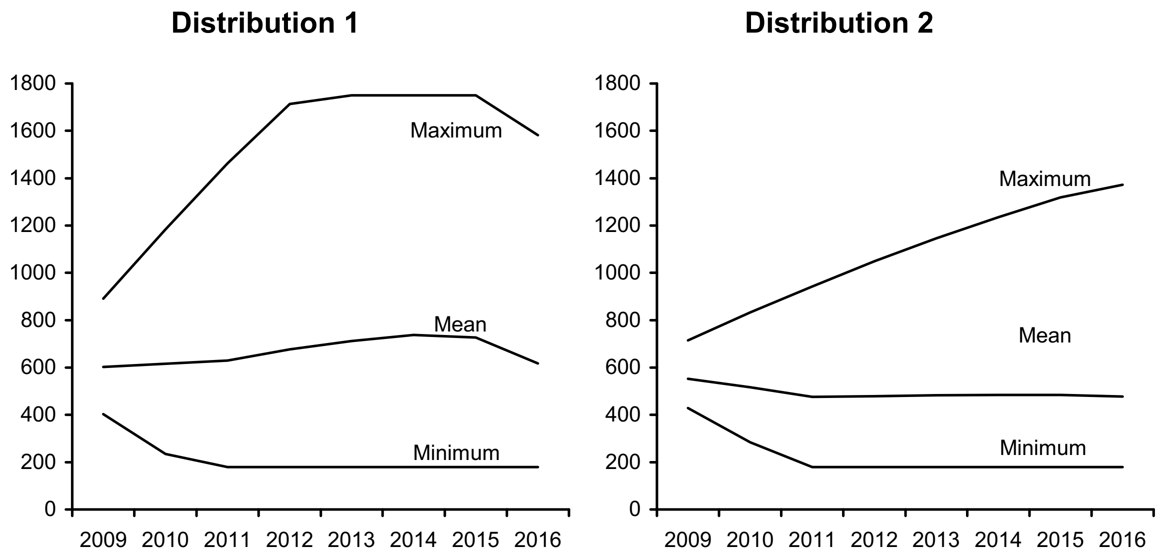
Figure D.4 **Price distributions for different inflow distribution assumptions**
Under scarcity-based pricing ($/kL)



[a] Maximum price. [b] Ninetieth percentile price. [c] Median price. [d] Tenth percentile price. [e] Minimum price.

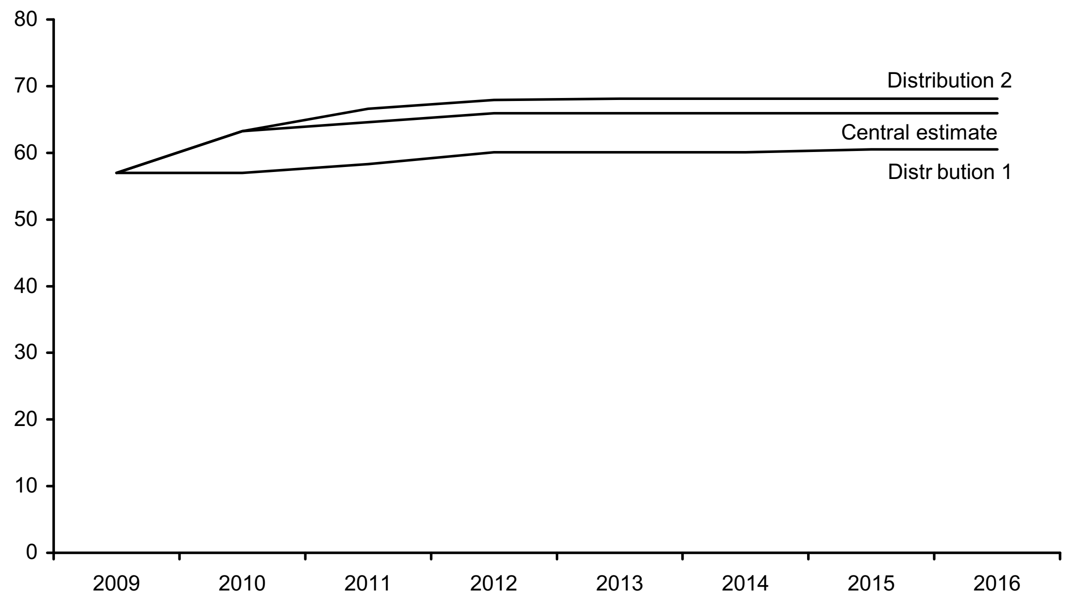
Data source: Modelling results.

Figure D.5 **Water in storage for different inflow distribution assumptions**
Under scarcity-based pricing (GL)



Data source: Modelling results.

Figure D.6 **Mean discounted, truncated investment expenditure for different inflow distribution assumptions**[a]
Under scarcity-based pricing ($ million)



[a] Includes the total cost of investment in all new supply sources.

Data source: Modelling results.

Table D.3 **Net social welfare costs of policy constraints for different inflow distribution assumptions**

Expected, net present value of costs ($ million) relative to scarcity-based pricing, for the next eight years

Policy	*Central estimate*	*Distribution of inflows*	
		Distribution 1	Distribution 2
Restrictions	522	549	466
Restrictions and LRMC	658	651	786
LRMC	94	97	97
Cost recovery	153	141	152

Source: Modelling results.

D.3 Weight attached to low inflow scenarios

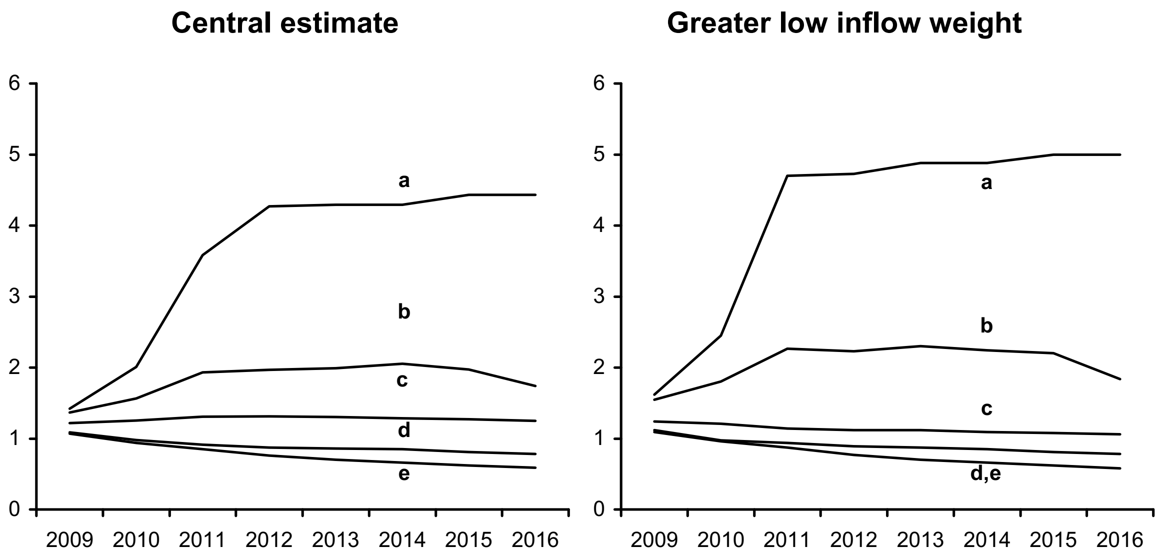
This sensitivity test involved running the model with additional weight attached to low inflow scenarios in the probability tree. This can be thought of as representing a decision maker that is particularly concerned about low inflow events. In order to implement this, an additional reduction was imposed on the level of inflows attached to low inflow events. This means that deviations in inflows that would only be expected to occur in 10 per cent of years (based on historical data) are given a 25 per cent weighting, effectively increasing the weight given to low inflow scenarios (table D.4). Results are shown in figures D.7, D.8 and D.9, and in table D.5.

Table D.4 **Inflow weights for sensitivity testing**

	Base case		*Alternative weighting*	
	Probability	Inflows (GL)	Probability	Inflows (GL)
Low	0.25	180	0.25	156
Medium	0.50	300	0.50	300
High	0.25	573	0.25	573

Figure D.7 **Price distributions for greater weighting of low inflows**

Under scarcity-based pricing ($/kL)

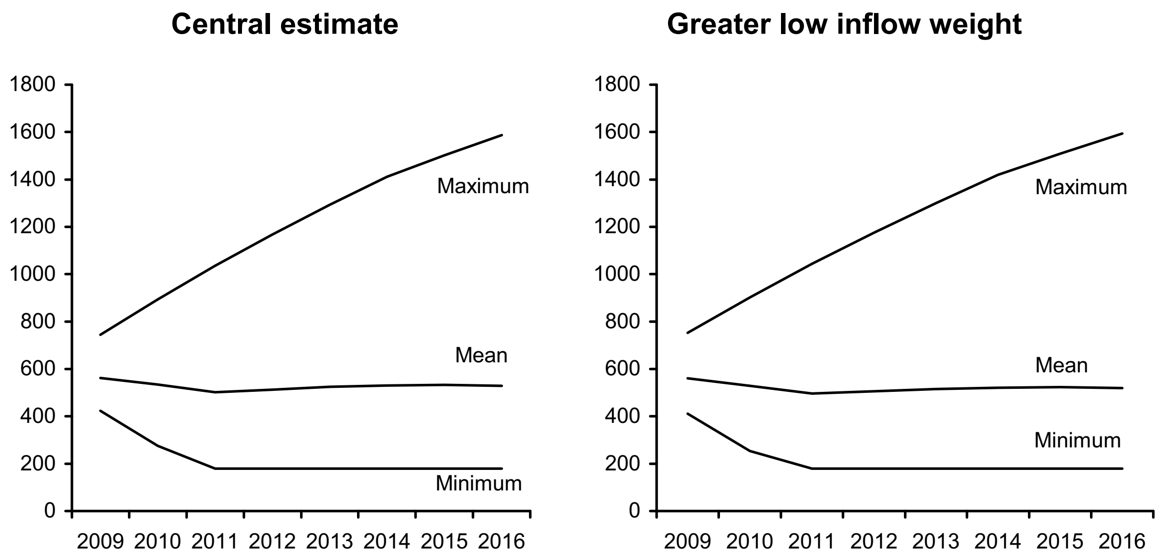


[a] Maximum price. [b] Ninetieth percentile price. [c] Median price. [d] Tenth percentile price. [e] Minimum price.

Data source: Modelling results.

Figure D.8 **Water in storage for greater weighting of low inflows**

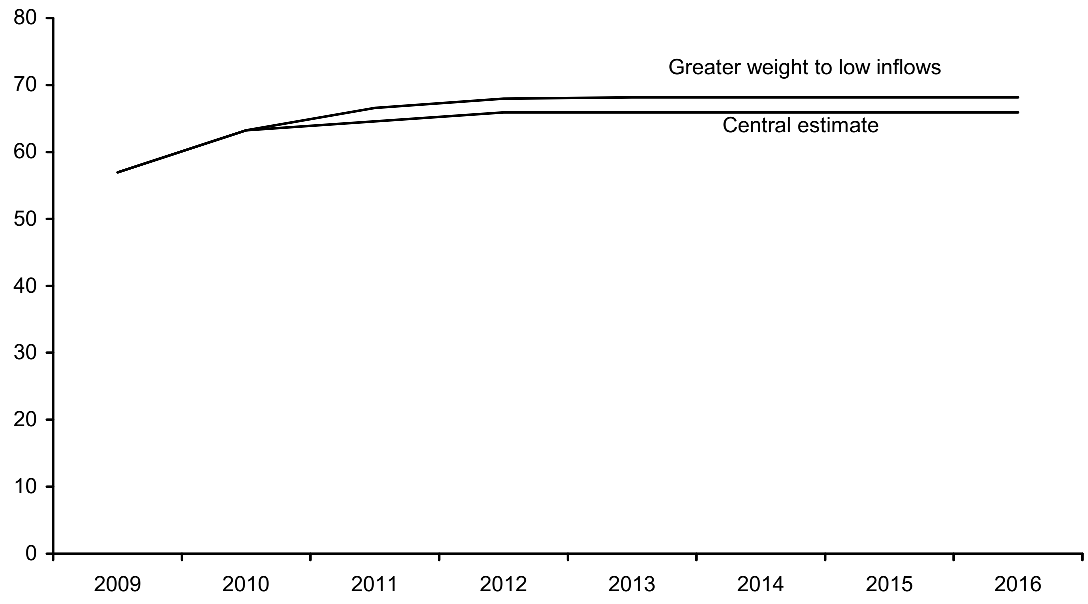
Under scarcity-based pricing (GL)



Data source: Modelling results.

Figure D.9 **Mean discounted, truncated investment expenditure for greater weighting of low inflows**[a]

Under scarcity-based pricing ($ million)



[a] Includes the total cost of investment in all new supply sources.

Data source: Modelling results.

Table D.5 **Net social welfare costs of policy constraints for greater weighting of low inflows**

Expected, net present value of costs ($ million) relative to scarcity-based pricing, for the next eight years

Policy	*Central estimate*	*Greater weighting of low inflows*
Restrictions	522	544
Restrictions and LRMC	658	698
LRMC	94	101
Cost recovery	153	243

Source: Modelling results.

D.4 Price elasticity of demand

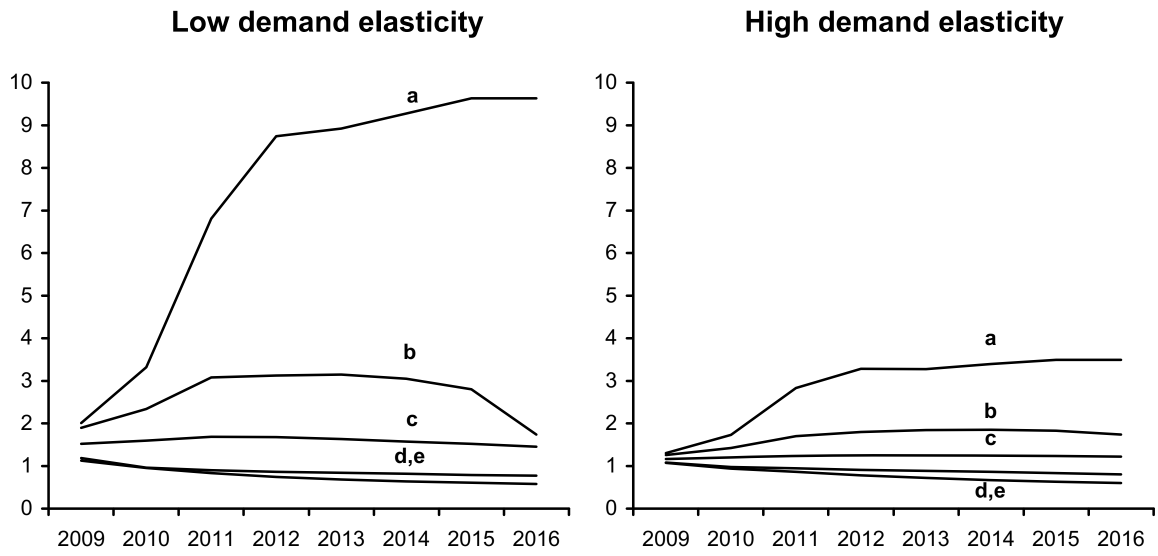
Sensitivity tests were performed in the demand elasticity, to examine how responsive the results are to different elasticity parameter values. A low elasticity (aggregate household elasticity of -0.1) and a high elasticity (aggregate household price elasticity of demand of -0.5) are tested (appendix B).

Testing the sensitivity of the price elasticity of demand can also be interpreted as a test on partial risk aversion in the consumption of water. The degree of partial risk aversion can be quantified using a measure analogous to the Arrow-Pratt measure of relative risk aversion, calculated as $\frac{-Q.u''(Q)}{u'(Q)}$ (Menezes and Hanson 1970). Q is the quantity of water consumed, and $u(Q)$ is the utility function for consumers. Partial risk aversion increases with lower demand elasticity (Table D.6). Results are shown in figures D.10, D.11 and D.12, and in table D.7.

Table D.6 **Partial risk aversion for various elasticity values**
Measured at the mean level of consumption

Price elasticity of demand	*Relative risk aversion*
Low	8.5
Central estimate	7.4
High	6.5

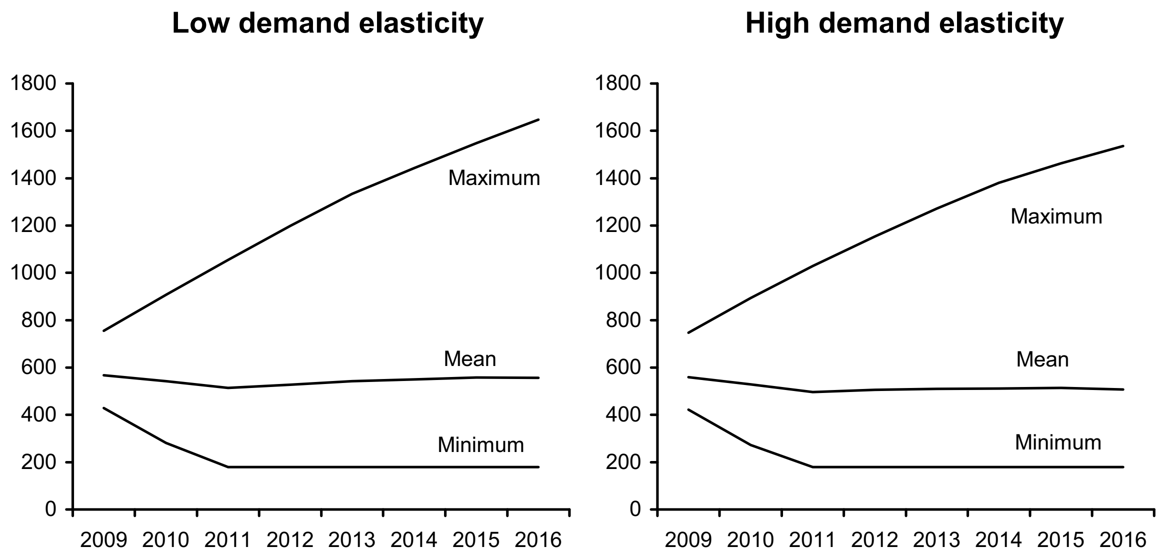
Figure D.10 **Price distributions for different demand elasticity values**
Under scarcity-based pricing ($/kL)



[a] Maximum price. [b] Ninetieth percentile price. [c] Median price. [d] Tenth percentile price. [e] Minimum price.

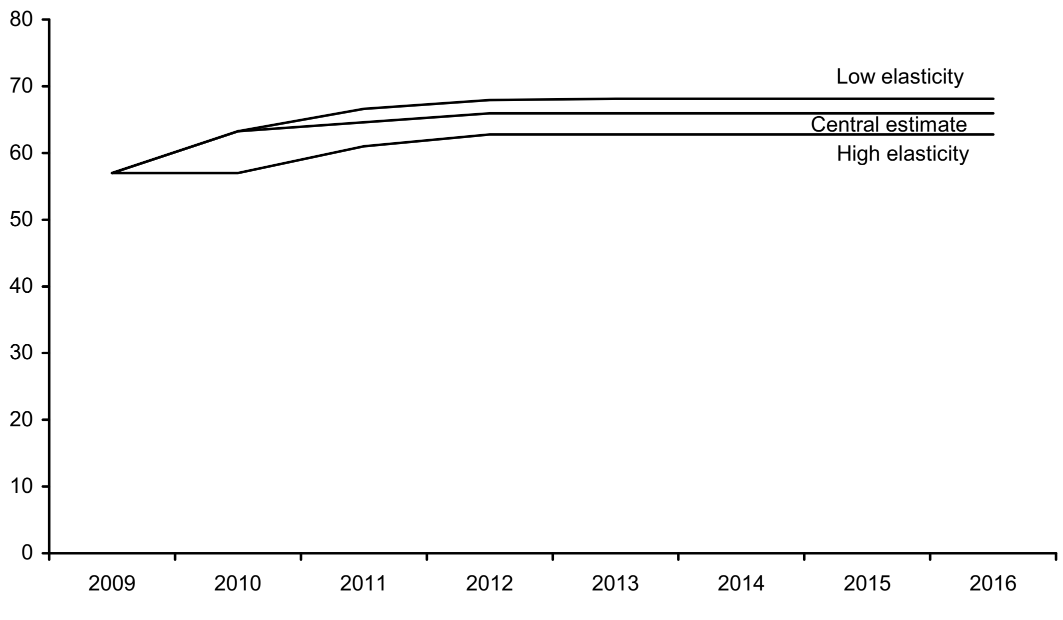
Data source: Modelling results.

Figure D.11 **Water in storage for different demand elasticity values**
Under scarcity-based pricing (GL)



Data source: Modelling results.

Figure D.12 **Mean discounted, truncated investment expenditure for different demand elasticity values**[a]
Under scarcity-based pricing ($ million)



[a] Includes the total cost of investment in all new supply sources.

Data source: Modelling results.

Table D.7 **Net social welfare costs of policy constraints for different demand elasticity values**

Expected, net present value of costs ($ million) relative to scarcity-based pricing, for the next eight years

Policy	*Central estimate*	*Demand elasticity*	
		Low (-0.10)	High (-0.5)
Restrictions	522	1 013	401
Restrictions and LRMC	658	1 573	548
LRMC	94	149	117
Cost recovery	153	225	134

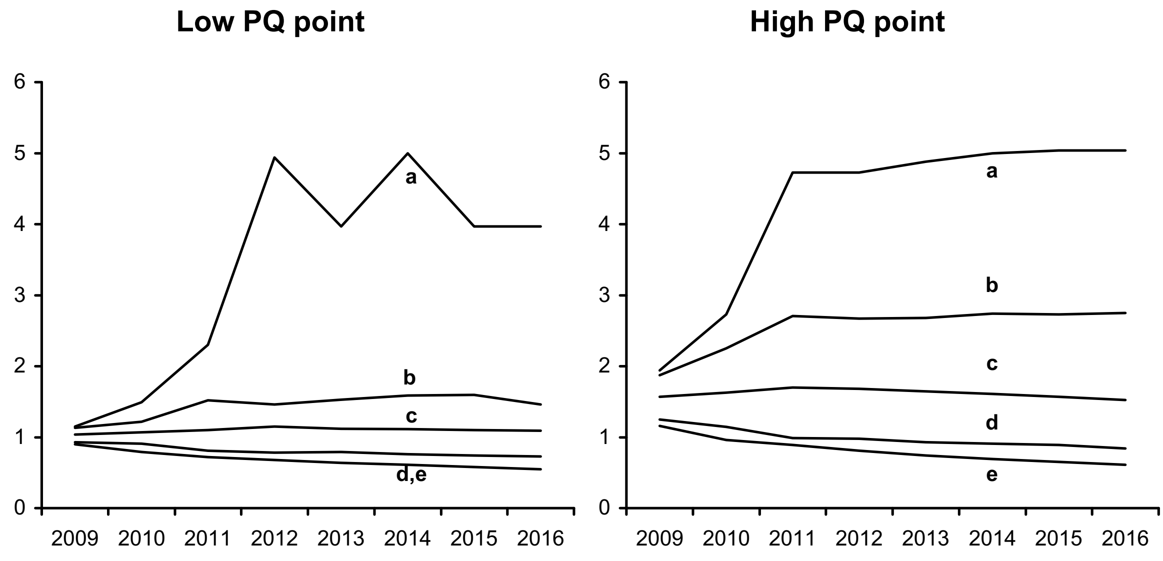
Source: Modelling results.

D.5 Price and quantity point used for demand calibration

The model demand functions (the quantity demanded for a given price) are calibrated using a price and quantity (PQ) reference point. Changes in consumer behaviour — for example, in response to public education or moral suasion initiatives — could change the location of the demand function in the future. This sensitivity test examined the impact of increasing and decreasing the quantity of water consumed at the calibration points by 10 per cent. Results are shown in figures D.13, D.14 and D.15, and in table D.8.

Figure D.13 **Price distributions for different demand calibrations**

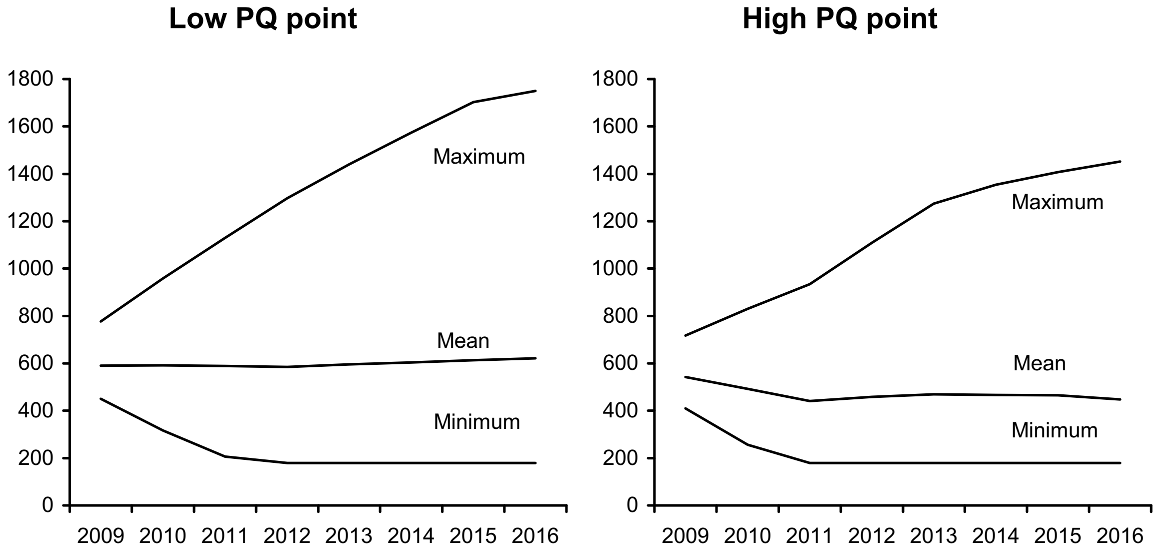
Under scarcity-based pricing ($/kL)



[a] Maximum price. [b] Ninetieth percentile price. [c] Median price. [d] Tenth percentile price. [e] Minimum price.

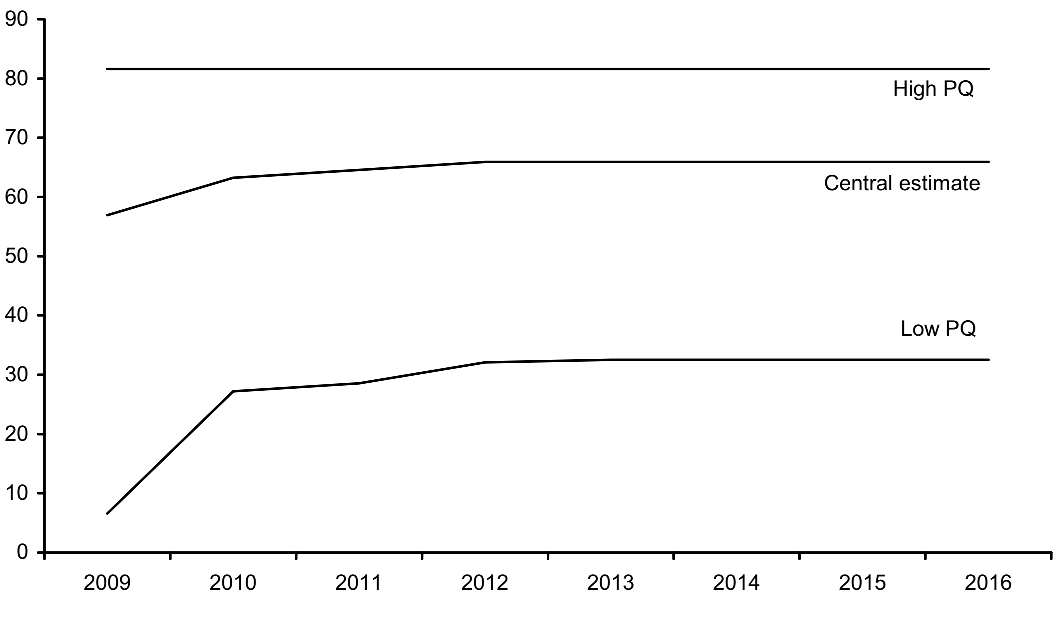
Data source: Modelling results.

Figure D.14 **Water in storage for different demand calibrations**
Under scarcity-based pricing (GL)



Data source: Modelling results.

Figure D.15 **Mean discounted, truncated investment expenditure for different demand calibrations**[a]
Under scarcity-based pricing ($ million)



a Includes the total cost of investment in all new supply sources.

Data source: Modelling results.

Table D.8 **Net social welfare costs of policy constraints for different demand calibrations**

Expected, net present value of costs ($ million) relative to scarcity-based pricing, for the next eight years

Policy	*Central estimate*	*Price and quantity for calibration*	
		Low (-10%)	High (+10%)
Restrictions	522	406	679
Restrictions and LRMC	658	564	813
LRMC	94	84	160
Cost recovery	153	117	293

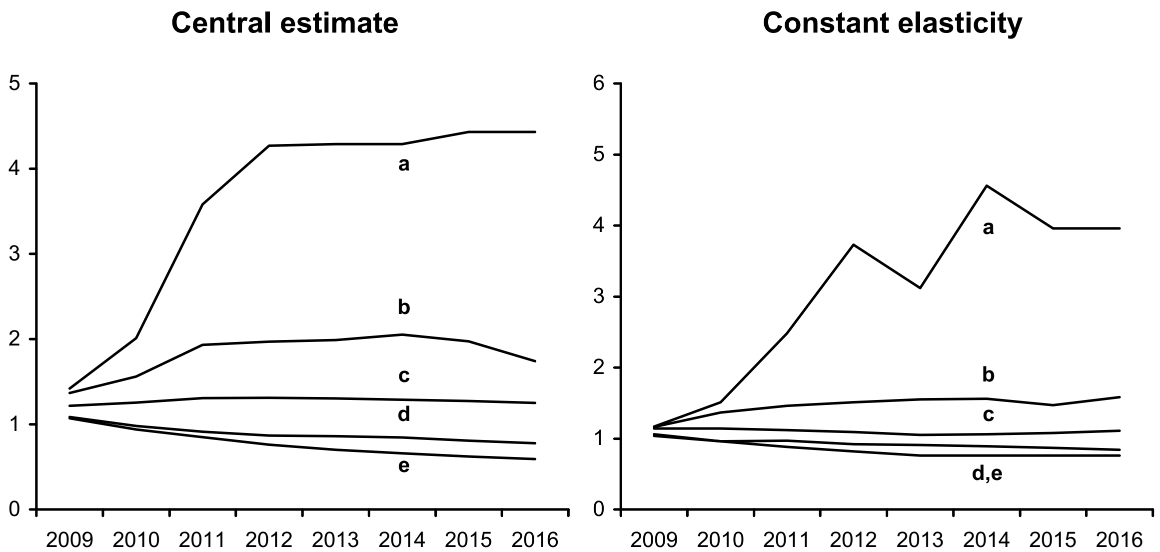
Source: Modelling results.

D.6 Constant price elasticity demand function

The model's sensitivity to demand specification was also examined. This was tested by substituting the linear demand function with a constant elasticity of demand function, calibrated to the same elasticity and demand point. Results are shown in figures D.16, D.17 and D.18, and in table D.9.

Figure D.16 **Price distributions for constant elasticity of demand**

Under scarcity-based pricing ($/kL)

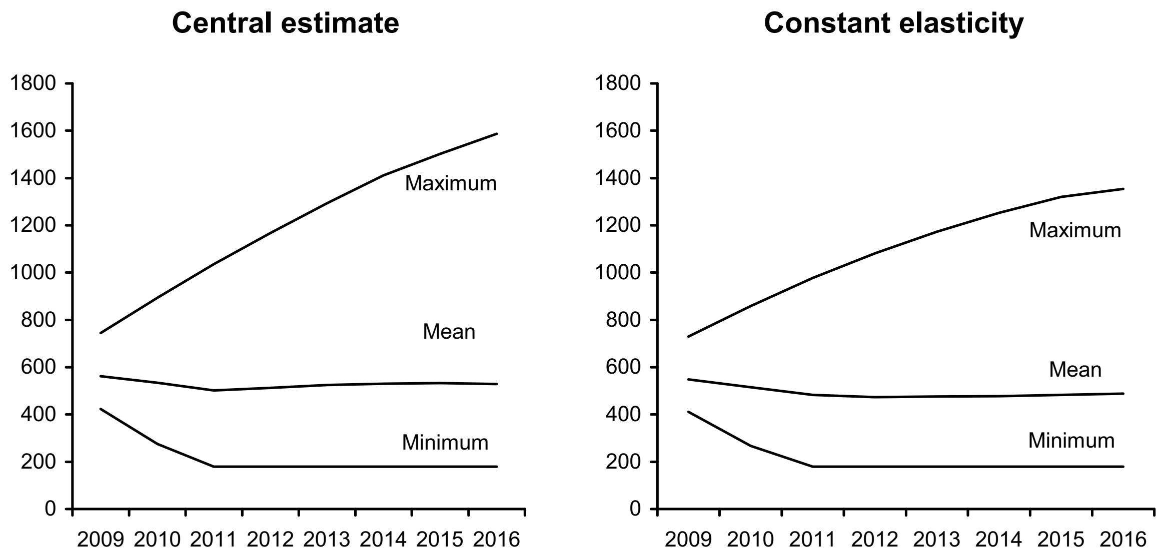


[a] Maximum price. [b] Ninetieth percentile price. [c] Median price. [d] Tenth percentile price. [e] Minimum price.

Data source: Modelling results.

Figure D.17 **Water in storage for constant elasticity of demand**

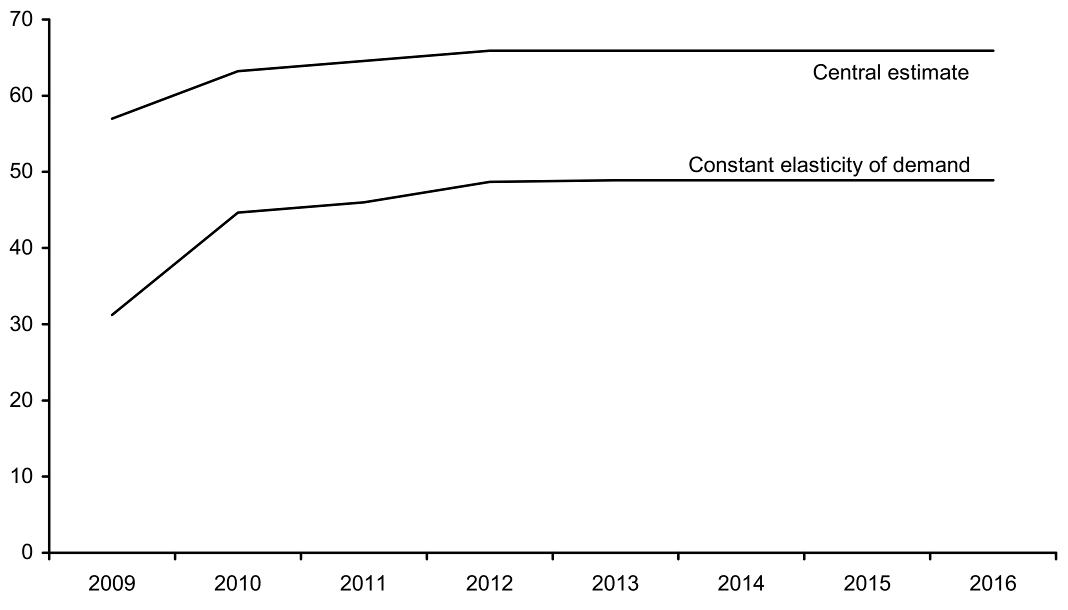
Under scarcity-based pricing (GL)



Data source: Modelling results.

Figure D.18 **Mean discounted, truncated investment expenditure for constant elasticity of demand**[a]

Under scarcity-based pricing ($ million)



a Includes the total cost of investment in all new supply sources.

Data source: Modelling results.

Table D.9 Net social welfare costs of policy constraints for different mean inflow assumptions

Expected, net present value of costs ($ million) relative to scarcity-based pricing, for the next eight years

Policy	*Central estimate*	*Constant elasticity demand function*
Restrictions	522	644
Restrictions and LRMC	658	822
LRMC	94	106
Cost recovery	153	256

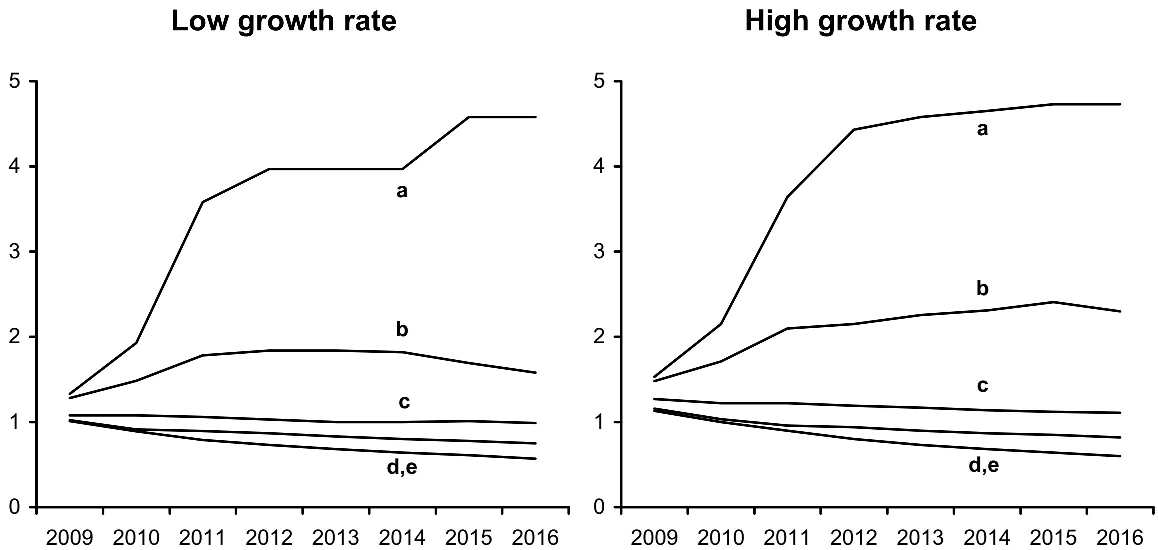
Source: Modelling results.

D.7 Growth rates of demand

The sensitivity of the model results to the growth rate of demand was tested by increasing and reducing the growth rate by 1 percentage point relative to the base case. Results are shown in figures D.19, D.20 and D.21, and in table D.10.

Figure D.19 Price distributions for different growth rates

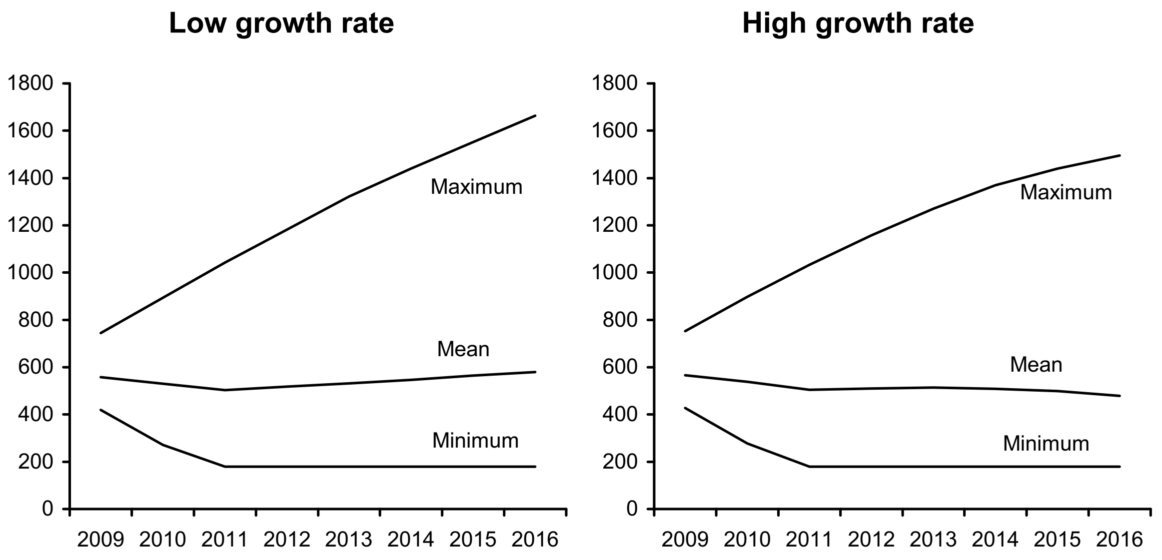
Under scarcity-based pricing ($/kL)



[a] Maximum price. [b] Ninetieth percentile price. [c] Median price. [d] Tenth percentile price. [e] Minimum price.

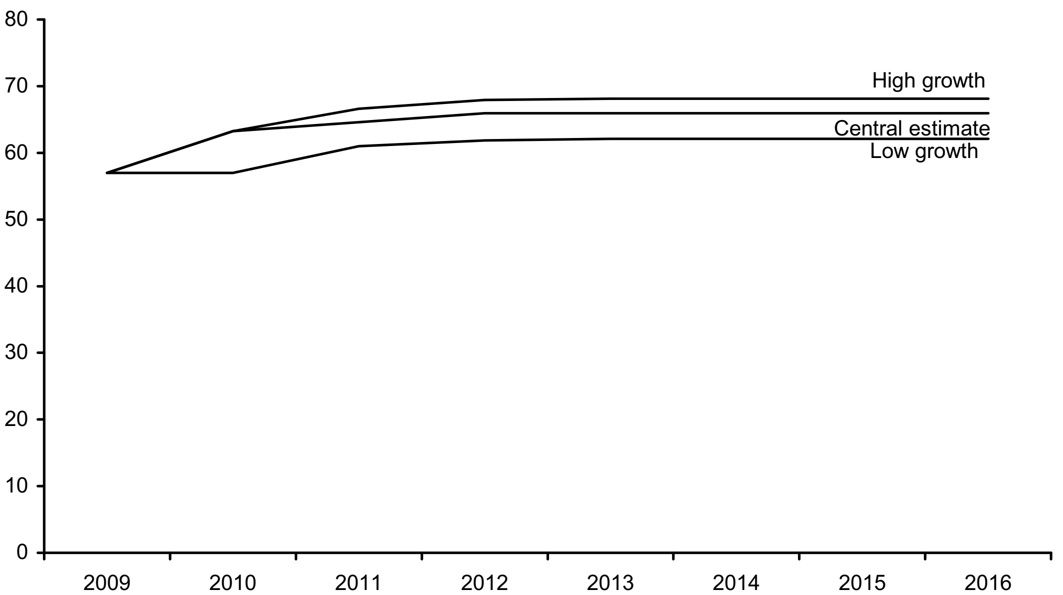
Data source: Modelling results.

Figure D.20 **Water in storage for different growth rates**
Under scarcity-based pricing (GL)



Data source: Modelling results.

Figure D.21 **Mean discounted, truncated investment expenditure for different growth rates**[a]
Under scarcity-based pricing ($ million)



[a] Includes the total cost of investment in all new supply sources.

Data source: Modelling results.

Table D.10 **Net social welfare costs of policy constraints for different growth rates**

Expected, net present value of costs ($ million) relative to scarcity-based pricing, for the next eight years

Policy	*Central estimate*	*Growth rates of demand*	
		Low (-1 ppt)[a]	High (+1 ppt)[a]
Restrictions	522	487	549
Restrictions and LRMC	658	637	583
LRMC	94	89	97
Cost recovery	153	134	176

[a] ppt.: percentage point

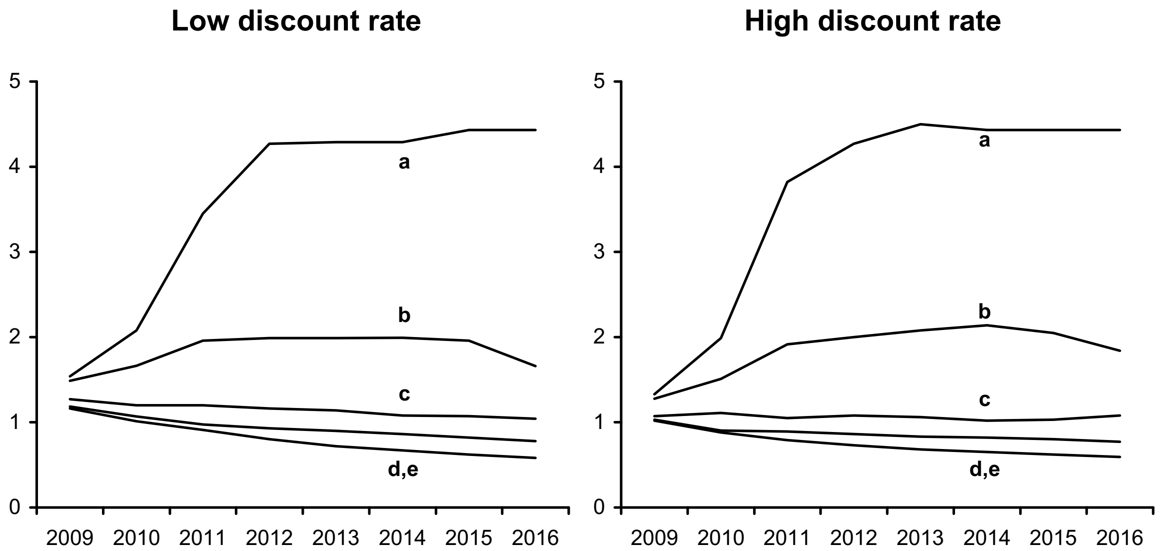
Source: Modelling results.

D.8 Discount rates

The sensitivity of the model results to the discount rate was tested by increasing and reducing the discount rate by 4 percentage points relative to the base case. Results are shown in figures D.22, D.23 and D.24, and in table D.11.

Figure D.22 **Price distributions for different discount rates**

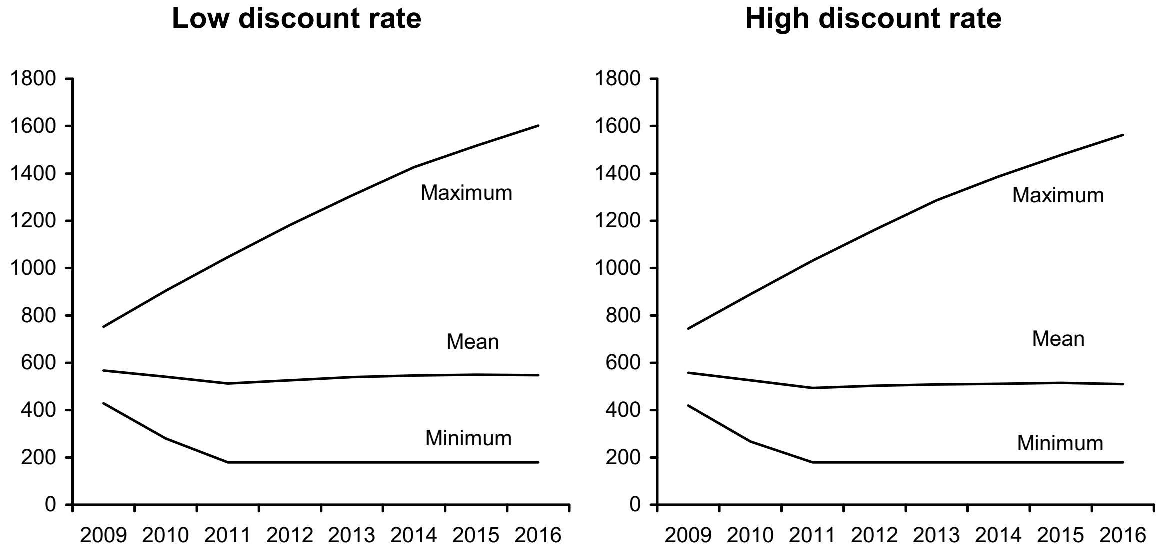
Under scarcity-based pricing ($/kL)



[a] Maximum price. [b] Ninetieth percentile price. [c] Median price. [d] Tenth percentile price. [e] Minimum price.

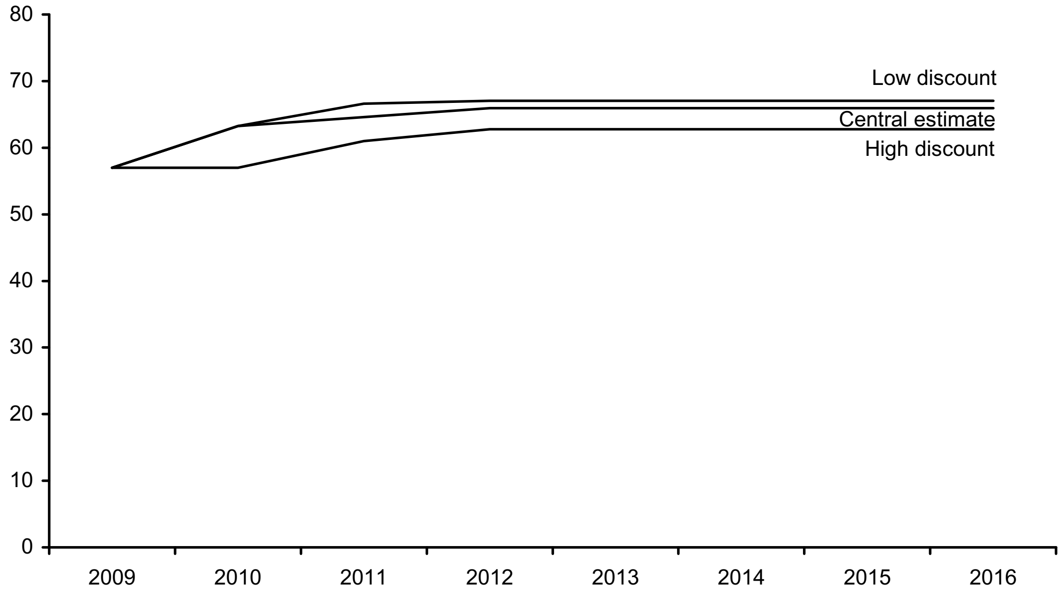
Data source: Modelling results.

Figure D.23 **Water in storage for different discount rates**
Under scarcity-based pricing (GL)



Data source: Modelling results.

Figure D.24 **Mean discounted, truncated investment expenditure for different discount rates[a]**
Under scarcity-based pricing ($ million)



[a] Includes the total cost of investment in all new supply sources.

Data source: Modelling results.

Table D.11 **Net social welfare costs of policy constraints for different discount rates**

Expected, net present value of costs ($ million) relative to scarcity-based pricing, for the next eight years

Policy	*Central estimate*	*Discount rates*	
		Low (2 per cent)	High (10 per cent)
Restrictions	522	540	507
Restrictions and LRMC	658	789	816
LRMC	94	98	92
Cost recovery	153	151	152

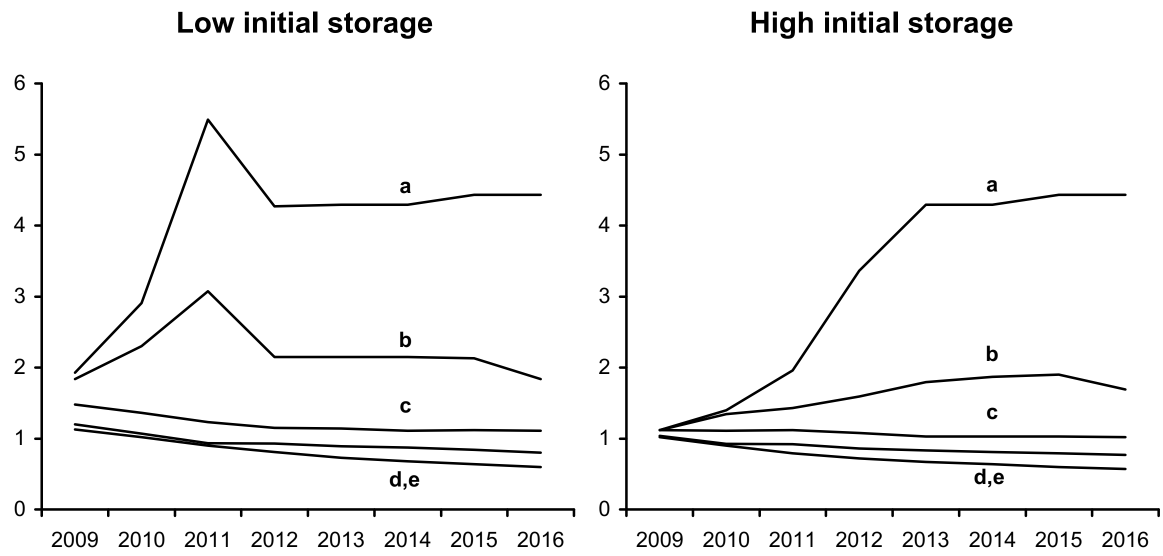
Source: Modelling results.

D.9 Initial storages

The sensitivity of the model results to the initial level of dam storage was tested by increasing and reducing initial storages by 20 per cent relative to the base case level of 35 per cent of capacity. Results are shown in figures D.25, D.26 and D.27, and in table D.12.

Figure D.25 **Price distributions for different initial storages**

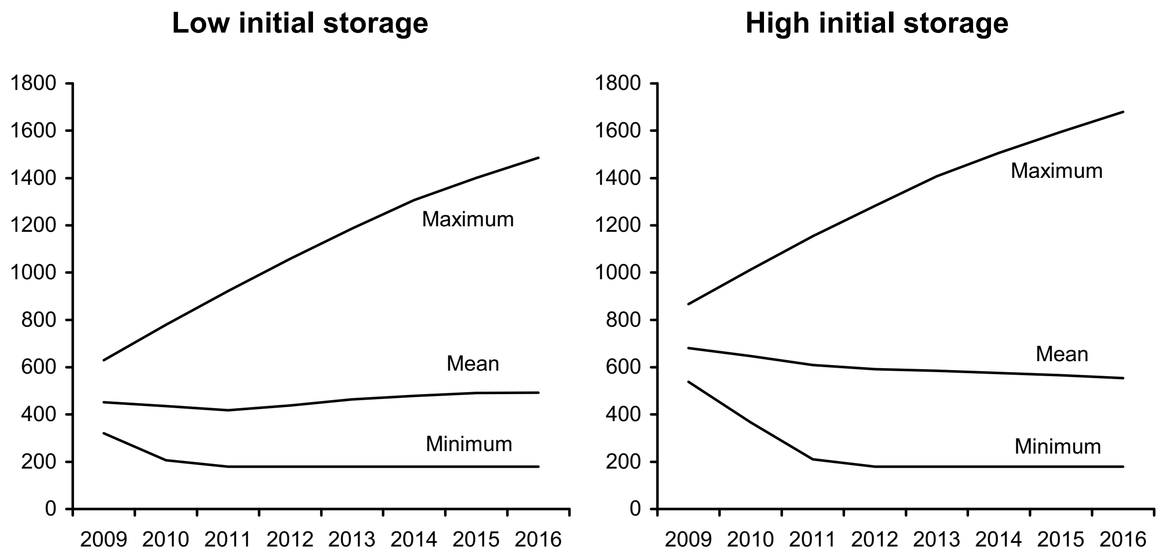
Under scarcity-based pricing ($/kL)



a Maximum price. **b** Ninetieth percentile price. **c** Median price. **d** Tenth percentile price. **e** Minimum price.

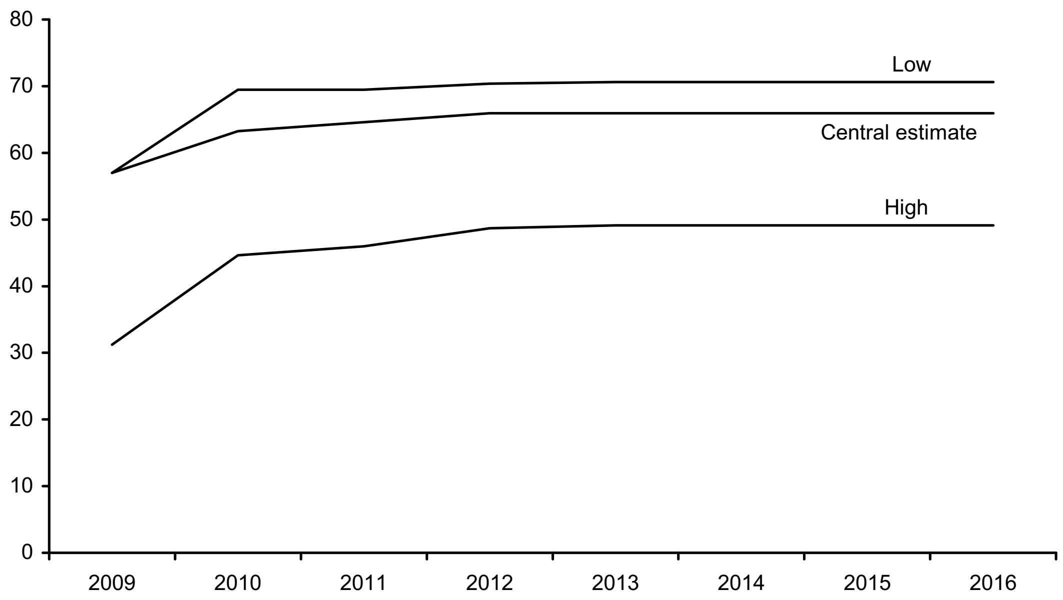
Data source: Modelling results.

Figure D.26 **Water in storage for different initial storages**
Under scarcity-based pricing (GL)



Data source: Modelling results.

Figure D.27 **Mean discounted, truncated investment expenditure for different initial storages**[a]
Under scarcity-based pricing ($ million)



a Includes the total cost of investment in all new supply sources.

Data source: Modelling results.

Table D.12 **Net social welfare costs of policy constraints for different initial storages**

Expected, net present value of costs ($ million) relative to scarcity-based pricing, for the next eight years

Policy	*Central estimate*	*Initial storage*	
		Low (-20 per cent)	High (+20 per cent)
Restrictions	522	641	339
Restrictions and LRMC	658	1 320	580
LRMC	94	116	87
Cost recovery	153	302	87

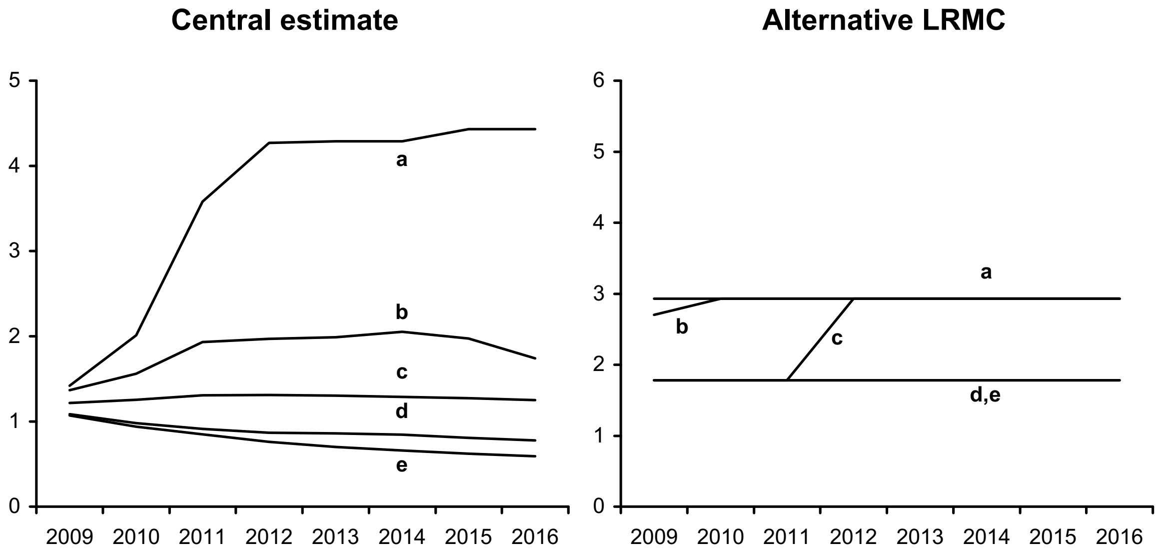
Source: Modelling results.

D.10 Alternative specification of long-run marginal cost pricing

An alternative specification of long-run marginal cost (LRMC) was also examined, in order to test the sensitivity of the LRMC simulations. The alternative specification involved setting all prices based on the levelised cost of the next cheapest source of supply. Initially, this is based on the cost of rural–urban trade. After the pipe has been commissioned, the price increases in line with the cost of desalination. Results are shown in figures D.28, D.29 and D.30, and in table D.13.

Figure D.28 **Price distributions for the alternative LRMC specification**

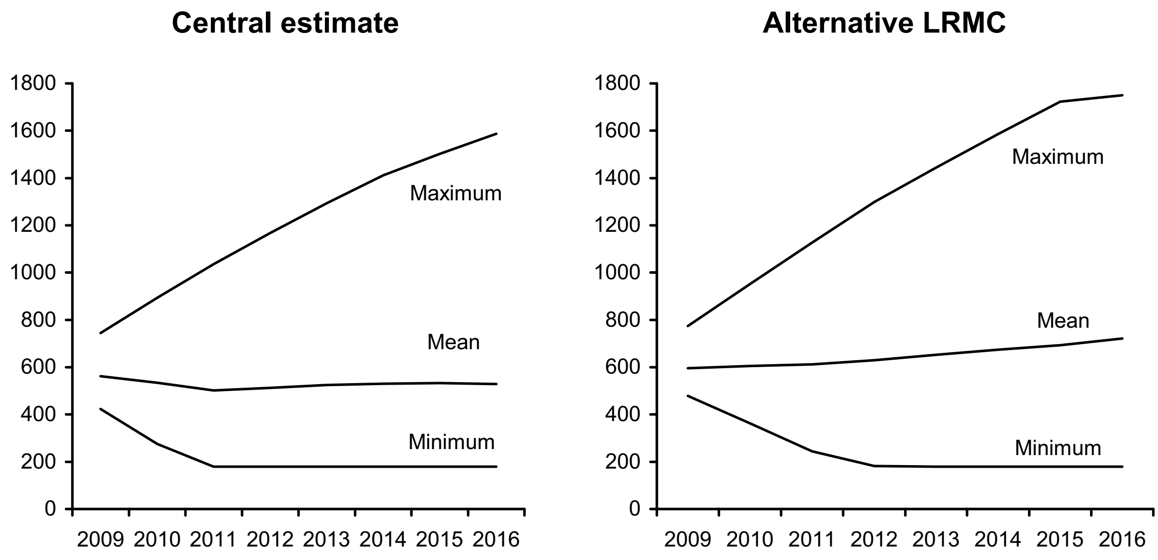
Under scarcity-based pricing ($/kL)



[a] Maximum price. [b] Ninetieth percentile price. [c] Median price. [d] Tenth percentile price. [e] Minimum price.

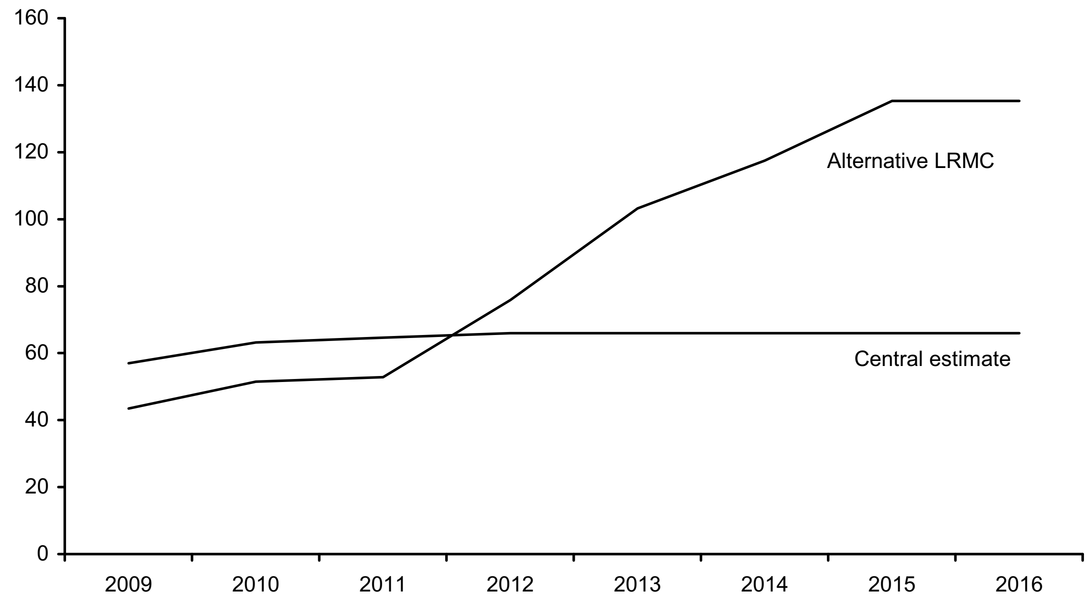
Data source: Modelling results.

Figure D.29 **Water in storage for the alternative LRMC specification**
Under scarcity-based pricing (GL)



Data source: Modelling results.

Figure D.30 **Mean discounted, truncated investment expenditure for the alternative LRMC specification[a]**
Under scarcity-based pricing ($ million)



[a] Includes the total cost of investment in all new supply sources.

Data source: Modelling results.

Table D.13 **Net social welfare costs of policy constraints for the alternative LRMC specification**

Expected, net present value of costs ($ million) relative to scarcity-based pricing, for the next eight years

Policy	*Central estimate*	*Alternative LRMC specification*
Restrictions	522	n.a.
Restrictions and LRMC	658	n.a.
LRMC	94	304
Cost recovery	153	n.a.

Source: Modelling results.

n.a.: not applicable

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